A Joint Factor Model for Bonds, Stocks, and Options *

Turan G. Bali† Heiner Beckmeyer‡ Amit Goyal§

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Abstract

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Keywords: factor model, IPCA, corporate bond, option, stock returns

JEL classification: G10, G11, G12

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†McDonough School of Business, Georgetown University. turan.bali@georgetown.edu.

‡University of Münster. heiner.beckmeyer@wiwi.uni-muenster.de.

§University of Lausanne and Swiss Finance Institute. amit.goyal@unil.ch.

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Abstract

Motivated by structural credit risk models, we propose a parsimonious reduced-form joint factor model for bonds, options, and stocks. By extending instrumented principal component analysis to accommodate heterogeneity in how firm characteristics instrument the sensitivity of bonds, options, and stocks, we find that our model is able to *jointly* explain the risk-return tradeoff for the three asset classes. Just five factors are sufficient to explain 17% of the total variation of bond, option, and stock returns; these five factors leave the returns of only eleven out of 219 characteristic-managed portfolios unexplained. Finally, we investigate the patterns of commonality in return predictability.

1 Introduction

We propose a joint factor model for stocks, bonds, and options, motivated by the theories developed in Du, Elkamhi, and Ericsson (2019), Geske (1979), and Merton (1974). In Merton's (1974) framework, stocks and bonds issued by the same firm represent claims on the same underlying assets of the firm. Specifically, equity (debt) securities can be viewed as a long (short) position in call (put) option on the firm's assets. Du, Elkamhi, and Ericsson (2019) augment the Merton (1974) model with time-varying and priced asset volatility, and show that this can explain both the level and the dynamics of credit spreads and equity volatilities. Geske (1979) shows that an option contract written on corporate securities, such as an equity option, can be viewed as an option on an option, or a compound option. Consequently, as long as the three markets are partially integrated, they share a common factor structure. Our paper is devoted to characterizing this factor structure.

There is an intensive discussion in the literature on the degree of integration between the bond and stock markets (see, for example, Choi and Kim, 2018, Chordia, Goyal, Nozawa, Subrahmanyam, and Tong, 2017, and Sandulescu, 2023). The literature has also documented that option trading activity can influence the prices of individual stocks and bonds issued by the firm (Easley, O'Hara, and Srinivas, 1998). Similarly, informed option trading demand pressure can have an impact on option prices (Gârleanu, Pedersen, and Poteshman, 2008). There is also substantial evidence of information flow between individual equity option and stock markets (An, Ang, Bali, and Cakici, 2014), as well as individual equity option and bond markets (Cao, Goyal, Xiao, and Zhan, 2023).[1] More specifically, variables constructed with option market information predict future returns of individual stocks (Neuhierl, Tang,

[1] An, Ang, Bali, and Cakici (2014), Bali and Hovakimian (2009), Cremers and Weinbaum (2010), and Xing, Zhang, and Zhao (2010) find a significantly positive cross-sectional relation between call-minus-put option implied volatility spreads and future returns of optionable stocks. Johnson and So (2012) find a positive relation between the ratio of trading volume in the stock to option trading volume and future stock returns. The findings of these studies suggest a link between investors' demand for options and future returns of the underlying stock. Cao, Goyal, Xiao, and Zhan (2023) propose a similar argument for the connection between equity options and future returns of corporate bonds.

Varneskov, and Zhou, 2023), and stock characteristics are important determinants of future option returns (Bali, Beckmeyer, Moerke, and Weigert, 2023) and future bond returns (Bali, Goyal, Huang, Jiang, and Wen, 2022).

As a result, we argue that option traders' expectations and their actual trades in the option markets have a significant impact on the future prices of individual stocks and bonds, which eventually influence the firms' asset returns. To be able to capture these complicated dynamics in the bond, option, and stock markets and their impact on future firm values, we extend the instrumented principal component analysis (IPCA) of Kelly, Pruitt, and Su (2019) to allow for heterogeneity in how firm characteristics inform the pricing of bonds, options, and stocks. Hence, our IPCA-based joint factor model produces a more realistic expected return benchmark for the firm.

We note that our objective is not to propose a new structural model. Instead, motivated by Geske (1979) and an extended version of the Merton (1974) model by Du, Elkamhi, and Ericsson (2019), we propose a reduced-form factor model that jointly prices stocks, bonds, and options. The joint factor model can be justified within a compound option pricing model with stochastic asset price and asset volatility dynamics (Doshi, Ericsson, Fournier, and Seo, 2022). Since it is difficult to accurately characterize asset value and asset volatility dynamics for individual firms, we rely on joint IPCA with a large set of bond, option, and stock characteristics to back out a joint risk factor model from the time-series and cross-section of bond, option, and stock returns.

Main Findings: We extend the IPCA framework of Kelly, Pruitt, and Su (2019) by accommodating asset class-level differences in how characteristics instrument the variation in factor sensitivities, while maintaining a joint factor structure for firms' bonds, options, and stocks. In our joint IPCA methodology, individual asset returns for all three asset classes are driven by the same K latent factors through time-varying factor loadings, which we parameterize as a linear function of observable firm characteristics. We allow this linear function to vary for each asset class and use a large number of firm-level characteristics, which incorporate information from the firms' bonds, options, and stocks.

Specifically, we use the stock-level characteristics from Jensen, Kelly, and Pedersen (2023) and the option-level characteristics from Bali, Beckmeyer, Moerke, and Weigert (2023). We use the set of bond-level characteristics provided by Dickerson, Mueller, and Robotti (2023), which we augment by several additional characteristics. In total, we end up with 254 firm characteristics, of which 38 are based on the bond, 63 on the option, and 153 on the stock. From this exhaustive list of characteristics, we use 163 firm characteristics that result in significant Sharpe ratios (SRs) for at least one asset class. We form characteristic managed portfolios (CMPs) from these characteristics and find that 120 generate a significant SR for one asset class, 33 for two, and nine generate a significant SR for all three bond, option, and stock portfolios. This preliminary analysis already shows that asset classes are partially integrated, with the same conditioning information producing valuable trading strategies across the three asset classes.

Our estimated joint IPCA describes returns of bonds, options, and stocks well. A five-factor model explains 17% of the total variation of overall asset returns; 12% for options, 16% for stocks, and 30% for bonds. We use a simple aggregation scheme for the returns of a hypothetical investor looking to hold an equal investment in a firm's bond, option, and stock. Based on this aggregation scheme, we find that adjusting aggregate returns for risk using our joint IPCA model leaves only 9.9% of alphas significant, with an average unconditional R^2 of 33.8%.

Because the joint IPCA estimates a single factor structure for the three asset classes, we can back out the implied mean-variance efficient (MVE) tangency portfolio. Cochrane (2009) shows that shocks to the MVE portfolio are directly proportional to shocks to the stochastic discount factor. Our five-factor joint IPCA model generates an annualized SR of 6.9 in-sample (IS) and 6.4 out-of-sample (OOS). We also show that $t + 1$ returns to the

MVE portfolio are related to the VIX in t (Martin, 2017), and that the joint factor structure allows the model to benefit from substantial diversification benefits, by investing in multiple assets of a firm. The MVE's subportfolios of bonds, options, and stocks are uncorrelated or even negatively correlated. Finally, we show that the net-of-fees SR remains large and above 2.2 for realistic-to-high levels of relative transaction costs (Frazzini, Israel, and Moskowitz, 2018; Choi, Huh, and Shin, 2021).

The common factor structure of joint IPCA is essential for explaining returns across bonds, options, and stocks. We assess how well the model captures unconditional alphas for the 163×3 CMPs of bonds, options, and stocks. Unconditionally, we find that 219 CMPs generate full-sample average returns that are statistically significant at the 5% level. After adjusting for risk using our five-factor joint IPCA model, only 11 alphas remain significant; 1 for bonds, 7 for options, and 3 for stocks.

Next, we investigate the commonality in return predictability in three different ways. We first analyze whether there is commonality in explanatory power for bonds, options, and stocks. We sort bonds, options, or stocks into decile portfolios by the model's respective R^2. When sorting by one asset class, we also document the resulting R^2 for each decile for the remaining two asset classes. If the markets are partially integrated, we expect that, for stocks that are well priced by the joint IPCA, bonds of the same firms and options on the same stock are well-priced too. Empirically, we find that in deciles sorted by bond's R^2, the R^2 spread of sorting options and stocks is 16% and 7%, respectively. For a sort on option R^2s, the bond and stock spread is 18% and 17%, respectively; and for a sort on stock R^2, the decile R^2 spread for bonds and options is 11% and 20%, respectively.

Second, we study which factors are most important for explaining returns of each asset class. In particular, are all five factors required for each asset class? To answer this question, we iteratively "turn off" the influence of each of the five factors and document the resulting drop in R^2 across asset classes. We find that two factors are responsible for roughly 86%

of the model's ability to explain bond returns, three factors for roughly 79% of the model's ability to explain option returns, and three factors for 87% of the model's explanatory power for stock returns. The five common factors improve the explanatory power for each asset class. The first factor captures the aggregate market effects and is equally important for explaining bond, option, and stock returns. The second factor captures a large degree of bond and stock return variation, while the third and fourth factors capture primarily option and stock return variation. The fifth factor captures bond and option variation. We also show that the most important characteristics to instrument variation in sensitivities to the joint latent factors are shared across asset classes. At the same time, this exercise highlights the importance of incorporating a vast range of characteristics of bonds, stocks, and options.

A third way of understanding the usefulness of the joint IPCA is to compare its performance with IPCA models estimated separately for bonds, options, and stocks. We have mentioned earlier that our joint IPCA leaves only 11 statistically significant alphas from a total of 219 CMPs. We find that each of the three single IPCA models, estimated separately for one asset class, perform worse. The five-factor bond-based IPCA fails to explain 193 alphas, the option-based IPCA 34, and the stock-based IPCA 173. We also consider a combined factor model, which uses two factors estimated separately for each asset class. This combined model fails to explain 31 alphas. Furthermore, we find that the joint IPCA's tangency portfolio outperforms the tangency portfolio implied by each of the single asset class IPCA models.

Finally, we also compare our joint IPCA with existing benchmark bond-, option-, and stock-level factor models put forth by the literature to explain returns within one of the three asset classes. We find that the bond market CAPM fails to explain the returns of 206 CMPs, the two-factor straddle model of Coval and Shumway (2001) a total of 171, and the five-factor model of Fama and French (2015) augmented with momentum leaves 210 CMP returns unexplained. Even a combination of the 9 factors (1 bond, 2 option, and 6 stock factors) fails to explain 182 alphas.

Since IPCA factors are latent in nature, it is useful to investigate the dynamics governing the joint five-factor structure. We first show that the five factors capture significant variation in macroeconomic fundamentals. As an example, we show that the second latent factor hedges macroeconomic uncertainty and intermediary risks, with a positive relation to the macroeconomic uncertainty measure of Jurado, Ludvigson, and Ng (2015), and a negative relation to the intermediary capital ratio of He, Kelly, and Manela (2017). The first IPCA factor is exposed to intermediary risks, and the fourth factor captures the spread between overall macroeconomic risks and the risks of the intermediary sector. We furthermore show that the five latent factors capture important information from the three asset classes that cannot be replicated by the three macroeconomic indicators or the nine benchmark factor models. For this, we propose a novel method to interpret latent factors, which replaces each factor's realizations by the fitted values from regressing it on a set of macroeconomic indicators and benchmark factors.

Related Literature: Our paper contributes to the literature investigating the integration of the bond and stock markets. If the two markets are (partially) integrated, risk premia should show up in both markets. Koijen, Lustig, and Van Nieuwerburgh (2017) show that some bond factors are priced in the cross-section of stock returns, whereas Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2017) argue that equity and corporate bond returns require a different set of risk factors, and Choi and Kim (2018) find that the risk premia of stock factors differ when using bond returns, accounting for the implied hedge ratio. On the modeling front, Du, Elkamhi, and Ericsson (2019) extend the Merton (1974) structural credit risk model with priced asset variance risk and show that this resolves the credit risk puzzle.

Doshi, Ericsson, Fournier, and Seo (2022) extend the compound option pricing model of Geske (1979) by allowing for idiosyncratic volatility and asset value jumps in the stochastic volatility model of Du, Elkamhi, and Ericsson (2019). While Collin-Dufresne, Junge, and

Trolle (2022) argue that CDX and SPX options are disintegrated, the model setup of Doshi, Ericsson, Fournier, and Seo (2022) allows the authors to jointly explain the level and time variation of both SPX and CDX options. Culp, Nozawa, and Veronesi (2018) derive the notion of "pseudo firms" from the insights of the Merton (1974) model using traded SPX option prices and show that the credit risk puzzle can be explained by a risk premium for tail and idiosyncratic asset risks. There is, however, little research on the joint integration of bond, option, and stock markets.[2] Our paper adds additional insights into the risk-return trade-off of bonds, options, and stocks from a reduced form factor model with a shared factor structure.

There is a vast literature on factor models for equity returns (see, for example Fama and French, 2015 and Hou, Xue, and Zhang, 2015), corporate bonds (Kelly, Palhares, and Pruitt, 2023), currency (Lustig, Roussanov, and Verdelhan, 2011), commodity futures (Szymanowska, de Roon, Nijman, and van den Goorbergh, 2014), and cryptocurrencies (Liu, Tsyvinski, and Wu, 2022). Despite the proliferation of factor models for stocks, bonds, and other asset classes, the literature on factor models for option returns is relatively sparse, with a few recent advances in Goyal and Saretto (2022) and Horenstein, Vasquez, and Xiao (2022). Kozak, Nagel, and Santosh (2020) advocate for a low-dimensional factor structure. We achieve parsimony by estimating a model of common latent risk factors across asset classes, which exploits the markets' partial integration. Just five factors are sufficient to accurately and jointly express the risk-return trade-off of bonds, options, and stocks.

[2] In a contemporaneous working paper to ours, Chen, Roussanov, Wang, and Zou (2024) estimate a conditional latent factor model to identify common risk factors for bonds, options, and stocks and find that the return predictors with zero loadings on the latent systematic risk factors receive a larger premium than the compensation for risk.

2 Theoretical Motivation

Following Bates (1996), Du, Elkamhi, and Ericsson (2019), Heston (1993), and Leland (1994), one can assume that the dynamics of a firm's asset value and asset variance are governed by a stochastic volatility-jump diffusion model. Although the market prices of diffusive and volatility risks are assumed to be constant in the aforementioned theoretical models, in practice they are known to exhibit significant time-series variation and nonlinearity. Thus, it is very challenging to provide an accurate characterization of firm value dynamics in a theoretical setting. As discussed by Du, Elkamhi, and Ericsson (2019) and Doshi, Ericsson, Fournier, and Seo (2022), since there is no closed form solution for the firm, equity, or debt value, earlier studies either calibrate or simulate the stochastic volatility models to obtain the empirical counterparts of asset price and asset volatility dynamics.

In a model where corporate securities are options on a firm's assets, option contracts on these can be viewed as options on options, or compound options (Geske, 1979). Doshi, Ericsson, Fournier, and Seo (2022) extend the compound option pricing model of Geske (1979) by allowing for idiosyncratic volatility and asset value jumps in the stochastic volatility model of Du, Elkamhi, and Ericsson (2019). Doshi, Ericsson, Fournier, and Seo (2022) also adopt the simulation approach in Du, Elkamhi, and Ericsson (2019) and show that their model jointly explains the level and time variation of both equity index (SPX) and credit index (CDX) option prices well OOS. In the model of Doshi, Ericsson, Fournier, and Seo (2022), aggregate unlevered asset return and variance shocks are the only two sources of priced risk. Thus, financial instruments such as the equity index, credit protection index, and equity/credit index options derive their risk premia from these two sources. However, each instrument has exposure to its own specific states of the world, and hence differs in its loading on the common sources of risk. As a result, while sources of risk are shared across markets, each instrument is priced quite differently.

Based on the above theoretical models, we propose a joint factor model for bonds, stocks, and options that can be justified within a compound option pricing model with stochastic asset price and asset volatility dynamics. As shown by Du, Elkamhi, and Ericsson (2019) and Doshi, Ericsson, Fournier, and Seo (2022), it is very difficult to estimate directly the asset value and asset volatility dynamics proposed in their model. Hence, both studies follow a calibration or simulation approach. In addition to the complications about the estimation of stochastic-volatility/jump type models for stock and bond returns, we argue for the presence of sizeable information flow between equity option and underlying stock markets. Thus, changes in equity option prices may have a significant impact on the future prices of individual stocks and bonds, which eventually influence the firm's asset returns. To incorporate these complex dynamics in the option, stock, and bond markets and their impact on future firm values, we rely on joint IPCA with a large set of stock, bond, and option characteristics to back out a joint risk factor model for the firm's asset returns. As in Doshi, Ericsson, Fournier, and Seo (2022), we explicitly allow for heterogeneity in the sensitivity of bonds, options, and stocks to the common set of risk factors.

The benefits of a joint factor model are manifold: first and foremost, we retain a parsimonious factor structure across many asset classes, yielding a lower number of common factors, as soon as the included asset classes are (partially) integrated. Kozak, Nagel, and Santosh (2020) advocate for focusing on a small number of factors. Estimating the *common* factor structure for many assets serves the purpose so that the resulting factor structure is valid for pricing all these asset classes. Second, a joint factor model allows us to estimate a tangency portfolio, which incorporates the covariance structure between asset classes. This in turn informs us about the dynamics of the stochastic discount factor, which spans the joint pricing of multiple asset classes. From a trading perspective, the tangency portfolio across asset classes informs researchers and practitioners alike about relative investment opportunities in the three markets. Third, we learn about commonalities and differences in the risk-return tradeoff of different asset classes in a unified model. By investigating the importance of the latent factors and observable characteristics in instrumenting betas at the asset class level, we can assess the relative degree of integration of bond, option, and stock returns of the same firm. Our proposed extension of IPCA (Joint IPCA) combines these benefits in a simple and intuitive model setup.

3 Econometric Methodology

One of our contributions is to extend the well-established IPCA by Kelly, Pruitt, and Su (2019). We accommodate asset class-level differences in how characteristics instrument the variation in factor sensitivities, while maintaining a joint factor structure across bonds, options, and stocks. Consider an asset i, which is part of one of three asset classes, $AC \in$ [Bonds, Options, Stocks]. We can express asset i's excess return R^{AC}_{it+1} as:

$$R^{AC}_{it+1} = \beta^{AC\,\prime}_{it} F_{t+1} + \varepsilon_{it+1}, \qquad \beta^{AC}_{it} = z'_{it}\Gamma^{AC}_{\beta}. \tag{1}$$

Individual returns are driven by K latent factors, F_{t+1}, through factor loadings β^{AC}_{it}, which we parameterize as a linear function of L observable characteristics z_{it}. The mapping function from characteristics to betas is given by an $L \times K$ matrix Γ^{AC}_{β} that is specific to asset class AC. If there are N_{t+1} assets with available data, then we can express the asset pricing equation as:

$$R^{AC}_{t+1} = \beta^{AC}_{t} F_{t+1} + \varepsilon^{AC}_{t+1}, \tag{2}$$

where $\beta^{AC}_{t} = Z_t\Gamma^{AC}_{\beta}$ is a $N_{t+1} \times K$ matrix of betas using the $N_{t+1} \times L$ matrix Z of characteristics.

It is easy to see that the factor sensitivity of the stock and bonds of the same firm need not be the same. In the classic Merton-type firm model (Merton, 1974), a firm's stock can be modeled as a call option on the firm's assets, while its debt is the combination of a risk-less bond and a written put. The sensitivities of these two option portfolios to the same set of factors will naturally differ. Likewise, delta-hedged equity option returns capture differences in the expectation and realization of variance and jump risks, among others. Their factor sensitivity is therefore also going to differ from the sensitivity of the underlying stock, which itself is not directly exposed to these higher-order terms.

The innovation in our paper, therefore, is to allow for differences in the mapping function Γ_β^{AC} for each asset class. Instead of forcing Γ_β to be the same for bonds, options and stocks, we allow for class-level variation in how asset characteristics inform us about the risk-return tradeoff.

Consider three $N_{t+1} \times 1$ vector of returns, R^B_{t+1}, R^O_{t+1}, and R^S_{t+1}, representing returns for bonds, options, and stocks, respectively. The factor sensitivities are given by $\beta_t^B = Z_t \Gamma_\beta^B$, $\beta_t^O = Z_t \Gamma_\beta^O$, and $\beta_t^S = Z_t \Gamma_\beta^S$ for the three asset classes. Note that since the set of L characteristics is the same for each asset class, we allow for stock/option characteristics to influence bond returns in addition to bond characteristics, and so on.

It will be convenient to stack the three return vectors together into one $3N_{t+1} \times 1$ vector R_{t+1} as:

$$R_{t+1} \equiv \begin{bmatrix} R^B_{t+1} \\ R^O_{t+1} \\ R^S_{t+1} \end{bmatrix} = \begin{bmatrix} \beta_t^B \\ \beta_t^O \\ \beta_t^S \end{bmatrix} F_{t+1} + \begin{bmatrix} \varepsilon^B_{t+1} \\ \varepsilon^O_{t+1} \\ \varepsilon^S_{t+1} \end{bmatrix} = \beta_t F_{t+1} + \varepsilon_{t+1},$$

with

$$\beta_t \equiv \begin{bmatrix} \beta_t^B \\ \beta_t^O \\ \beta_t^S \end{bmatrix} = \begin{bmatrix} Z_t\Gamma_\beta^B \\ Z_t\Gamma_\beta^O \\ Z_t\Gamma_\beta^S \end{bmatrix} = \mathcal{Z}_t \Gamma_\beta, \qquad \mathcal{Z}_t \equiv \begin{bmatrix} Z_t & 0 & 0 \\ 0 & Z_t & 0 \\ 0 & 0 & Z_t \end{bmatrix}, \; \Gamma_\beta \equiv \begin{bmatrix} \Gamma_\beta^B \\ \Gamma_\beta^O \\ \Gamma_\beta^S \end{bmatrix}, \tag{3}$$

where β_t is a $3N_{t+1} \times K$ matrix of loadings, $\mathcal{Z}_t$ is a $3N_{t+1} \times 3L$ matrix of stacked characteristics, and Γ_β is the $3L \times K$ mapping matrix from characteristics to loadings (0 is a $N_{t+1} \times L$ matrix of zeros). Eq. (3) is our central Joint IPCA asset pricing equation.

It is useful to define a $3L \times 3L$ matrix W_t as (0 below is a $L \times L$ matrix of zeros):

$$W_t = \mathcal{Z}_t'\mathcal{Z}_t/N_{t+1} = \begin{bmatrix} Z_t'Z_t & 0 & 0 \\ 0 & Z_t'Z_t & 0 \\ 0 & 0 & Z_t'Z_t \end{bmatrix} /N_{t+1}, \tag{4}$$

and a $3L \times 1$ matrix X_{t+1} as:

$$X_{t+1} \equiv \begin{bmatrix} X_{t+1}^B \\ X_{t+1}^O \\ X_{t+1}^S \end{bmatrix} = \mathcal{Z}_t' R_{t+1}/N_{t+1} = \begin{bmatrix} Z_t'R_{t+1}^B \\ Z_t'R_{t+1}^O \\ Z_t'R_{t+1}^S \end{bmatrix} /N_{t+1}. \tag{5}$$

It is readily seen that X_{t+1} are the returns of CMPs. Since we have L characteristics and three asset classes, we have $3L$ such portfolios.

With X_{t+1} and W_t at hand, the first order conditions for Eq. (3) are:

$$\widehat{F}_{t+1} = \left(\widehat{\Gamma}_\beta' W_t \widehat{\Gamma}_\beta\right)^{-1} \widehat{\Gamma}_\beta' X_{t+1} \tag{6}$$
$$\text{vec}\left(\widehat{\Gamma}_\beta\right) = \left(\sum_{t=1}^{T-1} W_t \otimes \widehat{F}_{t+1}\widehat{F}_{t+1}'\right)^{-1} \left(\sum_{t=1}^{T-1} X_{t+1} \otimes \widehat{F}_{t+1}\right).$$

While this system of first-order conditions still does not admit a closed-form solution, it is quickly solvable using an alternating least-squares procedure. Latent factor realizations are obtained from month-by-month cross-sectional regressions of the stacked vector of the excess returns of all assets R_{t+1} on β_t (Fama and MacBeth, 1973). Γ_β are the coefficients of regressing CMP returns on factors F_{t+1} interacted with asset characteristics $\mathcal{Z}_t$. Given the structure of the system, the estimation of Γ_βs is essentially three separate regressions, one for each asset class.

Identifying a unique set of parameters is important in latent factor models, as they are identified only up to a rotation. Models $\Gamma_\beta F_{t+1}$ and $\Gamma_\beta R R^{-1} F_{t+1}$ are identical for any rotation matrix R. Following Kelly, Pruitt, and Su (2019), we impose the normalization that $\Gamma_\beta'\Gamma_\beta$ is the identity matrix, that the unconditional second moment matrix of F_{t+1} is diagonal with descending diagonal entries, and that the time-series average of F_{t+1} is positive. The identification assumptions do not restrict the model's ability to explain returns of bonds, options, and stocks, but merely serve as a way to pin down unique parameters of the model.

4 Data

4.1 Returns & Characteristics

Returns. Our analyses use returns of three different asset classes. Excess stock returns (corrected for delisting) are from CRSP.

We obtain corporate bond data for the sample period from August 2002 to August 2022 using the dataset provided by Dickerson, Mueller, and Robotti (2023), which extends the WRDS Corporate Bond Database.[3] The corporate bond return in month t is defined as

$$R_t^B = \frac{P_t + A_t + C_t}{P_{t-1} + A_{t-1}} - 1, \tag{7}$$

where P_t is the last price at which bond was traded in month t, A_t is accrued interest on the same day of bond prices, and C_t the coupon payment in month t, if any. This dataset also alleviates the issue of faulty outliers in TRACE, which Dick-Nielsen, Feldhütter, Pedersen, and Stolborg (2023) document, by imputing the affected returns from TRACE with quote-implied returns.

Finally, we consider daily delta-hedged option returns following Bali, Beckmeyer, Moerke, and Weigert (2023). Let the option contract's value be denoted by O and the value of the underlying stock as S. Then, the delta-hedged dollar gain over the period $(t, t+1)$ is given

[3] The dataset can be downloaded at https://openbondassetpricing.com/.

by:

$$\Pi_{t+1} = O_{t+1} - O_t - \sum_{n=0}^{N-1} \Delta_{t_n} \left(S_{t_{n+1}} - S_{t_n}\right) - \sum_{n=0}^{N-1} \frac{R_{ft_n}}{365} \left(O_{t_n} - \Delta_{t_n} S_{t_n}\right). \tag{8}$$

We scale the dollar gain by the initial value of the investment portfolio (Cao and Han, 2013):

$$R^O_{t+1} = \frac{\Pi_{t+1}}{|\Delta_t S_t - O_t|}. \tag{9}$$

We retain call contracts that expire on the third Friday of the month after the next, or in roughly 50 days. To assure the validity of the option prices when entering the position, we select contracts that have a standard contract multiplier of 100, an offer greater than the bid, where the bid exceeds \$0.125, the bid-ask spread is less than 50% of the mid quote, the contract's implied volatility is available, and the quotes adhere to American option bounds. We also require that the contract has positive open interest or volume today, and that it was traded at least once in the seven days leading up to the trade initiation on the last trading day of month t. We carefully assure that filters are only applied at the time of trade initiation to avoid forward-looking information to affect our return estimates (Duarte, Jones, Mo, and Khorram, 2023).

For the *estimation* of the joint IPCA models, returns are winsorized at the 1% level per asset class. For *inference*, we use non-winsorized returns throughout this paper. We also scale returns of each asset class by their full-sample standard deviation, to increase their comparability and assure that they roughly contribute equally to the estimation of IPCA. Limited by the availability of bond return data through TRACE, we start our sample in August of 2002. Our sample period includes the great recession in 2008-2009, as well as the Covid-selloff at the beginning of 2020.

Contract Selection. For each firm×month observation, we select a representative bond and option. Without this step, given that many firms have multiple bonds and options, information about options and bonds would invariably dominate that of stocks in the joint model estimation. Choosing one bond and one option also facilitates the comparison of the model's ability to price assets of different classes. For the bond market, we follow Dick-Nielsen, Feldhütter, Pedersen, and Stolborg (2023) and calculate the aggregate corporate bond return for firm i in month $t+1$ as

$$R^B_{it+1} = \sum_{k \in \mathcal{R}} w^k_{it} R^k_{it+1}, \tag{10}$$

where $\mathcal{R}$ is the set of bonds with available returns in month $t+1$. We weight each bond by its market value at the end of month t. Whenever a firm has bonds with missing returns, Dick-Nielsen, Feldhütter, Pedersen, and Stolborg (2023) advocate for imputing its return from the observed average bond return, R^B_{it+1}, using a duration adjustment:

$$\widetilde{R}^B_{it+1} = R^B_{it+1} \frac{D^{\mathcal{A}}_t}{D^{\mathcal{R}}_t}, \tag{11}$$

where $D^{\mathcal{A}}_t$ is the weighted-average duration of all bonds of firm i, and $D^{\mathcal{R}}_t$ of bonds with valid returns in $t+1$. Bond characteristics are also aggregated by weighting with each bond's market value at the end of month t.

For options, we select all at-the-money calls, defined as $\left|\frac{\ln K/S}{iv \times \sqrt{ttm}}\right| \leq 1$,[4] that expire on the third Friday of month $t+2$. Among these calls, we use the contract with the strike price closest to the current value of the underlying as the representative contract for each firm.

Given that our objective is to study the joint factor structure across bonds, stocks, and options, we require that each included firm is either optionable *or* has a traded bond with valid transaction data in TRACE, during our sample period from August 2002 through August 2022. This restricts our sample to the largest and most liquid stocks. In total, we have valid bond returns for 1,565 unique firms, valid option returns for 5,755 unique firms and valid stock returns for 5,958 unique firms, yielding a total of 1.2 million asset×month

[4] Where iv is the contract's implied volatility, ttm its time-to-maturity, K its strike price, and S the price of its underlying stock.

observations.

Characteristics. Option-level characteristics are taken from Bali, Beckmeyer, Moerke, and Weigert (2023) and stock-level characteristics from Jensen, Kelly, and Pedersen (2023). We use the set of bond-level characteristics provided by Dickerson, Mueller, and Robotti (2023), which we augment by several additional characteristics. In total, we end up with 254 firm characteristics, of which 38 are based on the bond, 63 on the option, and 153 on the stock. For each characteristic, we impose a limit on how often it can be missing.[5] Specifically, we require that each characteristic is available for at least half of all observations. This filter drops seven characteristics. Following standard practice in the literature, we rank each characteristic cross-sectionally and standardize its values to lie between -0.5 and 0.5 for each month.

To weed out characteristics that essentially convey the same information, we identify those pairs that have correlation, $|\rho| \geq 95\%$. From each identified pair, we retain that characteristic which is available for more asset$\times$month observations. This drops 23 characteristics from our dataset, for a total of 224 firm characteristics (254 in total, 7 dropped for availability, 23 for correlation). A complete list of the 254 characteristics is provided in Appendix A, including their academic source, and an indicator whether it is included in the final dataset, and if not, for which reason.

4.2 Characteristic-Managed Portfolios Across Asset Classes

Our joint dataset covers a large number of bond, option, and stock characteristics. We now provide first evidence of the benefits associated with a joint consideration of this information set when making investment decisions in either of the three asset classes. For each of the $l = 1, \ldots, L$ characteristics, we compute the investment performance of the associated CMPs,

[5] See Beckmeyer and Wiedemann (2023), Bryzgalova, Lerner, Lettau, and Pelger (2022), and Freyberger, Höppner, Neuhierl, and Weber (2022) for a discussion of missing data in cross-sectional asset pricing.

calculated as in Eq. (5) using bonds, options, or stocks as the investable assets. Panel A of Table 1 reports the twelve CMPs of bonds with the largest absolute SRs. We also provide the resulting SRs for the CMPs of options and stocks using the same 12 characteristics. The remaining panels replicate this exercise for the top-12 CMPs of options and stocks.

Panel A of Table 1 shows that the SRs of CMPs of bonds are large. For example, a long-short bond portfolio sorted on the stock's short-term reversal (S_ret_1_0) achieves an IS SR of 1.55. In contrast, using the same information in the options or stock market fails to generate a significant SR.[6] The average absolute SR of the top-12 CMPs of bonds is 0.94. In comparison, the value-weighted bond market portfolio constructed from our sample has a SR of just 0.41 in the same sample period. Sorting bonds on the idiosyncratic volatility of the stock (S_iskew) generates a significant SR of between 0.80 and 1.11, depending on the factor model used. Sorting by the spread between the option's implied and the underlying's past realized volatility (O_S_ivrv_roll252D) generates a large and negative SR of −1.20. Interestingly, the bond's short-term reversal is the only bond-level characteristic to enter the top-12 CMPs of bonds with a Sharpe of 0.78.[7] The second most profitable bond characteristic for CMPs of bonds is bond duration at rank 28 with a SR of 0.61. This is already first indicative evidence of the importance of considering characteristics *of the firm*, not only of the bond, when deciding to invest in corporate bonds. We also find that the characteristics of the top-12 CMPs of bonds are also able to generate meaningful investment performance for CMPs of options and to a lesser degree for CMPs of stocks. For example, idiosyncratic skewness (calculated using Hou, Xue, and Zhang (2015) four-factor model) generates a significant SR for bonds, options, and Sharpe ratio for stocks significant at the 10% level. Among the twelve characteristics, the average absolute SR of CMPs of options is 1.18 and that of CMPs of stocks is 0.40. Nine of the top-12 bond characteristics generate a significant Sharpe ratio among options, and one (four) a significant Sharpe ratio among stocks at the 1% (10%)

[6] We test for statistical significance of the SR following Lo (2002).

[7] We use the short-term reversal adjusted for bond microstructure noise, as advocated by Dickerson, Mueller, and Robotti (2023).

significance level.

The SRs of CMPs of options (Panel B of Table 1) are significantly higher than those of the CMPs of bonds, with an average absolute SR of 3.25. Sorting on the difference between at-the-money call and put implied volatilities (O_S_civpiv, see Bali and Hovakimian (2009)) generates the largest absolute SR of 3.65, followed by risk-neutral kurtosis (O_S_rnk_30). Similarly, information about the option's embedded leverage and past return momentum (O_B_mom_roll252D, see Heston, Jones, Khorram, Li, and Mo (2023) and Beckmeyer, Filippou, and Zhou (2023)) also generate highly significant SR. None (two) out of the 12 most important characteristics for options also generate significant SRs for CMPs of bonds (stocks). Nine out of the top-12 characteristics for CMPs of options use option information in their construction.

The top-12 CMPs of stocks (Panel C of Table 1) have an average absolute SR of 1.16. In general, option-based information is most valuable for stock-based investments, with the implied short-selling fee (O_shrtfee, see Muravyev, Pearson, and Pollet, 2022) and option frictions (O_fric, see Hiraki and Skiadopoulos, 2020) both generating an absolute SR of 1.5. Muravyev, Pearson, and Pollet (2022) show that the implied shorting fee explains much of the outperformance of common stock-market anomalies. Sorting stocks on this characteristic alone is already a profitable investment strategy in the absence of fees and implementation costs. The same characteristic also generates a significant SR for CMP of options. Other informative characteristic's are the stock's mispricing (S_mispricing_mgmt Stambaugh and Yuan, 2017), the ratio of sales to book equity (S_sale_bev), and investment growth (S_inv_gr1).

Overall, out of the 224 characteristics, 62 generate an insignificant SR at the 10% level for all of the three asset classes, 120 have a significant SR for one asset class, 33 for two, and nine for all three (bonds, options, and stocks). We use these 163 (120 + 33 + 9 + a constant) characteristics as inputs in modeling the risk-return tradeoff for each firm. Of

Table 1: Characteristic-managed Portfolios

The table shows annualized SRs of the 12 characteristic-managed portfolios that generate the largest absolute SRs for bonds in Panel A, options in Panel B, and stocks in Panel C. ***, **, * denote significance at the 1%, 5%, 10% level using the significance test for SRs of Lo (2002).

	Bonds	Options	Stocks
Panel A: Top 12 Bond CMPs			
S_ret_1_0	1.55***	−0.09	−0.15
O_S_ivrv_roll252D	−1.20***	−2.38***	−0.21
S_iskew_hxz4_21d	1.11***	−0.98***	0.56*
O_S_modos_roll252D	−0.94***	−0.99***	−0.36
S_iskew_capm_21d	0.89***	−1.01***	0.40
S_rmax5_rvol_21d	0.87***	0.15	−0.50*
S_eq_dur	0.85***	−2.43***	0.20
S_ni_me	−0.80**	2.76***	−0.24
S_iskew_ff3_21d	0.80**	−0.98***	0.51
O_S_nopt	−0.78***	1.17***	0.16
B_strev	0.78***	−0.81***	0.53*
S_inv_gr1	−0.74**	0.47	−0.98***
Abs. Mean	0.94	1.18	0.40
Panel B: Top 12 Option CMPs			
O_S_civpiv	−0.02	−3.65***	1.30***
O_S_rnk_30	−0.29	3.47***	0.00
O_S_ivud_30	−0.24	3.35***	−0.11
O_S_ivarud_30	−0.37	3.35***	−0.10
O_S_atm_civpiv	−0.03	−3.29***	1.45***
O_C_embedlev	−0.44	3.29***	0.16
O_B_mom_roll252D	−0.18	3.23***	0.10
O_S_rnk_182	−0.37	3.21***	−0.14
S_eqnpo_me	−0.60	3.13***	0.08
S_cash_at	0.30	−3.05***	0.13
S_ni_ivol	0.40	−3.05***	0.29
O_C_theta	−0.42	2.93***	0.15
Abs. Mean	0.30	3.25	0.33
Panel C: Top 12 Stock CMPs			
O_S_shrtfee	0.21	2.64***	−1.51***
O_S_fric	−0.17	−2.53***	1.51***
O_S_atm_civpiv	−0.03	−3.29***	1.45***
O_S_vs_level	−0.29	−2.70***	1.44***
O_S_civpiv	−0.02	−3.65***	1.30***
S_mispricing_mgmt	0.23	0.77*	1.08***
S_sale_bev	−0.05	0.69	1.05***
S_inv_gr1	−0.74**	0.47	−0.98***
S_nncoa_gr1a	−0.67*	1.36***	−0.92***
S_noa_gr1a	−0.71*	1.52***	−0.91***
S_dolvol_126d	−0.51	1.60***	−0.90**
S_inv_gr1a	−0.66**	1.47***	−0.88***
Abs. Mean	0.36	1.89	1.16

these 163 characteristics, 4 (44, 114) are derived from the firm's bond (option, stock) plus the constant. We use the restricted set of characteristics to avoid overfitting on in-sample information. Our procedure represents an ex-ante feature selection step common in the field of machine learning. The estimation of joint IPCA outlined in Section 3 consequently seeks to explain the returns of the 163×3 CMPs that on their own offer the most valuable investment performance. We note that we have fitted models on the entire set of 224 characteristics with very similar results.

5 A Joint Factor Model

5.1 Performance of Joint IPCA

Performance Metrics. We evaluate the model's IS and OOS performance using the metrics proposed by Kelly, Palhares, and Pruitt (2023):

$$\begin{aligned}
\text{Total } R^2 &= 1 - \frac{\sum_{i,t}\left(R_{it+1} - \widehat{\beta}_{it}\widehat{F}_{t+1}\right)^2}{\sum_{i,t} R_{it+1}^2} \\
\text{XS } R^2 &= \frac{1}{T}\sum\nolimits_t R_t^2, \text{ where } R_t^2 = 1 - \frac{\sum_i \left(R_{it+1} - \widehat{\beta}_{it}\widehat{F}_{t+1}\right)^2}{\sum_i R_{it+1}^2} \\
\text{Relative Pricing Error} &= \frac{\sum_i \alpha_i^2}{\sum_i \overline{R}_i^2}, \text{ where } \alpha_i = \frac{1}{T}\sum\nolimits_t \left(R_{it+1} - \widehat{\beta}_{it}\widehat{F}_{t+1}\right).
\end{aligned} \tag{12}$$

Total R^2 quantifies the model's success in explaining average returns for the three asset classes. It aggregates information both over months t and across assets i and compares the amount of variation in asset returns explained by joint IPCA's that is not already explained by a simple benchmark of predicting a zero excess return. Gu, Kelly, and Xiu (2020) argue that a historical mean tends to underperform a zero-forecast for single stocks OOS, which inflates a competing model's Total R^2. Next, we quantify how well a model explains cross-sectional returns. XS R^2 is similar to the average R^2 of Fama and MacBeth (1973) cross-sectional regressions performed each month. Finally, we record the average relative pricing error, which denotes how well a candidate model explains differences in average returns across assets. We prefer models that generate large R^2s and small relative pricing errors.

To assess the model's ability to explain average returns OOS, we estimate Joint IPCA using information until month t, which gives us the time t estimate of $\widehat{\Gamma}_{\beta,t}^{AC}\ \forall\, AC$ and therefore each asset's $\widehat{\beta}_{it}$. Then, we calculate the OOS factor realizations $\widehat{F}_{t+1}$ with a cross-sectional regression, as described in Eq. (6), using betas estimated through time t and asset return information realized in $t+1$. Consequently, the factors are obtained using asset weights which are known already in month t. We require at least 90 months of historical data to estimate the model. Therefore, our OOS test begins in February 2010.

In- and Out-of-Sample Performance. We vary the number of latent factors K of the joint ICPA model. We consider $K \in [1, ..., 5]$. The general consensus in the literature is to focus on parsimonious models with a low number of factors (Kozak, Nagel, and Santosh, 2020). Fama and French (2018) and Kelly, Palhares, and Pruitt (2023) advocate for at most six and five factors for the U.S. equity and corporate bond markets, respectively. It is important to note that joint IPCA has the additional benefit of estimating factors designed to explain returns of all three asset classes simultaneously. If bond, option, and stock markets are (partially) integrated, this will require a lower total number of factors to explain average returns for all three asset classes, yielding a parsimonious factor model applicable to each of the asset classes.

We show in Table 2 the IS and OOS performance metrics described in Eq. (12). We calculate these metrics for all three asset classes combined as well as separately for bonds, options, and stocks. We find that a single factor explains 12% of the total return variation, which varies between 4% for options and 24% for bonds. The Total R^2 increases significantly as we increase the number of factors K. For $K = 5$, joint IPCA explains 17% of the total return variation across bonds, stocks, and options; 12% for options, 16% for stocks, and 30%

Table 2: In- and Out-Of-Sample Metrics

The table shows IS and OOS performance metrics of joint IPCA models with K factors. The definition of each performance metric is given in Eq. (12). We consider $K \in [1, ..., 5]$ factors. The IS period runs from August 2002 through August 2022, the OOS period starts in February 2010, as we require at least 90 months of training data.

$K \rightarrow$	1		2		3		4		5	
	IS	OOS	IS	OOS	IS	OOS	IS	OOS	IS	OOS
Panel A: Total R^2										
Total	0.12	0.11	0.14	0.12	0.15	0.13	0.16	0.14	0.17	0.15
Bonds	0.25	0.25	0.26	0.27	0.27	0.28	0.29	0.32	0.30	0.34
Options	0.04	0.03	0.08	0.06	0.10	0.08	0.12	0.10	0.12	0.10
Stocks	0.12	0.12	0.15	0.14	0.15	0.14	0.16	0.14	0.16	0.14
Panel B: XS R^2										
Total	0.08	0.08	0.11	0.10	0.12	0.11	0.13	0.12	0.13	0.12
Bonds	0.17	0.11	0.17	0.13	0.18	0.09	0.21	0.20	0.24	0.21
Options	0.03	0.01	0.06	0.04	0.08	0.06	0.10	0.08	0.10	0.08
Stocks	0.09	0.10	0.11	0.11	0.12	0.12	0.13	0.12	0.13	0.12
Panel C: Relative Pricing Error										
Total	0.97	0.99	0.93	0.95	0.90	0.91	0.87	0.87	0.86	0.87
Bonds	0.84	1.06	0.84	1.00	0.85	1.07	0.79	0.92	0.75	0.94
Options	0.98	0.99	0.94	0.95	0.90	0.90	0.87	0.87	0.87	0.87
Stocks	0.90	0.95	0.85	0.93	0.91	0.95	0.80	0.91	0.80	0.92

for bonds.

OOS Total R^2s are remarkably close to their IS counterparts. The parsimonious structure of joint IPCA guards against overfitting and explains bond, option, and stock returns well IS and OOS. For the same five-factor model, we continue to explain 15% of the variation in returns with no forward-looking information, which ranges from 10% for options to 14% for stocks and 34% for bonds.

The fraction of cross-sectional variation explained (XS R^2) is in general comparable to the Total R^2s. The magnitudes tend to be slightly smaller, but the general trend is that option returns are the hardest to price and bond returns the easiest to price. The same applies to the relative pricing errors in Panel C of Table 2.

We find that the joint IPCA model can explain as much variation of stock returns as reported by Kelly, Pruitt, and Su (2019). We focus on the sample of stocks that are optionable or that the firm's bonds are traded and recorded in the TRACE database. We therefore provide confirming evidence that the IPCA structure is able to explain stock return variation well, even in a current sample between 2002 and 2022. The R^2s for options are slightly larger than those reported in Goyal and Saretto (2022), but within the same ballpark. Finally, the performance metrics for bonds are lower than those reported by Kelly, Palhares, and Pruitt (2023), but again within the same ballpark. Given the best performance of a model with five joint factors, we will focus our subsequent analysis on the case of $K = 5$.

Expected vs. Realized Returns. Figure 1 shows that joint IPCA explains the realized returns of the CMPs described in Section 4.2 well. The IS fit is shown in the left panel, the OOS fit in the right panel. We consider $K = 5$ latent common factors. We compare the model-implied expected return for each CMP with the average realized excess return over the sample period. For comparability, we normalize all portfolios to have 10% annualized volatility. For CMPs of bond, option, and stocks, the figure shows that the Joint IPCA produces a scatter plot that is closely aligned with the 45°-line, demonstrating small IS and OOS pricing errors. The IS fit is best for bonds and stocks, with a slight tilt in the slope for options: realized option returns tend to be less variable than expected by the model. The OOS fit remains remarkably stable, with joint IPCA explaining average returns well with a near-symmetric dispersion around the 45°-line.

Explaining Aggregate Returns. Our factor model is constructed to explain returns across asset classes which puts us in the unique position to learn about the underlying joint factor structure. As an approximation, we now investigate the predictability of *aggregate returns*, for which we assume an equal investment in a firm's bond, option, and stock, requiring that we observe returns for all three asset classes:

$$R_{it+1}^{firm} = \left(R_{it+1}^{B} + R_{it+1}^{O} + R_{it+1}^{S}\right)/3. \tag{13}$$

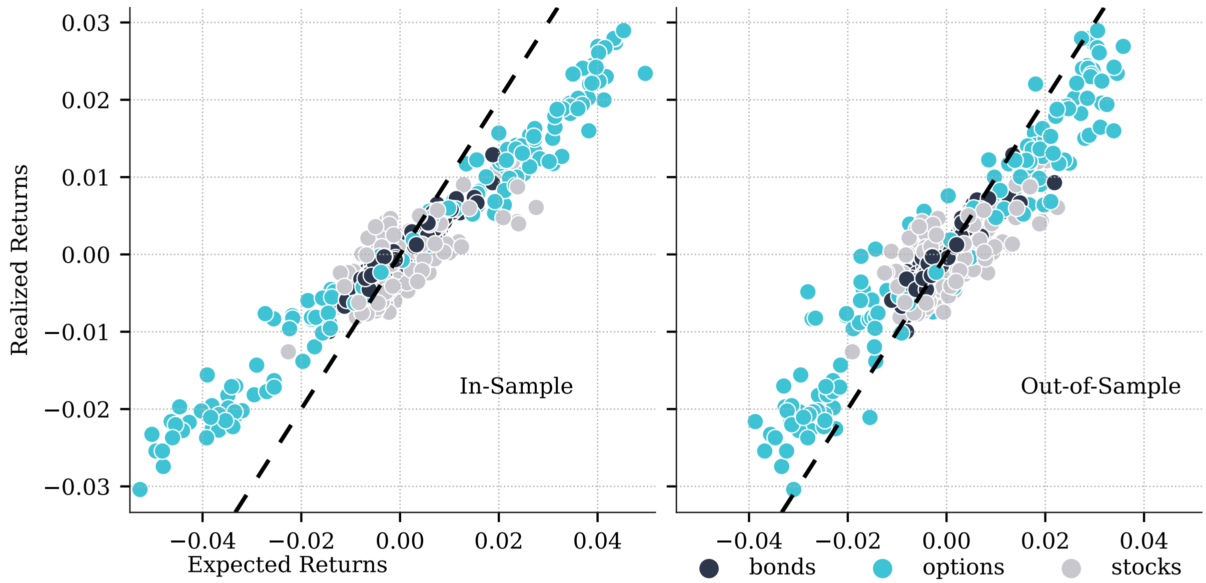

Figure 1: Expected vs. Realized Returns

The figure shows a scatter of returns expected by the five-factor joint IPCA model versus average realized returns of the 163×3 characteristic-managed portfolios described in Section 4.2 (163 for each asset class). In the left panel, we show the results for the IS period from August 2002 through August 2022. The right panel shows the results for iteratively fitting a model with no forward-looking information. The OOS period begins in February 2010. We distinguish CMPs of bonds, options, and stocks through different colors.

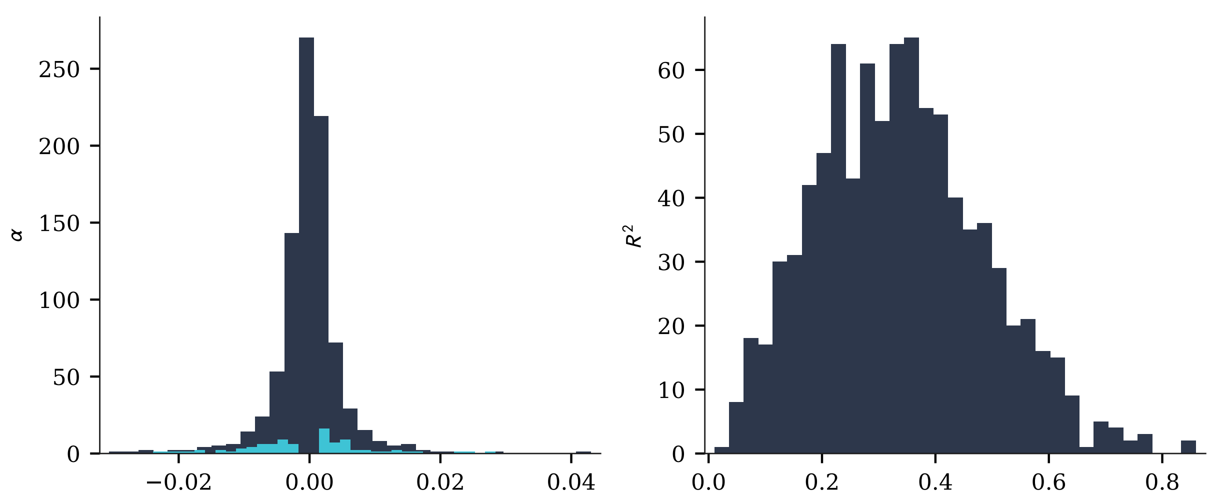

Figure 2: Unconditional αs and Time-Series R^2 of Aggregate Returns

The left panel of the figure shows average aggregate returns as defined in Eq. (13) in dark blue color and their unconditional αs that remain significant at the 5%-level after adjusting for risk using the $K = 5$ factor joint IPCA model in teal color. The right panel shows the resulting R^2s.

The left panel of Figure 2 shows the histogram of unconditional average aggregate returns in dark blue color and unconditional alphas after adjusting for risk using our $K = 5$ factor joint IPCA model in teal color. We only show the 9.9% of alphas that remain significant after the risk adjustment. Overall, the joint IPCA model explains aggregate returns well, which is also evident in the resulting R^2s shown in the right panel of Figure 2. The average firm-level R^2 is 33.8%.

5.2 Stochastic Discount Factor

We next seek to understand the implications that joint IPCA has for the stochastic discount factor. For that, we calculate the mean-variance efficient (MVE) tangency portfolio implied by a $K = 5$ joint factor model. Joint IPCA can exploit the covariance between the asset classes to form efficient mean-variance portfolios *across asset classes*. The MVE portfolio is of particular interest, as shocks to its returns are directly proportional to shocks to the firm-level stochastic discount factor M (Cochrane, 2009):

$$M_{t+1} - \mathbb{E}_t[M_{t+1}] = b \times \left(R_{t+1}^{MVE} - \mathbb{E}_t[R_{t+1}^{MVE}] \right), \tag{14}$$

Returns of the MVE portfolio inform us about the risks most correlated with the marginal utility of the marginal investor that is simultaneously active in bonds, options, and stocks.

Table 3: Sharpe Ratio of the Tangency Portfolio

The table shows the IS and OOS SRs of the tangency portfolio implied by a K-factor joint IPCA model. The IS period runs from August 2002 through August 2022, the OOS period starts in February 2010, as we require at least 90 months of historic data.

$K \rightarrow$	1	2	3	4	5
In-Sample	0.61	0.78	3.02	6.42	6.88
OOS	0.56	1.02	1.79	5.65	6.36

We report IS and OOS SRs of the tangency portfolios implied by a K-factor joint IPCA model in Table 3. We again consider $K \in [1, ..., 5]$. A single factor generates a SR of 0.61 IS and 0.56 OOS. Increasing the number of factors monotonically increases the resulting IS and OOS performance: a five-factor model generates a SR of 6.88 IS and 6.36 OOS, with only a

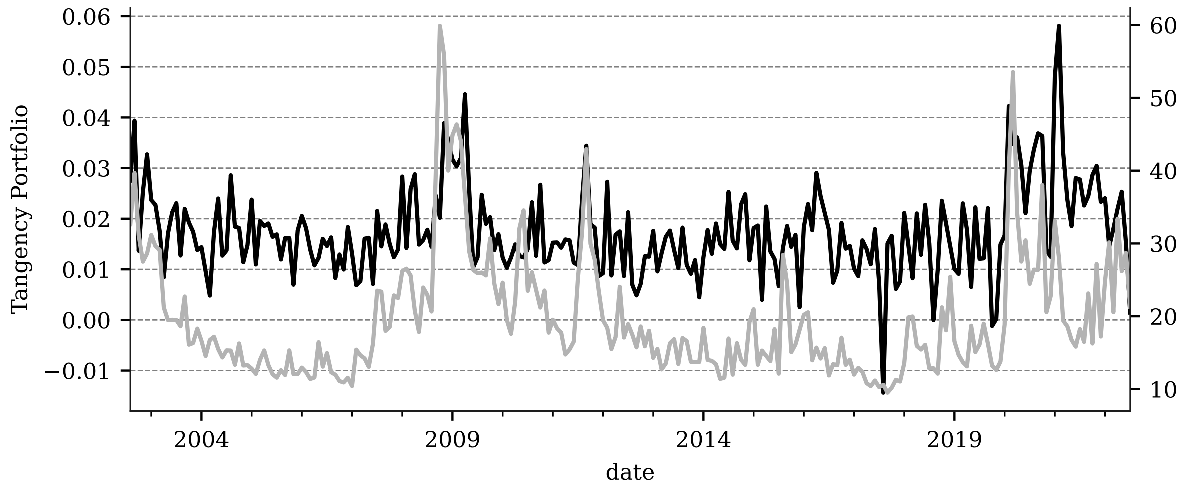

Figure 3: Tangency Portfolio Over Time

The figure shows the returns at time $t+1$ of the tangency portfolio for a $K = 5$ factor joint IPCA model in black. We overlay the VIX at time t in gray. The sample period runs from August 2002 through August 2022.

slight performance degradation OOS, suggesting a remarkable stability in the usefulness of the extracted information from the three asset classes. These SRs are significant at the 1% level using the statistical test of Lo (2002).[8]

Eq. (14) shows that shocks to the tangency portfolio are directly proportional to shocks to the stochastic discount factor. Figure 3 therefore overlays the tangency portfolio's returns in $t+1$ over time with the VIX at time t. Martin (2017) shows that an option portfolio similar to that of the VIX can be used to derive a lower bound on the expected market return. The correlation between the VIX_t and R^{MVE}_{t+1} is 0.55: whenever the VIX is low, so are the returns of the tangency portfolio in the next month. In times of crises, both the VIX and the returns to the tangency portfolio tend to spike. For example, in November of 2008, the tangency portfolio returns 3.9% with a VIX above 50. During the Covid-selloff at the beginning of 2020, we once again find large returns of the MVE portfolio, with the VIX above 50.

[8] In Section 6 we compare the SRs produced by joint IPCA to those obtained within the individual asset classes.

Decomposition to three asset classes. Since the MVE portfolio is a portfolio of latent factors, F_{t+1}, and the factors themselves are linear combinations of individual security returns (first row of Eq. (3)), we can express the tangency portfolio as:

$$R_{t+1}^{MVE} = \sum_{i \in \{B,O,S\}} w_{it} R_{it+1} = \sum_{i=1}^{N_{t+1}} w_{it}^{B} R_{it+1}^{B} + \sum_{i=1}^{N_{t+1}} w_{it}^{O} R_{it+1}^{O} + \sum_{i=1}^{N_{t+1}} w_{it}^{S} R_{it+1}^{S}, \qquad (15)$$

where w_{it} is asset i's weight in the MVE portfolio.

Using Eq. (15), we can identify the contribution of each asset class to the returns of the tangency portfolio. We decompose the returns of the tangency portfolio obtained with $K = 5$ latent factors in Panel A of Table 4. The portfolio has an average monthly return of 1.75% with a volatility of 0.88%, resulting in the SR of 6.88 as discussed above. The portfolio's return is positively skewed (0.86) and the portfolio has small drawdowns of at most 1.44%. The remaining columns of Panel A in Table 4 show how the performance is attributable to investments in corporate bonds, options, and stocks. The sub-portfolio of bonds has an average return of 0.13% (SR of 1.60). The sub-portfolio of options contributes an average return of 1.38% (SR of 5.83), and the sub-portfolio of stocks has an average return of 0.24% (SR of 2.22).

Trading in options (Ofek, Richardson, and Whitelaw, 2004) as well as in corporate bond markets is known to be expensive (Bessembinder, Spatt, and Venkataraman, 2020). To understand if the proposed tangency portfolio would be implementable in real-time, we measure the portfolio's monthly turnover and transaction costs. For this, we define the portfolio's relative turnover as:

$$\text{Turnover} = \sum\nolimits_{t} \left(\sum\nolimits_{i} |w_{it} - w_{it-1}| \right) \Big/ T. \qquad (16)$$

Transaction costs are assumed to be proportional in the amount of trading. Kelly, Palhares, and Pruitt (2023) choose the upper bound of the transaction cost estimates for corporate

Table 4: Return and Variance Decomposition of Joint Tangency Portfolios

The table shows a decomposition of the joint tangency portfolio's return profile. We provide average monthly returns, standard deviations (Std), annualized Sharpe ratios (SR), skewness (Skew), kurtosis (Kurt), the maximum drawdown (MDD), the relative turnover (TO) as defined in Eq. (16), and transaction costs in return units assuming relative implementation costs of 30bps (TC) in Panel A, both for the joint tangency portfolio and sub-portfolios invested in the corresponding bond, option, and stock components. Panel B provides correlation coefficients for the tangency portfolio and the sub-portfolios. The sample period ranges from August 2002 through August 2022.

	Joint	Bonds	Options	Stocks
		Panel A: Returns		
Return	1.75	0.13	1.38	0.24
Std	0.88	0.27	0.82	0.38
SR	6.88	1.60	5.83	2.22
Skew	0.86	3.47	1.30	−0.81
Kurt	2.69	19.95	4.22	1.21
MDD	−1.44	−1.40	−1.22	−3.34
TO	1.03	0.40	2.38	1.12
TC	1.18	0.12	0.72	0.34
		Panel B: Correlation		
Bonds	0.25			
Options	0.90	0.09		
Stocks	0.19	−0.32	−0.14	

bonds by Choi, Huh, and Shin (2021) who recommend a one-way cost of 17–19bps. Frazzini, Israel, and Moskowitz (2018) show that AQR's average implementation costs for trading stocks varies between 5 and 25bps for large and small caps, respectively, with significant variation over time. For options trading, Muravyev and Pearson (2020) suggest that institutional investors are able to achieve much better execution than implied by bid-ask spreads. We consider a relatively conservative level of transaction costs of 30bps – larger than the estimates proposed by Choi, Huh, and Shin (2021) and Frazzini, Israel, and Moskowitz (2018). For simplicity, we use the same estimates for bonds, options, and stocks. The tangency portfolio's monthly turnover is relatively high at 103%, resulting in total transaction costs of 1.18% per month. As a result, the net-of-fees SR reduces to 2.26.

We find that the net returns and SR of the sub-portfolios of bonds and stocks shrink significantly with these high levels of transaction costs. Nevertheless, Panel B of Table 4 shows that there are important diversification benefits of investing jointly in bonds, stocks,

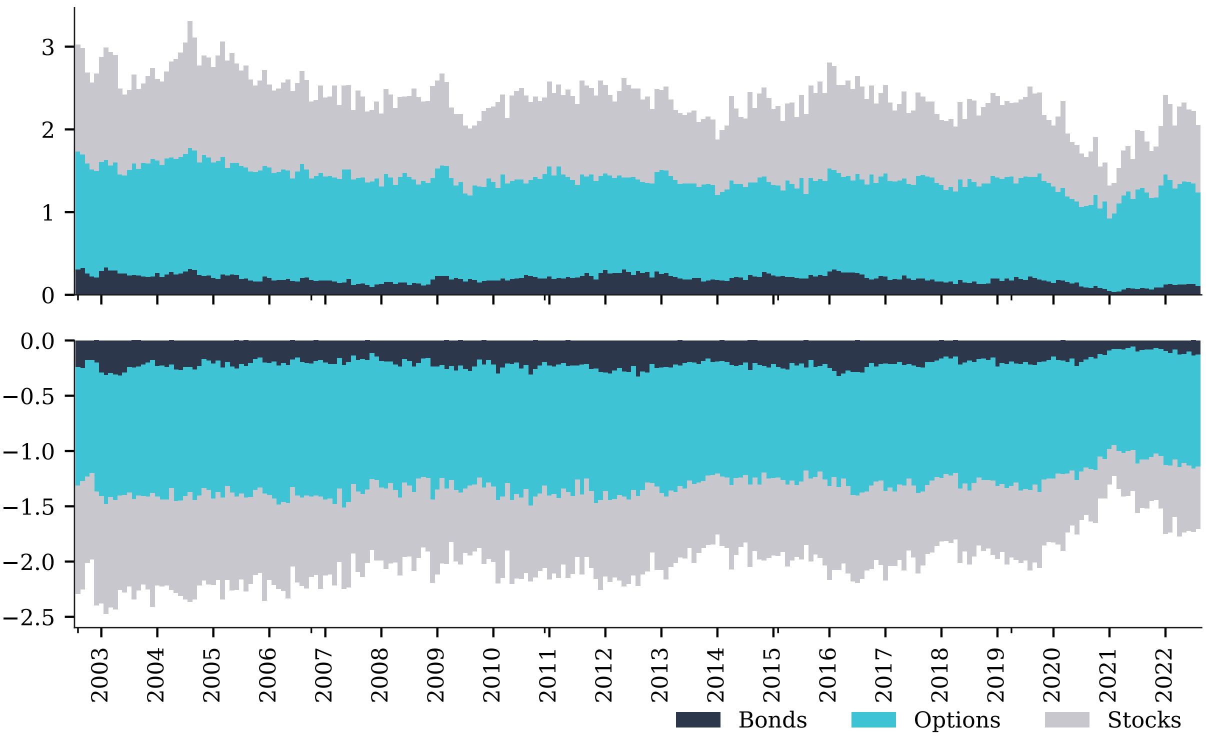

Figure 4: Tangency Portfolio Weights by Asset Class

The figure shows the distribution of long and short portfolio weights placed in each asset class over time, defined as $w^{\text{long}}_{AC,t} = \sum_{i \in AC} w_{it} \mathbb{1}_{w_{it}>0}$ and $w^{\text{short}}_{AC,t} = \sum_{i \in AC} w_{it} \mathbb{1}_{w_{it}<0}$, for $AC \in$ [bonds, options, stocks]. The sample period ranges from August 2002 through August 2022.

and options. We show the correlations between the tangency portfolio and the three sub-portfolios. The returns of the tangency portfolio are modestly correlated with the bond and stock sub-portfolios. The correlation with the option sub-portfolio is large at 90%, showcasing the relative outperformance of the options market.

The correlations between the different class-level sub-portfolios are very low or even negative, highlighting that investors can earn significant diversification benefits when incorporating information about the joint dependence structure of the three asset classes. The sub-portfolios of bonds and options are modestly correlated with a correlation of 0.09; sub-portfolios of bonds and stocks have a correlation of −0.32, and sub-portfolios of options and stocks have a correlation of −0.14.

We can also analyze the distribution of portfolio weights placed in each of the three asset classes. We separately consider long and short portfolio weights within each class AC, i.e., $w_{AC,t}^{\text{long}} = \sum_{i \in AC} w_{it} \mathbb{1}_{w_{it}>0}$ and $w_{AC,t}^{\text{short}} = \sum_{i \in AC} w_{it} \mathbb{1}_{w_{it}<0}$. Figure 4 shows that, on average, the tangency portfolio favors a symmetric but small long and short investment in corporate bonds. For options, we find a slight tilt towards shorting options, potentially to harness the variance risk premium embedded in equity options (Goyal and Saretto, 2009). The asset class weights in stocks are fairly symmetric. Overall, class-specific weights are relatively stable over time.

6 The Integration of Bonds, Options, and Stocks

A central question is the degree to which the different markets for bonds, options, and stocks are integrated. Table 2 already shows that the joint IPCA factor model simultaneously explains the returns of bonds, options, and stocks well. In this section, we dig deeper into common sources of return predictability.

6.1 Commonality in Predictability

Integrated Predictions. As a first step, we investigate if explanatory power of the factor model is *shared* across asset classes. If markets are (partially) integrated, we expect to observe shared patterns of predictability, with more predictable bond and option returns whenever the firm's stock return is easy to predict, for example. For each firm in our sample, we compute the Total R^2 using a joint $K = 5$ factor model separately for the firm's bond, option, and stock. To be able to compute a meaningful measure of variation, we require the data for a firm to be available for at least 24 months.

Next, we sort firms into decile portfolios by their bonds' Total R^2 and record the average bond, option, and stock R^2 for each decile. We also show the R^2 spread, as the difference between the average R^2 for the least predictable decile (1) and the most predictable decile

Table 5: Sorts on Asset Class Predictability

The table shows commonality in asset class-level predictability, measured by the Total R^2. As an example, in Panel A, we sort firms into deciles by their bond-level predictability and record the average R^2 of a $K = 5$ factor joint IPCA model. We then also record the average R^2 for the remaining two asset classes. We consider firms with at least 24 months of return data available. We repeat the procedure by sorting on option R^2 in Panel B and stock R^2 in Panel C.

	1	2	3	4	5	6	7	8	9	10	10-1
Panel A: Portfolios Sorted by Bonds' Total R^2											
Bonds	−0.34	0.17	0.25	0.32	0.39	0.44	0.50	0.56	0.62	0.72	1.06
Options	0.18	0.22	0.23	0.20	0.25	0.26	0.28	0.28	0.31	0.34	0.16
Stocks	0.23	0.24	0.26	0.25	0.29	0.29	0.32	0.30	0.29	0.30	0.07
Panel B: Portfolios Sorted by Options' Total R^2											
Bonds	0.30	0.31	0.37	0.33	0.38	0.41	0.38	0.42	0.42	0.48	0.18
Options	−0.07	0.03	0.08	0.12	0.16	0.20	0.25	0.30	0.37	0.50	0.57
Stocks	0.16	0.18	0.20	0.20	0.22	0.24	0.26	0.27	0.30	0.33	0.17
Panel B: Portfolios Sorted by Stocks' Total R^2											
Bonds	0.33	−0.12	0.37	0.32	0.36	0.37	0.39	0.39	0.42	0.43	0.11
Options	0.12	0.13	0.15	0.14	0.16	0.18	0.19	0.21	0.23	0.32	0.20
Stocks	−0.03	0.07	0.11	0.14	0.18	0.22	0.26	0.31	0.37	0.47	0.51

(10). Panel A of Table 5 shows the results for this sort. We find that the explanatory power of the joint IPCA model is negative at −34% for decile 1 bonds but increases significantly to 72% for decile 10 bonds. By construction, the 10−1 predictability is high at 106%. More interesting for our purposes, we also find a positive 10−1 spread in Total R^2 for the two other asset classes. For example, options of firms in decile 1 have an average R^2 of 18%, compared to an average R^2 of 34% for options of firms in decile 10. For stocks, the 10−1 spread is somewhat smaller at 7%. This result shows that sorting on how well joint IPCA explains bond returns also produces a meaningful explanatory spread for options and stocks.

We repeat this exercise by sorting on options explanatory power and show the results in Panel B of Table 5. The 10−1 explanatory spread for options is 57% and ranges between −0.07% and 50% from the bottom to the top decile. We find a strong commonality in the explanatory pattern between options, stocks and bonds: sorting on option Total R^2 produces a bond spread of 18% and a stock spread of 17%. Finally, Panel C of Table 5 sorts on stock

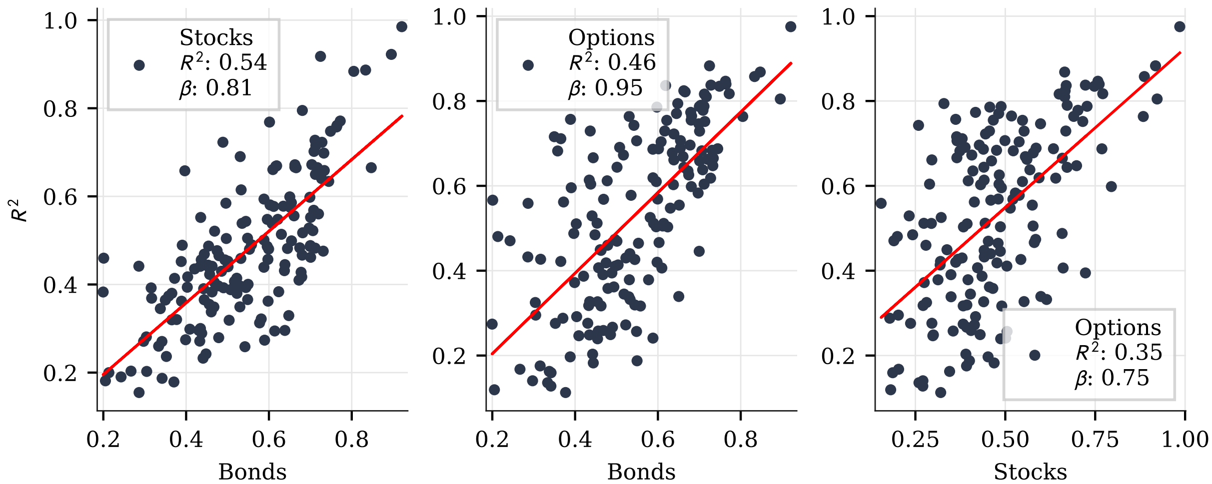

Figure 5: Commonality in the Predictability of Characteristic-Managed Portfolios

The figure shows the Total R^2 for a $K = 5$ factor joint IPCA model for the 163×3 CMPs of bonds, options, and stocks. The left plot is a scatter of the resulting R^2s for CMPs of bonds compared to CMPs of stocks. The middle (right) plot repeats this exercise for CMPs of bonds (stocks) on the x-axis and options on the y-axis.

Total R^2. We find that the $10-1$ spread for stock is 51%, for options is 20%, and for bonds is 11%, suggesting that stocks and options tend to be more integrated than stocks and bonds.

As another manifestation of commonality across the three asset classes, we calculate the Total R^2 of the 163 CMPs for each asset class and show bivariate scatter plots of these R^2s of CMPs of one asset class versus that of CMPs of another asset class in Figure 5. A strong correlation is evident in these plots showing that the explanatory power is shared across CMPs of bonds, options, and stocks. For example, regressing the R^2s of CMPs of stocks on the R^2s of CMPs of bonds gives a slope coefficient (β) of 0.81 (left panel). This regression explains 54% of the variation in explanatory R^2s. We also find a large agreement in the explanatory power of CMP returns for CMPs of bonds vs. options in the middle and stocks vs. options in the right panel of Figure 5.

Factor Importance. Our $K = 5$ joint IPCA factors jointly explain the returns of bonds, options, and stocks. But, are all five factors required for each asset class? For instance, if factors F1-F2 are important for bonds, F3-F4 for options, and F5 for stocks, then this would

Table 6: Factor Influence on Explaining Bond, Option, and Stock Returns

The table shows the relative reduction in Total R^2 when "turning off" the influence of the kth factor. We do so by setting each of that factor's realizations to zero. The left panel shows the impact of turning off factors one-by-one, the right panel shows the cumulative effect. Factors are ordered with the first factor having the largest average return.

K	Factor Influence				Cumulative Factor Influence			
↓	Bonds	Options	Stocks	Total	Bonds	Options	Stocks	Total
1	−0.27	−0.30	−0.22	−0.25	−0.27	−0.30	−0.22	−0.25
2	−0.58	−0.21	−0.53	−0.48	−0.86	−0.51	−0.75	−0.72
3	−0.01	−0.27	−0.08	−0.11	−0.87	−0.79	−0.83	−0.83
4	−0.03	−0.11	−0.12	−0.10	−0.90	−0.91	−0.96	−0.94
5	−0.12	−0.09	−0.04	−0.06	−1.00	−1.00	−1.00	−1.00

imply weak integration across the three asset classes (even if the three asset classes share the same set of characteristics to model time-varying betas). The evidence so far in this section, showing commonality in the explanatory power across the three asset classes, suggests that this is an unlikely possibility. Nevertheless, we formally investigate the commonality picked up by the five factors.

To do so, we iteratively "turn off" the influence of a factor by setting its realizations to zero. We then document the resulting drop in Total R^2 across asset classes, as well as individually for the subsample of bonds, options, and stocks. We start with the factor with the highest mean return (factor F1) and work our way down to the factor with the smallest mean return (factor F5). We provide the results on the *relative* reduction in Total R^2 in Table 6. The panel on the left shows the results when restricting the influence of one factor at a time, the panel on the right shows the cumulative effect, i.e., the second row shows the effect of simultaneously turning off factors F1 and F2.

Table 6 shows that turning off the influence of the first factor decreases the bond R^2 by 27% (from 30% to 22%), the option R^2 by 30%, and the stock R^2 by 22%, for a reduction in the overall R^2 by 25%. This first factor seems to be equally important for the three asset classes, seemingly capturing market effects across bonds, stocks, and options. Factor F2 is most important for bonds and stocks. Turning it off reduces the bond R^2 by 58% and the stock R^2 by 53%. The third factor is most important for options, and is responsible for 27% of joint IPCA's explanatory power over option returns. Excluding it also reduces the stock R^2 by 8%. F4 is also important for explaining stock and option returns. Excluding it reduces the R^2 by 11% and 12% for options and stocks, respectively. The fifth factor is mostly informative about bond and option returns.

The most important factors for explaining option returns are F1, F2 and F3, which together make up 79 percent of the model's option return predictability. Consistent with the commonality patterns shown in Table 5, the three factors are also responsible for 87 percent of the model's bond return predictability as well as for 83 percent of the model's stock return predictability, and thus for 83 percent of the model's total explanatory power. The three most important factors for stock return predictability are factors F1, F2, and F4. They capture 87 percent of the model's explanatory power for stocks, 62 percent for options, and 88 percent for bonds. These results again indicate a large degree of integration between the three markets. The last column "Total" shows that all five factors are important for explaining firm returns, with the "market factor" F1, the "bond-stock factor" F2 and the two "option-stock factors" F3 and F4 being most important.

Characteristics Importance. The specification in Eq. (3) has two main ingredients: latent factor realizations F_{t+1} and the Γ_β matrix, which maps observable characteristics to heterogeneity in factor sensitivities (betas). Combining the information from Γ_β and the factors, we can understand the relative importance of each input characteristic in describing expected returns. For this, we introduce an importance and a sensitivity measure, which extend the characteristic importance proposed by Kelly, Pruitt, and Su (2019). The lth row of Γ_β, γ_l, describes how characteristic l influences an asset's sensitivity to each of the K factors. Combined with the average return of the K factors this informs us about a characteristic's *Importance* defined as:

$$\mathit{Importance}(z_l) = |\gamma_l|' \,\overline{F}. \tag{17}$$

Importance measures the average absolute influence of the lth characteristic on an asset's β, weighted by each factor's average influence on expected returns. Similarly, we can assess the *Sensitivity* of the model's expected returns to a unit change in characteristic l:

$$\mathit{Sensitivity}(z_l) = \gamma_l' \overline{F}. \tag{18}$$

As characteristics are rank-standardized between -0.5 and 0.5, *Sensitivity* gives us the model-expected return spread of two stocks with a unit difference in l and with otherwise equal characteristics.

In Panel A of Table 7, we provide the ranks of *Importance* and the values for *Sensitivity* for the 12 most important characteristics for explaining bond returns.[9] Among the 12 most important characteristics for explaining bond returns are three bond characteristics (one being the constant), one option characteristic, and eight stock characteristics. The most important characteristic for bond returns is its rating (B_rating), followed by the bond market (const), the stock's short-term reversal (S_ret_1_0), and the bond's duration (B_DURATION). For example, a higher short-term reversal in the stock is associated with larger expected bond returns of 0.45% over the next month.

It is interesting to note that while the bond's short-term reversal is the sole bond characteristic in the list of top-12 CMPs of bonds in Section 4.2, information about the bond's rating (B_rating), duration (B_duration) and the overall bond market return (const) are important for modeling expected bond returns. The three characteristics generate an average

[9]Table 1 shows the most important characteristics that have the highest SRs for CMPs. However, a characteristic that generates high SR need not necessarily be the one that is important for describing the risk-return tradeoff in a joint IPCA model.

Table 7: Importance of Characteristics

The table shows *Importance* ranks (Eq. (17)) and *Sensitivity* values (Eq. (18)) for the top-12 most important characteristics, measured by their *Importance*, for bonds (Panel A), options (B), and stocks (C).

	Bonds		Options		Stocks	
	Imp.	Sens.	Imp.	Sens.	Imp.	Sens.
Panel A: Top Characteristics for Bonds						
B_rating	1	0.29	57	0.11	57	−0.06
const	2	0.22	12	−0.10	1	0.12
S_ret_1_0	3	0.45	43	−0.15	23	−0.09
B_duration	4	0.00	106	−0.04	87	0.02
S_cop_at	5	0.44	62	0.09	44	−0.03
S_at_turnover	6	−0.09	55	0.07	60	0.06
S_at_me	7	0.04	21	0.29	22	0.05
S_dolvol_126d	8	0.09	24	−0.20	4	0.07
O_S_toi	9	−0.23	17	0.34	50	−0.07
S_oaccruals_at	10	0.06	45	0.14	63	−0.05
S_rvol_21d	11	0.22	18	0.29	21	−0.05
B_mom6_1	12	−0.12	95	−0.05	94	0.04
Panel B: Top Characteristics for Options						
O_C_embedlev	23	0.01	1	1.04	40	−0.08
O_C_theta	82	−0.01	2	0.73	67	0.01
O_S_rnv_30	41	−0.11	3	0.67	37	−0.09
O_S_rns_30	19	0.08	4	−0.62	71	0.02
S_ivol_capm_252d	156	−0.01	5	0.57	9	0.11
O_S_rns_182	13	0.03	6	−0.58	35	−0.09
O_S_ivarud_30	33	−0.15	7	0.54	34	−0.09
S_market_equity	26	−0.02	8	−0.44	15	0.02
O_C_delta	80	0.01	9	0.33	77	−0.05
O_S_demand_pressure	56	0.08	10	−0.39	81	−0.02
O_S_ivud_30	105	0.02	11	0.36	95	0.04
const	2	0.22	12	−0.10	1	0.12
Panel C: Top Characteristics for Stocks						
const	2	0.22	12	−0.10	1	0.12
S_zero_trades_126d	39	0.13	16	−0.35	2	0.34
O_S_demand_roll252D	17	0.04	56	−0.02	3	0.26
S_dolvol_126d	8	0.09	24	−0.20	4	0.07
S_corr_1260d	97	−0.06	27	0.19	5	0.03
S_beta_60m	44	0.05	77	0.07	6	0.08
S_cop_atl1	57	−0.09	82	0.05	7	0.13
S_mispricing_mgmt	53	−0.01	60	0.10	8	−0.20
S_ivol_capm_252d	156	−0.01	5	0.57	9	0.11
S_prc	100	−0.05	85	0.07	10	−0.11
S_rmax5_21d	28	−0.11	14	0.35	11	−0.18
S_zero_trades_21d	20	0.03	19	0.32	12	−0.19

return spread of 0.29%, 0.00%, and 0.22%, respectively. Interestingly, while a bond's duration has an average return of zero, it is highly important for joint IPCA to describe variation in bond returns. We also report the *Importance* rank and *Sensitivity* for the other two asset classes in Panel A of Table 7. Of the 12 most important characteristics for explaining bond returns, the highest importance rank for explaining option returns is the constant on rank 12. The median rank among options is 44. The stock's assets-to-market ratio (S_at_me), dollar trading volume (S_dolvol_126d), and realized volatility (S_rvol_21d) are also among the 30 most important characteristics for explaining option returns. Stock trading volume is the fourth most important determinant of expected stock returns, the stock's assets-to-market ratio and realized volatility are ranked 22nd and 21st, respectively, for modeling expected stock returns.

In Panel B of Table 7, we show the most influential characteristics for explaining option returns. Most important are the option's embedded leverage and theta. Both characteristics are not only vital for explaining variation in options' sensitivity to the common risk factors, but their implied return spreads are also large at 1.04% and 0.73%. Risk-neutral moments (O_S_rnv_30, O_S_rns_30, O_S_rns_182), the stock's idiosyncratic volatility (S_ivol) and market capitalization (S_market_equity), as well as the relative trading in the options vs. the stock market are also important determinants of expected option returns. Five (three) out of the top-12 characteristics for explaining option returns also show up among the 30 most important characteristics for modeling expected bond (stock) returns.

The most important characteristics for explaining stock returns are almost exclusively derived from the stock itself (Panel C of Table 7). Only the past option volume relative to the stock's market capitalization (O_S_demand_roll252D) enters the top-12 as an option-based characteristic. Other important characteristics are the stock's liquidity (S_zero_trades_126d), dollar trading volume (S_dolvol_126d), beta (S_beta_60m), mispricing (S_mispricing_mgmt) and MAX returns (S_rmax5_21d, see Bali, Cakici, and Whitelaw, 2011). Five (seven) of the top-12 characteristics for explaining stock returns are also among the top-30 characteristics for bonds (options).

Overall, the overlap in characteristics explaining returns of all three asset classes shows that the joint factor structure that we extract provides a parsimonious factor model with the ability to simultaneously explain the returns of the three asset classes. Information about returns and characteristics from the two other asset classes is beneficial to understand the risk-return trade-off of the third asset class.

6.2 Joint vs. Single IPCA

The joint factor structure that we extract with joint IPCA is beneficial to explain returns of bonds, options, and stocks, while maintaining a parsimonious model with a low number of latent factors. We now highlight that joint IPCA is better able to explain average returns than individual IPCA models estimated for a single asset class and compare $K = 5$ factor models.

Unconditional Alphas. We compare our joint IPCA versus three individual IPCA models estimated for each asset class in their explanatory power for 163×3 CMPs described in Section 4.2. We calculate and compare unconditional alphas as this provides a common testing ground for how well the latent factors are able to explain returns of each of the three asset classes. The results are presented in Table 8.

37 CMPs of bonds, 142 of options, and 40 CMPs of stocks have significant average returns for a total of 219 CMPs with significant full-sample returns, or a little under half of the 163×3 CMPs that we analyze. We use Newey and West (1987) standard errors with 12 lags to account for serial correlation and heteroskedasticity in returns.

Joint IPCA does an impressive job of explaining average returns across asset classes: it leaves only 11 alphas unexplained: 1 for bonds, 7 for options, and 3 for stocks. Looking at single IPCA models estimated on individual asset classes, we find that the five bond-based factors fail to explain 23 CMPs of bonds, 139 CMPs of options, and 31 CMPs of stocks, for

Table 8: Unconditional Alphas of Joint and Single IPCA Models

The table shows how many of the 163×3 CMPs defined in Section 4.2 have significant average returns per asset class, and how many unconditional alphas remain significant after adjusting for risk either using the joint IPCA model, or individual IPCA models estimated for a single asset class. We also consider a combined model with two factors estimated from each asset class $(2+2+2)$. We use Newey and West (1987) standard errors with 12 lags.

		Unconditional alphas				
	Average	Joint	Single IPCA			
	returns	IPCA	5 Bond	5 Option	5 Stock	$2+2+2$
Bonds	37	1	23	6	17	10
Options	142	7	139	20	129	14
Stocks	40	3	31	8	27	7
Σ	219	11	193	34	173	31

a total of 193 CMPs with significant alphas. The option-level IPCA performs significantly better: it leaves 6, 20, and 8 CMP alphas of bonds, options, and stocks, respectively, as statistically significant. The model estimated exclusively on stock returns performs slightly better than its bond counterpart: the single IPCA with five stock-level factors fails to explain 17, 129, and 27 CMPs of bonds, options, and stocks, respectively, or a total of 173 CMPs. Impressively, our joint IPCA model manages to explain more CMP returns of the same asset class than the single IPCA models were estimated on, highlighting the importance of incorporating the integration structure between the three asset classes. Finally, we also consider a model that extracts two factors per asset class. This model performs better than the individual IPCA models but still leaves a total of 31 CMP returns unexplained.

Sharpe Ratios. Next, we analyze how well each of the models describes the mean-variance frontier. Table 9 shows that the tangency portfolio of the joint IPCA has an annualized SR of 6.88, compared to 1.37 for the bond-based, 5.90 for the option-based, and 2.30 for the stock-based IPCA model. The combined model with two factors estimated from each of the asset classes has a tangency portfolio with a SR of 5.99. Interestingly, the Sharpe ratios of the class-level sub-portfolios of joint IPCA are comparable, or in the case of bonds even exceed the Sharpe ratios of the tangency portfolios of the individual IPCA models (see Table 4).

Table 9: Comparison of Sharpe Ratios for Joint vs. Single IPCA Models

The table compares the SRs of the tangency portfolios for the joint IPCA model as well as individual IPCA models estimated for a single asset class. We assess the statistical significance of joint IPCA's outperformance, by regressing the returns of its tangency portfolio on a constant and each of the single IPCA tangency portfolio returns, after fixing each portfolio's full-sample standard deviation to 10% per year. The resulting t-statistics in parenthesis are computed with Newey and West (1987) standard errors with 12 lags. We also consider a combined model with two factors estimated from each asset class $(2+2+2)$.

	Joint	Single IPCA			
	IPCA	5 Bond	5 Option	5 Stock	$2+2+2$
Sharpe Ratio	6.88	1.37	5.90	2.30	5.99
Outperformance		4.59	0.82	3.81	0.74
		(12.92)	(6.76)	(8.00)	(6.06)

The second row of Table 9 shows that the joint IPCA's tangency portfolio significantly outperforms its competitors. For this, we fix the full-sample standard deviation of the returns of each tangency portfolio to 10% per year, and regress the difference between joint IPCA's tangency portfolio returns and the tangency portfolio returns of the single IPCA models on a constant. The outperformance is measured by the alpha estimates and corresponding t-statistics that are computed with Newey and West (1987) standard errors with twelve lags. In all cases we find a highly significant SR outperformance of the joint tangency portfolio, ranging between 0.74% and 4.59%.

Latent Factor Correlation. As a final comparison between joint and single IPCA models, we compute the correlations between the five joint factors and each of the five single factors. This allows us to understand common patterns in joint and single factors. Differences in the included information may inform us about the reason for joint IPCA's outperformance.

Consistent with the analysis of the influence of each factor in Table 6, we find in Table 10 that "market factor" F1 is highly correlated with one or more single bond, option and stock factors. Likewise, F2, which is responsible for a large part of joint IPCA's ability to explain bond and stock return variation, is highly correlated to bond factor B3 and stock factors S4 and S5. Joint factor F3 is highly correlated with option factor O5 and stock factors S2 and S4, while joint factor F4 is highly correlated with option factors O2 and O3, stock

Table 10: Correlation of Joint and Single Latent Factors

The table shows the Pearson correlation coefficient between the five joint IPCA factors (F1 to F5) and each of the five latent factors obtained from individual IPCA models estimated on a single asset class. The largest absolute correlation coefficient per row is highlighted in boldface.

	F1	F2	F3	F4	F5
			Panel A: Latent Bond Factors		
$B1$	**0.41**	−0.31	−0.14	−0.10	0.03
$B2$	**0.58**	−0.53	−0.10	−0.15	−0.12
$B3$	−0.36	**0.59**	0.06	−0.35	−0.03
$B4$	0.07	−0.04	0.18	−0.44	**-0.62**
$B5$	0.13	−0.35	0.24	0.08	**-0.37**
			Panel B: Latent Option Factors		
$O1$	**0.46**	0.07	0.02	0.03	0.00
$O2$	0.08	0.07	−0.31	**-0.44**	0.02
$O3$	−0.04	0.03	0.13	**0.29**	0.02
$O4$	0.03	**-0.10**	0.07	0.01	−0.04
$O5$	0.39	**-0.62**	0.53	0.18	−0.38
			Panel C: Latent Stock Factors		
$S1$	−0.08	**0.15**	−0.03	−0.06	−0.07
$S2$	0.11	−0.19	**0.42**	−0.02	−0.24
$S3$	0.03	−0.14	−0.10	**0.36**	0.02
$S4$	0.38	**-0.56**	0.37	−0.36	0.17
$S5$	−0.60	**0.62**	0.17	0.15	−0.14

factor S3, and bond factors B3 and B4. Finally, joint factor F5 is most correlated with bond factors B4 and B5, but is also related to option factor O5. In summary, the joint factors capture information across latent factors estimated from single asset classes, and combine this information to explain returns of bonds, stocks, and options, in a parsimonious factor structure.

7 Interpreting Latent Factors

7.1 Macroeconomic Sensitivity

It is instructive to understand how the joint latent factors, capable of pricing bonds, options and stocks simultaneously, relate to observable macroeconomic indicators. We, therefore, regress each of the $K = 5$ latent factors on several established macroeconomic variables in Table 11. We include innovations in the Chicago Fed National Activity Index (CFNAI), which is a leading indicator of U.S. economic activity extracted from a broad range of individual macroeconomic variables, innovations in the macroeconomic uncertainty measure (UNC) of Jurado, Ludvigson, and Ng (2015), and in the intermediary capital ratio (ICR) of He, Kelly, and Manela (2017).

We find that our first IPCA factor (the "market factor") is positively exposed to the intermediary capital ratio. A one standard deviation increase in ICR increases its returns by 89bps. Factor F2 is positively related to innovations in UNC and negatively to innovations in ICR. This is consistent with predictions of the intertemporal capital asset pricing model (ICAPM) (Merton, 1973), in that low (high) levels of UNC (ICR) signal better investment opportunities in the future. The latent factors are constructed to have a positive full-sample mean, suggesting that exposure to the second latent factor exposes the investor to considerable macroeconomic risks (Maio and Santa-Clara, 2012).

Table 11: Regressing Latent Factors on Macroeconomic Indicators

The table shows the results of regressing each of the $K = 5$ latent factors on innovations of the Chicago Fed National Activity Index (CFNAI), the macroeconomic uncertainty index (UNC) of Jurado, Ludvigson, and Ng (2015), and the intermediary capital ratio (ICR) of He, Kelly, and Manela (2017). The three macroeconomic indicators are normalized by their full-sample standard deviation. *** (**, *) denote statistical significance at the 1% (5%, 10%) level. We use Newey and West (1987) standard errors with twelve lags.

	F1	F2	F3	F4	F5
const.	2.69***	0.88***	0.16	0.11	0.03
CFNAI	0.03	−0.01	−0.03	0.10	−0.03
UNC	−0.07	0.29***	−0.25**	−0.28**	0.43***
ICR	0.89***	−1.91***	0.18	−0.38**	0.21
Adj.R^2	0.23	0.53	0.06	0.04	0.09

The third factor is negatively exposed to UNC, suggesting suggesting that it is a hedge against macroeconomic uncertainty. F4 is negatively exposed to innovations in UNC and ICR, therefore capturing the spread between overall macroeconomic uncertainty and risks of the intermediary sector. Finally, factor F5 is positively related to UNC, yielding larger returns in times of elevated macroeconomic uncertainty.

7.2 Joint IPCA and Benchmark Factor Models

We have thus far shown that the joint IPCA model outperforms IPCA models estimated for single asset classes. We now compare the joint model with the performance of various benchmark factor models, which have been proposed in the literature for either of the three asset classes. To the best of our knowledge, joint IPCA is the first attempt at finding a *joint* factor model, capable of pricing bonds, options, and stocks simultaneously. We include the Fama and French (2015) five-factor model augmented with momentum (Carhart, 1997) as the leading factor model for the stock market (FF6) and the bond market CAPM for bonds (MKTB). There is still no consensus about the "best" factor model for options, such that we resort to the two straddle-based factors proposed by Coval and Shumway (2001) (CS). We also consider a combination of the three factor models, which in total includes nine factors $(6 + 1 + 2)$.

Table 12: Unconditional Alphas of Joint IPCA vs. Benchmark Factor Models

The table shows how many of the 3×163 characteristic-managed portfolios defined in Section 4.2 have significant average returns per asset class, and how many unconditional alphas remain significant after adjusting for risk either using the joint IPCA model, or benchmark factor models. We consider the Fama and French (2015) five-factor model augmented with momentum (Carhart, 1997) (FF6), bond market CAPM (MKTB), and the two straddle-based factors inspired by Coval and Shumway (2001) (CS). We also consider a combination of the three factor models (Comb.), which in total includes nine factors (6 + 1 + 2). We use Newey and West (1987) standard errors with twelve lags.

		Unconditional alphas				
	Average	Joint	Benchmark Factors			
	returns	IPCA	MKTB	CS	FF6	Comb.
Bonds	37	1	32	21	33	21
Options	142	7	138	125	142	128
Stocks	40	3	36	25	35	33
Σ	219	11	206	171	210	182

We repeat the unconditional alpha analysis of Table 8 for the comparison between the joint IPCA model and the three benchmark factor models in Table 12. We have already shown that the joint IPCA leaves a statistically significant alpha in only 11 out of 219 CMPs. The performance of the benchmark factor models are much worse. The CS option model fares best but still leaves 171 CMP returns unexplained, 21 for bonds, 125 for options, and 25 for stocks. The bond CAPM performs even worse: it fails to explain the returns of 206 out of 219 CMPs. The FF6 stock model fails to explain the returns of 210 CMPs. Even a combination of the nine factors into a single model fails to explain the returns of 182 CMPs, of which 21 are of bonds, 128 of options, and 33 of stocks.

Table 13: Regressing Latent Factors on Benchmark Factors

The table shows the results of regressing each of the $K = 5$ latent factors on a number of benchmark factors. We consider the Fama and French (2015) five-factor model augmented with momentum (Carhart, 1997) (FF6), bond market CAPM (MKTB), and the two straddle-based factors inspired by Coval and Shumway (2001) (CS). The benchmark factors are normalized by their full-sample standard deviation. *** (**, *) denote statistical significance at the 1% (5%, 10%) level. We use Newey and West (1987) standard errors with twelve lags.

	F1	F2	F3	F4	F5
const.	2.36***	1.21***	−0.00	−0.03	0.05
STRADDLE_INDEX	0.03	−0.09	0.14	0.19	0.09
STRADDLE_STOCK	0.04	0.03	−0.62***	−0.52***	0.06
MKT-RF	0.23**	−0.92***	0.60***	−0.58***	0.99***
SMB	0.30***	−0.32***	0.23***	−0.46***	0.43***
HML	−0.10	−0.17	0.04	0.36***	0.10
RMW	−0.03	0.10	−0.22***	0.05	−0.20***
CMA	−0.04	−0.14*	−0.19***	0.10	0.17**
MOM	−0.25	0.10	0.41***	0.47**	−0.28***
MKTB	0.76***	−1.06***	−0.68***	0.45**	−1.56***
Adj.R^2	0.58	0.80	0.44	0.24	0.67

In Table 13, we show results from regressing each of the $K = 5$ latent factors on the nine benchmark factors from the three models described above. One of the advantages of our joint IPCA specification is that it is able to extract information from the three asset classes simultaneously. In contrast, factor models have typically been confined to extracting information from a single asset class. Another advantage of the latent factor specification is that it does not require prior knowledge about which characteristic-sorted factors drive return differences in the cross-section. Instead, we extract a statistically optimal set of factors. The results in Table 13 impressively highlight this advantage: we cannot find a clear mapping between the five latent factors and the benchmark factors. The regressions' adjusted R^2s vary between 24% for F4 and 80% for F2. F1 seems to capture market-level effects, particularly among stocks and bonds, with a significantly positive coefficient for both the stock market MKT-RF and bond market MKTB. This echoes the results of Table 6 that F1 is important for explaining variation in bond, option, and stock returns jointly. F2 also seems to capture variation in the stock and bond market, which is also consistent with the factor exlusion exercise in Table 6. Factors F3 and F4 are related to the straddle factor from individual stocks, and most stock-level factors. Finally, F5 is most exposed to the bond-market. These results showcase that the latent factor structure extracts information beyond of what is captured in established benchmark models.

7.3 Replacing Factors

As a final analysis towards interpreting the $K = 5$ latent factors of our joint IPCA specification, we perform a factor-replacement exercise. For this, we first calculate the drop in the Total R^2 when setting all realizations of each factor separately to zero. We have discussed the implications of this exercise in Table 6. Denote the resulting Total R^2 as R^2_{zero}. Then, we regress each of the kth latent factor on a constant and either the three macroeconomic indicators CFNAI, UNC and ICR, or the nine benchmark factors discussed above, subsumed in matrix $\boldsymbol{X}$:

$$F_{t,k} = \alpha_k + \boldsymbol{\beta}_k \boldsymbol{X}_t + \varepsilon_{t,k} \tag{19}$$

We replace the realizations of the kth factor with the fitted values from this regression:

$$\hat{F}_{t,k} = \alpha_k + \boldsymbol{\beta}_k \boldsymbol{X}_t, \tag{20}$$

and record the resulting Total R^2 when replacing factor kth's realizations with either macroeconomic information or information from the nine benchmark factors. Denote the resulting Total R^2 as R^2_X. Finally, in Figure 6, we show the reduction in the model's Total R^2 relative to setting the realizations of the kth factor to zero:

$$\text{Relative Reduction} = \frac{R_X^2 - R^2}{R_{\text{zero}}^2 - R^2}. \quad (21)$$

A value of 1 indicates that replacing the factor's realizations with its projections on macroeconomic information/benchmark factors produces a Total R^2 as low as that achieved by simply setting its realizations to zero. A value of 0 instead indicates that replacing the factor's realizations works well and produces no loss in explanatory power. This exercise allows us to quantify the relative importance of the information embedded in macroeconomic indicators and benchmark factors for the five latent and shared factors.

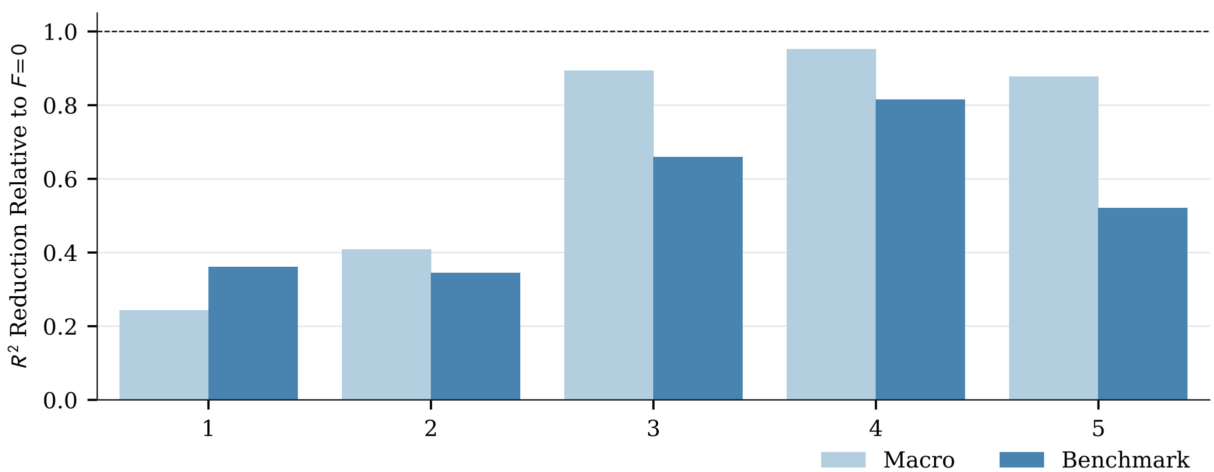

Figure 6: Replacing Latent Factors with Macroeconomic Indicators or Benchmark Factors

The figure shows the relative reduction in the Total R^2 (Eq. (21)) when replacing latent factor realizations with fitted values from regressing it on macroeconomic indicators or benchmark factors extracted from bonds, options, and stocks.

Figure 6 shows that replacing the latent factors with their fitted values always reduces the model's ability to explain returns of bonds, options, and stocks (all Relative Reduction values are greater than zero). This again highlights that the joint IPCA factors optimally describe the risk-return trade-off across the three asset classes and pick up on important variation unexplained by macroeconomic information and information extracted from the asset classes individually. The figure also shows that the first and second factor, which explains most variation of bond, option, and stock returns, can be replicated reasonably well by macroeconomic information and by benchmark factors, with Relative Reduction values between 0.25 and 0.40. This is despite the fact that benchmark factors explain a larger fraction of the factors' return variation (58% and 80% in Table 13 versus 23% and 53% in Table 11), and highlights the importance of the factor replacement exercise to interpret the latent factors. The three remaining factors are only poorly replicated using macroeconomic information or benchmark factors, with Relative Reduction values far exceeding 50%. The latent factors convey information about return variation across bonds, stocks, and options not picked up by the benchmark factors nor the macroeconomic indicators.

8 Conclusion

We propose a factor model to jointly describe the risk-return tradeoff for bonds, options, and stocks. Just five shared factors are able to explain between 12% and 30% of the return variation of bonds, options, and stocks, and describe the conditional and unconditional pricing of each asset class well. The resulting tangency portfolio exploits important diversification benefits enjoyed when simultaneously modeling the risk-return tradeoff for the three asset classes, and achieves an in-sample (out-of-sample) Sharpe ratio of 6.9 (6.4).

The parsimonious factor structure of joint IPCA better explains average returns across asset classes. Of 219 CMPs that have a significant average return over our sample period between August of 2002 and August of 2022, our five-factor joint IPCA model leaves only eleven unexplained. In contrast, a five-factor bond-only IPCA model leaves 193 unexplained. Option- and stock-only IPCA models leave 34 and 173 unexplained, respectively, and even a model, which combines latent factors extracted individually from the three asset classes fails to explain the returns of 31 CMPs. We also compare joint IPCA with prominent benchmark factor models put forth in the literature for the three asset classes. Combining the bond market factor, two option, and six stock factors fails to explain the return patterns of 182

CMPs, lagging far behind the explanatory power of joint IPCA.

We investigate patterns of commonality and find a high degree of integration among bonds, options, and stocks. Interestingly, we also find a high degree of integration between bonds and options, lending empirical credence to the idea of Merton (1974) structural credit risk model that bonds (and stocks) are options on a firm's assets and thus share many of the properties of equity options. While most research has thus far focused on the integration of bond and stock markets (see Du, Elkamhi, and Ericsson, 2019, as an example), our results call for the additional consideration of options and how the trading activity in equity options relates not only to the underlying stock but also to corporate bonds of the same firm. Doshi, Ericsson, Fournier, and Seo (2022) is a first step in this direction at the index level.

References

An, Byeong-Je, Andrew Ang, Turan G. Bali, and Nusret Cakici, 2014, The joint cross section of stocks and options, *Journal of Finance* 69, 2279–2337.

Bali, Turan G, Heiner Beckmeyer, Mathis Moerke, and Florian Weigert, 2023, Option return predictability with machine learning and big data, *Review of Financial Studies* 36, 3548–3602.

Bali, Turan G, Nusret Cakici, and Robert F Whitelaw, 2011, Maxing out: Stocks as lotteries and the cross-section of expected returns, *Journal of Financial Economics* 99, 427–446.

Bali, Turan G., Amit Goyal, Dashan Huang, Fuwei Jiang, and Quan Wen, 2022, Predicting corporate bond returns: Merton meets machine learning, *Georgetown McDonough School of Business Research Paper (3686164)* pp. 20–110.

Bali, Turan G., and Armen Hovakimian, 2009, Volatility spreads and expected stock returns, *Management Science* 55, 1797–1812.

Bates, David S., 1996, Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options, *Review of Financial Studies* 9, 69–107.

Beckmeyer, Heiner, Ilias Filippou, and Guofu Zhou, 2023, A New Option Momentum: Compensation for Risk, *Available at SSRN*.

Beckmeyer, Heiner, and Timo Wiedemann, 2023, Recovering missing firm characteristics with attention-based machine learning, *Available at SSRN 4003455*.

Bessembinder, Hendrik, Chester Spatt, and Kumar Venkataraman, 2020, A survey of the microstructure of fixed-income markets, *Journal of Financial and Quantitative Analysis* 55, 1–45.

Bryzgalova, Svetlana, Sven Lerner, Martin Lettau, and Markus Pelger, 2022, Missing financial data, *Available at SSRN 4106794*.

Cao, Jie, Amit Goyal, Xiao Xiao, and Xintong Zhan, 2023, Implied volatility changes and corporate bond returns, *Management Science* 69, 1375–1397.

Cao, Jie, and Bing Han, 2013, Cross section of option returns and idiosyncratic stock volatility, *Journal of Financial Economics* 108, 231–249.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Chen, Zhongtian, Nikolai L Roussanov, Xiaoliang Wang, and Dongchen Zou, 2024, Common Risk Factors in the Returns on Stocks, Bonds (and Options), Redux, Working paper.

Choi, Jaewon, Yesol Huh, and Sean Seunghun Shin, 2021, Customer liquidity provision: Implications for corporate bond transaction costs, *Available at SSRN 2848344*.

Choi, Jaewon, and Yongjun Kim, 2018, Anomalies and market (dis) integration, *Journal of Monetary Economics* 100, 16–34.

Chordia, Tarun, Amit Goyal, Yoshio Nozawa, Avanidhar Subrahmanyam, and Qing Tong, 2017, Are capital market anomalies common to equity and corporate bond markets? An empirical investigation, *Journal of Financial and Quantitative Analysis* 52, 1301–1342.

Cochrane, John, 2009, *Asset pricing: Revised edition*. (Princeton university press).

Collin-Dufresne, Pierre, Benjamin Junge, and Anders B. Trolle, 2022, How integrated are credit and equity markets? Evidence from index options, *Swiss Finance Institute Research Paper*.

Coval, Joshua D., and Tyler Shumway, 2001, Expected option returns, *Journal of Finance* 56, 983–1009.

Cremers, Martijn, and David Weinbaum, 2010, Deviations from put-call parity and stock return predictability, *Journal of Financial and Quantitative Analysis* 45, 335–367.

Culp, Christopher L., Yoshio Nozawa, and Pietro Veronesi, 2018, Option-based credit spreads, *American Economic Review* 108, 454–488.

Dick-Nielsen, Jens, Peter Feldhütter, Lasse Heje Pedersen, and Christian Stolborg, 2023, Corporate bond factors: Replication failures and a new framework, *Available at SSRN*.

Dickerson, Alexander, Philippe Mueller, and Cesare Robotti, 2023, Priced risk in corporate bonds, *Journal of Financial Economics* 150, 103707.

Doshi, Hitesh, Jan Ericsson, Mathieu Fournier, and Sang Byung Seo, 2022, Asset Variance Risk and Compound Option Prices, *Available at SSRN 3885357*.

Du, Du, Redouane Elkamhi, and Jan Ericsson, 2019, Time-Varying Asset Volatility and the Credit Spread Puzzle, *Journal of Finance* 74, 1841–1885.

Duarte, Jefferson, Christopher S Jones, Haitao Mo, and Mehdi Khorram, 2023, Too Good to Be True: Look-ahead Bias in Empirical Option Research, Working paper.

Easley, David, Maureen O'Hara, and P.S. Srinivas, 1998, Option volume and stock prices: Evidence on where informed traders trade, *Journal of Finance* 53, 431–465.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F., and Kenneth R. French, 2018, Choosing factors, *Journal of Financial Economics* 128, 234–252.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Frazzini, Andrea, Ronen Israel, and Tobias J. Moskowitz, 2018, Trading costs, *Available at SSRN 3229719*.

Freyberger, Joachim, Björn Höppner, Andreas Neuhierl, and Michael Weber, 2022, Missing data in asset pricing panels, Working paper, National Bureau of Economic Research.

Gârleanu, Nicolae, Lasse Heje Pedersen, and Allen M. Poteshman, 2008, Demand-based option pricing, *Review of Financial Studies* 22, 4259–4299.

Geske, Robert, 1979, The valuation of compound options, *Journal of Financial Economics* 7, 63–81.

Goyal, Amit, and Alessio Saretto, 2009, Cross-section of option returns and volatility, *Journal of Financial Economics* 94, 310–326.

Goyal, Amit, and Alessio Saretto, 2022, Are Equity Option Returns Abnormal? IPCA Says No, *IPCA Says No (August 19, 2022)*.

Gu, Shihao, Bryan T. Kelly, and Dacheng Xiu, 2020, Empirical asset pricing via machine learning, *Review of Financial Studies* 33, 2223–2273.

He, Zhiguo, Bryan T. Kelly, and Asaf Manela, 2017, Intermediary asset pricing: New evidence from many asset classes, *Journal of Financial Economics* 126, 1–35.

Heston, Steven L., 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, *Review of Financial Studies* 6, 327–343.

Heston, Steven L., Christopher S. Jones, Mehdi Khorram, Shuaiqi Li, and Haitao Mo, 2023, Option Momentum, *Journal of Finance, Forthcoming* 78, 3141–3192.

Hiraki, Kazuhiro, and George Skiadopoulos, 2020, The Contribution of Frictions to Expected Returns, Working paper.

Horenstein, Alex R., Aurelio Vasquez, and Xiao Xiao, 2022, Common factors in equity option returns, *Available at SSRN 3290363*.

Hou, Kewei, Chen Xue, and Lu Zhang, 2015, Digesting anomalies: An investment approach, *Review of Financial Studies* 28, 650–705.

Jensen, Theis Ingerslev, Bryan T. Kelly, and Lasse Heje Pedersen, 2023, Is there a replication crisis in finance? *Journal of Finance* 78, 2465–2518.

Johnson, Travis L., and Eric C. So, 2012, The option to stock volume ratio and future returns, *Journal of Financial Economics* 106, 262–286.

Jurado, Kyle, Sydney C Ludvigson, and Serena Ng, 2015, Measuring uncertainty, *American Economic Review* 105, 1177–1216.

Kelly, Bryan, Diogo Palhares, and Seth Pruitt, 2023, Modeling corporate bond returns, *The Journal of Finance* 78, 1967–2008.

Kelly, Bryan T., Seth Pruitt, and Yinan Su, 2019, Characteristics are covariances: A unified model of risk and return, *Journal of Financial Economics* 134, 501–524.

Koijen, Ralph S.J., Hanno Lustig, and Stijn Van Nieuwerburgh, 2017, The cross-section and time series of stock and bond returns, *Journal of Monetary Economics* 88, 50–69.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2020, Shrinking the cross-section, *Journal of Financial Economics* 135, 271–292.

Leland, Hayne E., 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.

Liu, Yukun, Aleh Tsyvinski, and Xi Wu, 2022, Common risk factors in cryptocurrency, *Journal of Finance* 77, 1133–1177.

Lo, Andrew W., 2002, The statistics of Sharpe ratios, *Financial Analysts Journal* 58, 36–52.

Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan, 2011, Common risk factors in currency markets, *Review of Financial Studies* 24, 3731–3777.

Maio, Paulo, and Pedro Santa-Clara, 2012, Multifactor models and their consistency with the ICAPM, *Journal of Financial Economics* 106, 586–613.

Martin, Ian, 2017, What is the Expected Return on the Market? *Quarterly Journal of Economics* 132, 367–433.

Merton, Robert C, 1973, An intertemporal capital asset pricing model, *Econometrica: Journal of the Econometric Society* pp. 867–887.

Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–470.

Muravyev, Dmitriy, and Neil D. Pearson, 2020, Options trading costs are lower than you think, *Review of Financial Studies* 33, 4973–5014.

Muravyev, Dmitriy, Neil D. Pearson, and Joshua Matthew Pollet, 2022, Anomalies and Their Short-Sale Costs, *Available at SSRN 4266059*.

Neuhierl, Andreas, Xiaoxiao Tang, Rasmus Tangsgaard Varneskov, and Guofu Zhou, 2023, Option characteristics as cross-sectional predictors, *Available at SSRN 3795486*.

Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703–708.

Ofek, Eli, Matthew Richardson, and Robert F. Whitelaw, 2004, Limited arbitrage and short sales restrictions: Evidence from the options markets, *Journal of Financial Economics* 74, 305–342.

Sandulescu, Mirela, 2023, How Integrated Are Corporate Bond and Stock Markets? Working paper, Swiss Finance Institute.

Stambaugh, Robert F, and Yu Yuan, 2017, Mispricing factors, *The review of financial studies* 30, 1270–1315.

Szymanowska, Marta, Frans de Roon, Theo Nijman, and Rob van den Goorbergh, 2014, An anatomy of commodity futures risk premia, *Journal of Finance* 69, 453–482.

Xing, Yuhang, Xiaoyan Zhang, and Rui Zhao, 2010, What does the individual option volatility smirk tell us about future equity returns? *Journal of Financial and Quantitative Analysis* 45, 641–662.

A Firm Characteristics

The following table shows the whole set of 264 firm-level characteristics extracted from the firm's bonds, options, and stock. Alongside the characteristic's name, we provide a short description, its original source in the literature, whether it was extracted from information of the firm's bond, option, or stock. We also provide the reason for dropping the characteristic in the estimation of joint IPCA in Eq. (3). For bond-level characteristics, a name in capital letters indicates that we adjust for microstructure noise in bond transaction prices (Dickerson, Mueller, and Robotti, 2023).

Characteristic	Instrument	Description	Source	Dropped?
B_BONDPRC	Bonds	Price	Dickerson, Robotti, Robotti (2023)	insignificant
B_BOND_VALUE	Bonds	Value	Dickerson, Robotti, Robotti (2023)	correlation
B_BOND_YIELD	Bonds	Yield	Dickerson, Robotti, Robotti (2023)	correlation
B_CONVEXITY	Bonds	Convexity	Dickerson, Robotti, Robotti (2023)	correlation
B_CS	Bonds	Credit spread	Dickerson, Robotti, Robotti (2023)	insignificant
B_DURATION	Bonds	Duration	Dickerson, Robotti, Robotti (2023)	
B_ILLIQ	Bonds	Illiquidity	Dickerson, Robotti, Robotti (2023)	insignificant
B_STREV	Bonds	Short-term reversal	Dickerson, Robotti, Robotti (2023)	
B_beta_CFNAI	Bonds	Beta to Chicago Fed National Activity Index (controls for bond market)		insignificant
B_beta_ICR	Bonds	Beta to Intermediary Capital Ratio of He, Kelly, Manela (controls for bond market)		insignificant
B_beta_MKTB	Bonds	Beta to value-weighted bond market		insignificant
B_beta_dfy	Bonds	Beta to default spread of Welch, Goyal (controls for bond market)		insignificant
B_beta_tms	Bonds	Beta to term spread of Welch, Goyal (controls for bond market)		insignificant
B_beta_unc	Bonds	Beta to macroeconomic uncertainty of Jurado, Ludvigson, Ng (controls for bond market)		insignificant
B_beta_vix	Bonds	Beta to Cboe's VIX (controls for bond market)		insignificant
B_bond_age	Bonds	Bond's age		availability
B_bond_amount_out	Bonds	Amount outstanding	Dickerson, Robotti, Robotti (2023)	insignificant
B_es90	Bonds	Expected shortfall (90th percentile)		correlation
B_es95	Bonds	Expected shortfall (95th percentile)		insignificant
B_kurt12	Bonds	Return kurtosis measured over past 12 months		insignificant
B_kurt24	Bonds	Return kurtosis measured over past 24 months		insignificant
B_kurt36	Bonds	Return kurtosis measured over past 36 months		insignificant
B_ltrev36_12	Bonds	Long-term reversal (36-12)		insignificant
B_ltrev48_12	Bonds	Long-term reversal (48-12)	Dickerson, Robotti, Robotti (2023)	insignificant
B_mom12_1	Bonds	12-1 momentum		insignificant
B_mom3_1	Bonds	3-1 momentum		insignificant
B_mom6_1	Bonds	6-1 momentum	Dickerson, Robotti, Robotti (2023)	
B_mom9_1	Bonds	9-1 momentum		insignificant
B_n_trades_month	Bonds	Number of trades in previous month	Dickerson, Robotti, Robotti (2023)	insignificant
B_rating	Bonds	Rating	Dickerson, Robotti, Robotti (2023)	
B_skew12	Bonds	Return skewness measured over past 12 months		insignificant
B_skew24	Bonds	Return skewness measured over past 24 months		insignificant
B_skew36	Bonds	Return skewness measured over past 36 months		insignificant
B_std12	Bonds	Return standard deviation measured over past 12 months		insignificant
B_std24	Bonds	Return standard deviation measured over past 24 months		insignificant
B_std36	Bonds	Return standard deviation measured over past 36 months		insignificant
B_var90	Bonds	Value at risk (90th percentile)		insignificant
B_var95	Bonds	Value at risk (95th percentile)		insignificant
O_B_amihud_roll252D	bucket	Amihud illiquidity measure for options in bucket, rolling over past 252 days	Amihud (2002)	correlation
O_B_hkurt_roll252D	bucket	Historical kurtosis, rolling over past 252 days		correlation
O_B_hskew_roll252D	bucket	Historical skewness, rolling over past 252 days		
O_B_hstd_roll252D	bucket	Historical standard deviation, rolling over past 252 days		
O_B_illiq_roll252D	bucket	Autocorrelation of returns, measured over past 252 days	Bao, Pan, and Wang (2011)	correlation
O_B_ivrank_roll252D	bucket	IV rank measured over past 252 days		
O_B_mom_roll252D	bucket	Momentum, rolling over past 252 days	Heston, Jones, Khorram, Li, and Mo (2023)	
O_B_oi_share	bucket	Open interest share of options in bucket		
O_B_pfht_roll252D	bucket	Modified illiquidity measure based on zero returns, measured over past 252 days	Fong, Holden, and Trzcinka (2017)	correlation
O_B_pifht_roll252D	bucket	Extended FHT measure based on zero returns, measured over past 252 days		correlation
O_B_piroll_roll252D	bucket	Extended Roll's measure, measured over past 252 days	Goyenko, Holden, and Trzcinka (2009)	
O_B_pzeros_roll252D	bucket	Zero return days, measured over past 252 days	Lesmond, Ogden, and Trzcinka (1999)	insignificant
O_B_roll_roll252D	bucket	Roll's measure, measured over past 252 days	Roll (1974)	insignificant
O_B_stdamihud_roll252D	bucket	Standard deviation of Amihud illiquidity measure for options in bucket, rolling over past 252 days		
O_B_vol_share	bucket	Dollar volume share of options in bucket		
O_C_delta	contract	OM's delta		
O_C_embedlev	contract	Embedded leverage	Karakaya (2014)	
O_C_gamma	contract	OM's gamma multiplied by underlying close		correlation
O_C_impl_volatility	contract	IV		correlation
O_C_mid	contract	Option's mid price		insignificant
O_C_open_interest	contract	Open interest (dollar)		
O_C_option_age	contract	Option age in days		insignificant
O_C_spread	contract	Option's bid ask spread (relative)		

Continued on Next Page

Characteristic	Instrument	Description	Source	Dropped?
O_C_theta	contract	OM's theta divided by underlying close		
O_C_time_value	contract	Option's time value (mid - exercise_value)		insignificant
O_C_vega	contract	OM's vega divided by underlying close		insignificant
O_S_ailliq	underlying	Absolute illiquidity	Cao and Wei (2010)	
O_S_atm_civpiv	underlying	Atm call minus put IV measured from IV surface		
O_S_atm_dcivpiv	underlying	Change in atm call minus put IV measured from IV surface	An, Ang, Bali, and Cakici (2014)	
O_S_civpiv	underlying	Near-the-money call minus put IV	Bali and Hovakimian (2009)	
O_S_dciv	underlying	Change in atm call IV	An, Ang, Bali, and Cakici (2014)	
O_S_demand_pressure	underlying	Demand pressure	Cao, Vasquez, Xiao, and Zhan (2019)	
O_S_demand_roll252D	underlying	Option volume measured over past 252 days vs. market capitalization		
O_S_dos_roll252D	underlying	Dollar option vs. dollar stock volume, measured over past 252 days		
O_S_dpiv	underlying	Change in atm put IV	An, Ang, Bali, and Cakici (2014)	
O_S_fric	underlying	Contribution of market frictions to expected returns	Hiraki and Skiadopoulos (2020)	
O_S_ivarud_182	underlying	Up vs. down IV ratio with 182-day expiration	Huang and Li (2019)	correlation
O_S_ivarud_30	underlying	Up vs. down IV ratio with 30-day expiration	Huang and Li (2019)	
O_S_ivd	underlying	IV duration	Schlag, Thimme, and Weber (2020)	
O_S_ivrv_roll252D	underlying	IV minus RV, realized measured over past 252 days	Bali and Hovakimian (2009)	
O_S_ivslope	underlying	IV slope	Vasquez (2017)	insignificant
O_S_ivud_182	underlying	Up minus down IV with 182-day expiration		
O_S_ivud_30	underlying	Up minus down IV with 30-day expiration		
O_S_ivvol_roll252D	underlying	Volatility of IV, measured over past 252 days	Baltussen, van Bekkum, and van der Grient (2018)	insignificant
O_S_modos_roll252D	underlying	Modified stock vs. option volume, measured over past 252 days	Johnson and So (2012)	
O_S_nopt	underlying	Number of options trading on the underlying		
O_S_pba	underlying	Volume-weighted bid-ask spread	Cao and Wei (2010)	
O_S_pcratio_roll252D	underlying	Put vs. call volume, measured over past 252 days	Blau, Nguyen, and Whitby (2014)	
O_S_pilliq	underlying	Relative illiquidity	Cao and Wei (2010)	
O_S_rnk_182	underlying	Risk neutral kurtosis with 182-day expiration		
O_S_rnk_30	underlying	Risk neutral kurtosis with 30-day expiration		
O_S_rns_182	underlying	Risk neutral skewness with 182-day expiration	Borochin, Chang, and Wu (2020)	
O_S_rns_30	underlying	Risk neutral skewness with 30-day expiration	Borochin, Chang, and Wu (2020)	
O_S_rnv_182	underlying	Risk neutral volatility with 182-day expiration		correlation
O_S_rnv_30	underlying	Risk neutral volatility with 30-day expiration		
O_S_shrtfee	underlying	Implied short-selling fee	Muravyev, Pearson, and Pollet (2021)	
O_S_skewiv	underlying	IV skew	Xing, Zhang, and Zhoa (2010)	
O_S_tlm_182	underlying	Option-implied tail loss with 182-day expiration	Vilkov and Xiao (2012)	availability
O_S_tlm_30	underlying	Option-implied tail loss with 30-day expiration	Vilkov and Xiao (2012)	
O_S_toi	underlying	Total open interest		
O_S_tv	underlying	Total volume		insignificant
O_S_vs_change	underlying	Change in weighted put-call spread	Cremers and Weinbaum (2010)	
O_S_vs_level	underlying	Weighted put-call spread	Cremers and Weinbaum (2010)	
S_age	Stock	Firm age	Jiang Lee and Zhang (2005)	
S_aliq_at	Stock	Liquidity of book assets	Ortiz-Molina and Phillips (2014)	
S_aliq_mat	Stock	Liquidity of market assets	Ortiz-Molina and Phillips (2014)	insignificant
S_ami_126d	Stock	Amihud Measure	Amihud (2002)	correlation
S_at_be	Stock	Book leverage	Fama and French (1992)	
S_at_gr1	Stock	Asset Growth	Cooper Gulen and Schill (2008)	
S_at_me	Stock	Assets-to-market	Fama and French (1992)	
S_at_turnover	Stock	Capital turnover	Haugen and Baker (1996)	
S_be_gr1a	Stock	Change in common equity	Richardson et al. (2005)	
S_be_me	Stock	Book-to-market equity	Rosenberg Reid and Lanstein (1985)	
S_beta_60m	Stock	Market Beta	Fama and MacBeth (1973)	
S_beta_dimson_21d	Stock	Dimson beta	Dimson (1979)	insignificant
S_betabab_1260d	Stock	Frazzini-Pedersen market beta	Frazzini and Pedersen (2014)	insignificant
S_betadown_252d	Stock	Downside beta	Ang Chen and Xing (2006)	insignificant
S_bev_mev	Stock	Book-to-market enterprise value	Penman Richardson and Tuna (2007)	
S_bidaskhl_21d	Stock	The high-low bid-ask spread	Corwin and Schultz (2012)	
S_capex_abn	Stock	Abnormal corporate investment	Titman Wei and Xie (2004)	
S_capx_gr1	Stock	CAPEX growth (1 year)	Xie (2001)	
S_capx_gr2	Stock	CAPEX growth (2 years)	Anderson and Garcia-Feijoo (2006)	
S_capx_gr3	Stock	CAPEX growth (3 years)	Anderson and Garcia-Feijoo (2006)	
S_cash_at	Stock	Cash-to-assets	Palazzo (2012)	

Continued on Next Page

Characteristic	Instrument	Description	Source	Dropped?
S_chcsho_12m	Stock	Net stock issues	Pontiff and Woodgate (2008)	
S_coa_gr1a	Stock	Change in current operating assets	Richardson et al. (2005)	
S_col_gr1a	Stock	Change in current operating liabilities	Richardson et al. (2005)	insignificant
S_cop_at	Stock	Cash-based operating profits-to-book assets		
S_cop_atl1	Stock	Cash-based operating profits-to-lagged book assets	Ball et al. (2016)	
S_corr_1260d	Stock	Market correlation	Assness, Frazzini, Gormsen, Pedersen (2020)	
S_coskew_21d	Stock	Coskewness	Harvey and Siddique (2000)	
S_cowc_gr1a	Stock	Change in current operating working capital	Richardson et al. (2005)	insignificant
S_dbnetis_at	Stock	Net debt issuance	Bradshaw Richardson and Sloan (2006)	
S_debt_gr3	Stock	Growth in book debt (3 years)	Lyandres Sun and Zhang (2008)	
S_debt_me	Stock	Debt-to-market	Bhandari (1988)	
S_dgp_dsale	Stock	Change gross margin minus change sales	Abarbanell and Bushee (1998)	
S_div12m_me	Stock	Dividend yield	Litzenberger and Ramaswamy (1979)	
S_dolvol_126d	Stock	Dollar trading volume	Brennan Chordia and Subrahmanyam (1998)	
S_dolvol_var_126d	Stock	Coefficient of variation for dollar trading volume	Chordia Subrahmanyam and Anshuman (2001)	correlation
S_dsale_dinv	Stock	Change sales minus change Inventory	Abarbanell and Bushee (1998)	
S_dsale_drec	Stock	Change sales minus change receivables	Abarbanell and Bushee (1998)	
S_dsale_dsga	Stock	Change sales minus change SG&A	Abarbanell and Bushee (1998)	insignificant
S_earnings_variability	Stock	Earnings variability	Francis et al. (2004)	
S_ebit_bev	Stock	Return on net operating assets	Soliman (2008)	
S_ebit_sale	Stock	Profit margin	Soliman (2008)	
S_ebitda_mev	Stock	Ebitda-to-market enterprise value	Loughran and Wellman (2011)	
S_emp_gr1	Stock	Hiring rate	Belo Lin and Bazdresch (2014)	
S_eq_dur	Stock	Equity duration	Dechow Sloan and Soliman (2004)	
S_eqnetis_at	Stock	Net equity issuance	Bradshaw Richardson and Sloan (2006)	
S_eqnpo_12m	Stock	Equity net payout	Daniel and Titman (2006)	
S_eqnpo_me	Stock	Net payout yield	Boudoukh et al. (2007)	
S_eqpo_me	Stock	Payout yield	Boudoukh et al. (2007)	
S_f_score	Stock	Pitroski F-score	Piotroski (2000)	
S_fcf_me	Stock	Free cash flow-to-price	Lakonishok Shleifer and Vishny (1994)	
S_fnl_gr1a	Stock	Change in financial liabilities	Richardson et al. (2005)	
S_gp_at	Stock	Gross profits-to-assets	Novy-Marx (2013)	
S_gp_atl1	Stock	Gross profits-to-lagged assets		correlation
S_intrinsic_value	Stock	Intrinsic value-to-market	Frankel and Lee (1998)	
S_inv_gr1	Stock	Inventory growth	Belo and Lin (2011)	
S_inv_gr1a	Stock	Inventory change	Thomas and Zhang (2002)	
S_iskew_capm_21d	Stock	Idiosyncratic skewness from the CAPM		
S_iskew_ff3_21d	Stock	Idiosyncratic skewness from the Fama-French 3-factor model	Bali Engle and Murray (2016)	
S_iskew_hxz4_21d	Stock	Idiosyncratic skewness from the q-factor model		
S_ivol_capm_21d	Stock	Idiosyncratic volatility from the CAPM (21 days)		correlation
S_ivol_capm_252d	Stock	Idiosyncratic volatility from the CAPM (252 days)	Ali Hwang and Trombley (2003)	
S_ivol_ff3_21d	Stock	Idiosyncratic volatility from the Fama-French 3-factor model	Ang et al. (2006)	correlation
S_ivol_hxz4_21d	Stock	Idiosyncratic volatility from the q-factor model		
S_kz_index	Stock	Kaplan-Zingales index	Lamont Polk and Saa-Requejo (2001)	insignificant
S_lnoa_gr1a	Stock	Change in long-term net operating assets	Fairfield Whisenant and Yohn (2003)	
S_lti_gr1a	Stock	Change in long-term investments	Richardson et al. (2005)	insignificant
S_market_equity	Stock	Market Equity	Banz (1981)	
S_mispricing_mgmt	Stock	Mispricing factor: Management	Stambaugh and Yuan (2016)	
S_mispricing_perf	Stock	Mispricing factor: Performance	Stambaugh and Yuan (2016)	
S_ncoa_gr1a	Stock	Change in noncurrent operating assets	Richardson et al. (2005)	
S_ncol_gr1a	Stock	Change in noncurrent operating liabilities	Richardson et al. (2005)	
S_netdebt_me	Stock	Net debt-to-price	Penman Richardson and Tuna (2007)	
S_netis_at	Stock	Net total issuance	Bradshaw Richardson and Sloan (2006)	
S_nfna_gr1a	Stock	Change in net financial assets	Richardson et al. (2005)	
S_ni_ar1	Stock	Earnings persistence	Francis et al. (2004)	
S_ni_be	Stock	Return on equity	Haugen and Baker (1996)	
S_ni_inc8q	Stock	Number of consecutive quarters with earnings increases	Barth Elliott and Finn (1999)	
S_ni_ivol	Stock	Earnings volatility	Francis et al. (2004)	
S_ni_me	Stock	Earnings-to-price	Basu (1983)	
S_niq_at	Stock	Quarterly return on assets	Balakrishnan Bartov and Faurel (2010)	
S_niq_at_chg1	Stock	Change in quarterly return on assets		

Continued on Next Page

Characteristic	Instrument	Description	Source	Dropped?
S_niq_be	Stock	Quarterly return on equity	Hou Xue and Zhang (2015)	
S_niq_be_chg1	Stock	Change in quarterly return on equity		insignificant
S_niq_su	Stock	Standardized earnings surprise	Foster Olsen and Shevlin (1984)	
S_nncoa_gr1a	Stock	Change in net noncurrent operating assets	Richardson et al. (2005)	
S_noa_at	Stock	Net operating assets	Hirshleifer et al. (2004)	
S_noa_gr1a	Stock	Change in net operating assets	Hirshleifer et al. (2004)	
S_o_score	Stock	Ohlson O-score	Dichev (1998)	
S_oaccruals_at	Stock	Operating accruals	Sloan (1996)	
S_oaccruals_ni	Stock	Percent operating accruals	Hafzalla Lundholm and Van Winkle (2011)	
S_ocf_at	Stock	Operating cash flow to assets	Bouchard, Krüger, Landier and Thesmar (2019)	
S_ocf_at_chg1	Stock	Change in operating cash flow to assets	Bouchard, Krüger, Landier and Thesmar (2019)	
S_ocf_me	Stock	Operating cash flow-to-market	Desai Rajgopal and Venkatachalam (2004)	
S_ocfq_saleq_std	Stock	Cash flow volatility	Huang (2009)	
S_op_at	Stock	Operating profits-to-book assets		
S_op_atl1	Stock	Operating profits-to-lagged book assets	Ball et al. (2016)	correlation
S_ope_be	Stock	Operating profits-to-book equity	Fama and French (2015)	
S_ope_bel1	Stock	Operating profits-to-lagged book equity		correlation
S_opex_at	Stock	Operating leverage	Novy-Marx (2011)	correlation
S_pi_nix	Stock	Taxable income-to-book income	Lev and Nissim (2004)	
S_ppeinv_gr1a	Stock	Change PPE and Inventory	Lyandres Sun and Zhang (2008)	
S_prc	Stock	Price per share	Miller and Scholes (1982)	
S_prc_highprc_252d	Stock	Current price to high price over last year	George and Hwang (2004)	
S_qmj	Stock	Quality minus Junk: Composite	Assness, Frazzini and Pedersen (2018)	
S_qmj_growth	Stock	Quality minus Junk: Growth	Assness, Frazzini and Pedersen (2018)	insignificant
S_qmj_prof	Stock	Quality minus Junk: Profitability	Assness, Frazzini and Pedersen (2018)	
S_qmj_safety	Stock	Quality minus Junk: Safety	Assness, Frazzini and Pedersen (2018)	
S_rd5_at	Stock	R&D capital-to-book assets	Li (2011)	availability
S_rd_me	Stock	R&D-to-market	Chan Lakonishok and Sougiannis (2001)	
S_rd_sale	Stock	R&D-to-sales	Chan Lakonishok and Sougiannis (2001)	
S_resff3_12_1	Stock	Residual momentum t-12 to t-1	Blitz Huij and Martens (2011)	insignificant
S_resff3_6_1	Stock	Residual momentum t-6 to t-1	Blitz Huij and Martens (2011)	
S_ret_12_1	Stock	Price momentum t-12 to t-1	Fama and French (1996)	insignificant
S_ret_12_7	Stock	Price momentum t-12 to t-7	Novy-Marx (2012)	insignificant
S_ret_1_0	Stock	Short-term reversal	Jegadeesh (1990)	
S_ret_3_1	Stock	Price momentum t-3 to t-1	Jegedeesh and Titman (1993)	
S_ret_60_12	Stock	Long-term reversal	De Bondt and Thaler (1985)	
S_ret_6_1	Stock	Price momentum t-6 to t-1	Jegadeesh and Titman (1993)	insignificant
S_ret_9_1	Stock	Price momentum t-9 to t-1	Jegedeesh and Titman (1993)	insignificant
S_rmax1_21d	Stock	Maximum daily return	Bali Cakici and Whitelaw (2011)	
S_rmax5_21d	Stock	Highest 5 days of return	Bali, Brown, Murray and Tang (2017)	
S_rmax5_rvol_21d	Stock	Highest 5 days of return scaled by volatility	Assness, Frazzini, Gormsen, Pedersen (2020)	
S_rskew_21d	Stock	Total skewness	Bali Engle and Murray (2016)	
S_rvol_21d	Stock	Return volatility	Ang et al. (2006)	
S_sale_bev	Stock	Assets turnover	Soliman (2008)	
S_sale_emp_gr1	Stock	Labor force efficiency	Abarbanell and Bushee (1998)	insignificant
S_sale_gr1	Stock	Sales Growth (1 year)	Lakonishok Shleifer and Vishny (1994)	
S_sale_gr3	Stock	Sales Growth (3 years)	Lakonishok Shleifer and Vishny (1994)	insignificant
S_sale_me	Stock	Sales-to-market	Barbee Mukherji and Raines (1996)	
S_saleq_gr1	Stock	Sales growth (1 quarter)		insignificant
S_saleq_su	Stock	Standardized Revenue surprise	Jegadeesh and Livnat (2006)	insignificant
S_seas_11_15an	Stock	Years 11-15 lagged returns, annual	Heston and Sadka (2008)	availability
S_seas_11_15na	Stock	Years 11-15 lagged returns, nonannual	Heston and Sadka (2008)	availability
S_seas_16_20an	Stock	Years 16-20 lagged returns, annual	Heston and Sadka (2008)	availability
S_seas_16_20na	Stock	Years 16-20 lagged returns, nonannual	Heston and Sadka (2008)	availability
S_seas_1_1an	Stock	Year 1-lagged return, annual	Heston and Sadka (2008)	insignificant
S_seas_1_1na	Stock	Year 1-lagged return, nonannual	Heston and Sadka (2008)	
S_seas_2_5an	Stock	Years 2-5 lagged returns, annual	Heston and Sadka (2008)	
S_seas_2_5na	Stock	Years 2-5 lagged returns, nonannual	Heston and Sadka (2008)	insignificant
S_seas_6_10an	Stock	Years 6-10 lagged returns, annual	Heston and Sadka (2008)	
S_seas_6_10na	Stock	Years 6-10 lagged returns, nonannual	Heston and Sadka (2008)	
S_sti_gr1a	Stock	Change in short-term investments	Richardson et al. (2005)	

Continued on Next Page

Characteristic	Instrument	Description	Source	Dropped?
S_taccruals_at	Stock	Total accruals	Richardson et al. (2005)	
S_taccruals_ni	Stock	Percent total accruals	Hafzalla Lundholm and Van Winkle (2011)	
S_tangibility	Stock	Asset tangibility	Hahn and Lee (2009)	
S_tax_gr1a	Stock	Tax expense surprise	Thomas and Zhang (2011)	insignificant
S_turnover_126d	Stock	Share turnover	Datar Naik and Radcliffe (1998)	correlation
S_turnover_var_126d	Stock	Coefficient of variation for share turnover	Chordia Subrahmanyam and Anshuman (2001)	
S_z_score	Stock	Altman Z-score	Dichev (1998)	
S_zero_trades_126d	Stock	Number of zero trades with turnover as tiebreaker (6 months)	Liu (2006)	
S_zero_trades_21d	Stock	Number of zero trades with turnover as tiebreaker (1 month)	Liu (2006)	
S_zero_trades_252d	Stock	Number of zero trades with turnover as tiebreaker (12 months)	Liu (2006)	correlation

Done.