Optimal Portfolio Choice with Absorbing State Markov Chains

Andrew Ang, Henry Shen, Jeff Shen, and Rui Zhao

This Version: June 28, 2023

Abstract

We seek to develop a model of optimal asset allocation with a market that has the potential to decouple. There are three Markov regimes: a regime where the market remains fully investable, a second regime where the market may become potentially decouple, and a third regime where the market becomes decoupled and investors lose all capital. The regimes are persistent, which induces time-varying hedging demands for an investor rebalancing regularly. The investor wishes to hold the potentially decoupled market as it can provide a source of returns that can be partially liquidated to provide intermediate consumption. With the framework, we compute certainty equivalents of foregoing investment in the potentially decoupling market and investigate a range of comparative statics including varying the probability of decoupling.

1

1. Introduction

Markets sometimes decouple from the world finance system, and during those periods of segmentation, investors can lose partial or all access to their capital. In March 2022, Russia was declared "uninvestable" by major index providers after Russia's links with global markets were severed, its foreign reserves frozen, and major Western countries imposed sanctions as a consequence for Russia invading Ukraine in February 2022. In particular, FTSE Russell declared Russia an "unclassified market" and removed it from its indexes on March 7, 2022, and MSCI classified Russia as a "standalone market" and removed Russia from the MSCI Emerging Markets Index (EM) on March 9, 2022.[1] When the Russian market decoupled, investors effectively lost access to their Russian investments at close to zero value.

In this paper, we examine how the possibility of a market becoming decoupled can affect optimal asset allocation. We model the event of becoming decoupled as an absorbing state in a Markov chain—conditional on the market changing status to decoupled, we assume investors lose their capital in the decoupled market. Before that event happens, investors may optimally allocate to the market that has the possibility of decoupling because it currently provides a potential return stream or a source of diversification. In our model, an investor receives utility over a consumption stream, which is paid out of the investment portfolio similar to the spending rule of an endowment or a retired person. Investors can partially fund their consumption by liquidating positions in that market so long as it does not transition to becoming decoupled.

The model is constructed as follows. For simplicity, we work with two assets: a market that may decouple, which we take in hypothetical exercises to represent an emerging market country index return, and we take MSCI Emerging Market Index as the second asset. The investor rebalances at the monthly frequency and consumes a constant fraction of their portfolio each month. We vary the investor's horizon but concentrate on 1-, 5-, and 10-year horizons. We specify three Markov states (or regimes): a "Fully Investable" state where there

---

is no probability of the market decoupling over the next month, a "Potentially Decouple" state where the market can transition to becoming decoupled or transition back to the first Fully Investable state, and the third "Decoupled" regime where the market experiences a -100% return and only the Emerging Market index is available as an investment going forward. The Decoupled regime is modeled as an absorbing state of the Markov chain.

The probability of a market decoupling is constant transitioning from the second Potentially Decouple regime to the third Decoupled regime, but the first Fully Investable and second Potentially Decouple regimes are persistent, following Hamilton (1989). This introduces time-varying transition probabilities and predictability—conditional on being in the Fully Investable regime, for example, the system is more likely to remain in the Fully Investable regime next period than transition to the Potentially Decouple regime, and in the Fully Investable regime, there is no probability of becoming decoupled over the next period. In the Potentially Decouple regime, we are more likely to remain in that regime or transition back to the Fully Investable regime as the probability of decoupling over the next month is small. As Merton (1971) and others note, this time-varying predictability induces hedging demands, where there is a difference between the investor's one-period ahead and long-horizon optimal holdings. In the hypothetical calibrations, we show that hedging demands can be positive with a 5% probability of becoming decoupled each year.

We compute certainty equivalents (or willingness-to-pay) of *not* being able to invest in a market that may transition to being decoupled, following Kandel and Stambaugh (1996), Campbell and Viceira (1999), and many others. The certainty equivalent is the sure amount of wealth a risk-averse investor must receive to compensate them for not being able to invest in the potentially decoupling market. Equivalently, this is the maximum amount of money an investor would be willing to pay to invest in the potentially decoupling market. Put another way, not investing in the potentially decoupling market is a portfolio restriction on the investor's opportunity set. Imposing any constraint on the agent's asset allocation problem is costly, and the certainty equivalent is the sure amount of money to compensate the investor for being subject to the restriction of not investing in the potentially decoupling market.

We find the certainty equivalents can be large. For one calibrated model that starts with a myopic 35% target allocation to a market which may decouple, the one-year certainty equivalent is 0.23 cents per dollar of wealth rising to a very large 5.49 cents for an investor with a 10-year horizon for an investor starting in the Fully Investable regime. With no leverage limits imposed, the certainty equivalent cost falls to 0.24 cents at the 10-year horizon, which is still significant relative to the typical expected return of a multi-asset portfolio (see, for example, He et al., 2022).

The model allows us to compute certainty equivalents as we vary probabilities of decoupling, spending rules, and leverage. Certainty equivalents are a non-linear function of the probability of becoming decoupled—in fact, at the one-year horizon, certainty equivalents can *increase* as the probability of decoupling increases because of hedging demands, consistent with the optimal holdings of the potentially decoupling asset increasing with horizon. Even with large probabilities of becoming decoupled of 0.2 to 0.3 per year, the willingness-to-pay to invest in the potentially decoupling market can exceed three cents per dollar of wealth for the 10-year horizon. As the consumption rates increase, willingness-to-pay decreases as there is effectively less time for the decoupled market to contribute to portfolio returns, but the willingness-to-pay is still approximately two cents per dollar of wealth for a 10% annual payout rate.

Our framework falls into a large optimal asset allocation literature beginning with Samuelson (1969). A key finding in the literature, which is also exhibited in our setting, is that time-varying expected returns can imply differences between one-period (or myopic) portfolio weights and long-horizon portfolio weights in settings where an investor can optimally rebalance each period (see, for example, Kandel and Stambaugh, 1996; Campbell and Viceira, 1999; Campbell, Chan, and Viceira, 2003). Our model follows Ang and Bekaert (2002) who were the first to examine portfolio choice with persistent Markov regimes, but do not specify one regime where a market decouples as an absorbing state.[2] This literature

---

[2] See also Ang and Bekaert (2004), Guidolin and Timmermann (2008), Tu (2010), Das et al. (2022), and Lewin and Campani (2022). Related studies are Liu, Longstaff, and Pan (2003), and Das and Uppal (2004) who examine the implications of jump processes for optimal asset allocation, except our jump downward is irreversible and set at -100%.

does not consider the specific setting of our calibrations to MSCI Russia or MSCI China index returns with the MSCI EM index, and does not model decoupling as an absorbing Markov chain. Another literature following Brown, Goetzmann, and Ross (1995) documents that securities markets can and do disappear and this affects estimates of risk and return, and Bekaert and Harvey (1995) show that countries undergo periods of integration and segmentation with international markets. These papers do not focus on implications for optimal asset allocation for countries that may potentially decouple.

The rest of this paper is organized as follows. We focus the methodology and calibrations to a hypothetical example of a potentially decoupling market representing an emerging market and EM MSCI index returns. Section 2 starts with estimating a two-state Markov regime-switching model. We introduce the probability of decoupling in Section 3 and describe the optimal asset allocation problem in Section 4. We report empirical results in in Section 5. In Section 6, we repeat the analysis for another hypothetical emerging market example and EM index returns. Section 7 concludes.

2. Two State Markov Model

We describe summary statistics of an emerging market country and the MSCI EM index and estimate a two-state Markov model—which represent the Fully Investable and Potentially Decouple regimes where the probability of becoming decoupled is zero. This sets the stage for Section 3, where we introduce a non-zero probability of decoupling.

2.1 Data

Exhibit 1 graphs cumulated returns of the MSCI China Index returns and the MSCI EM Index from January 1993 to February 2023. Over this period, the mean arithmetic return of China is a low 3.6% per year compared to 6.2% for the EM index. China returns have also been more volatile, at 32.1% per year, relative to EM returns at 21.9%. China's spectacular economic growth taking it from below 10% of US GDP in 1980 to overtaking the US and

becoming the largest economy in the world in 2014 measured in purchasing power parity terms according to the World Bank,[3] has not been reflected in large China stock market gains.

Exhibit 1: Cumulated Returns of MSCI China and MSCI Emerging Markets Indexes
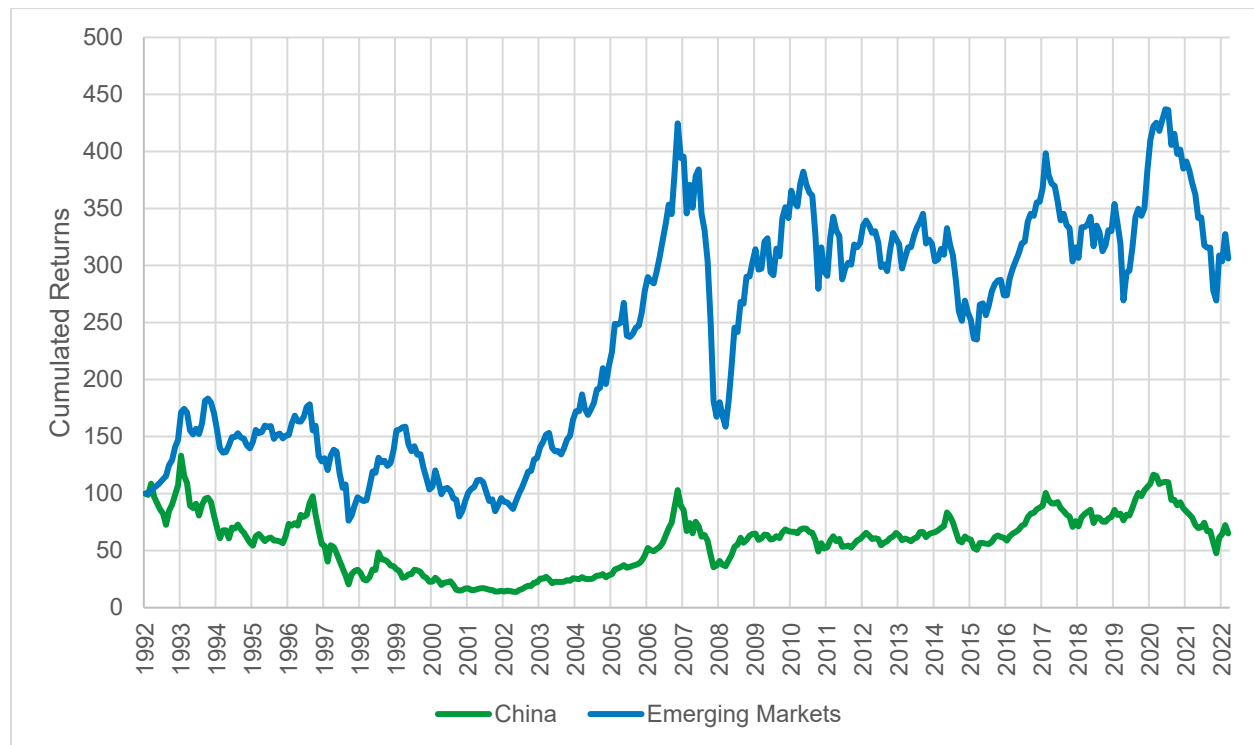
The figures shown relate to past performance. Past performance is not a reliable indicator of current or future results. Index performance returns do not reflect any management fees, transaction costs or expenses. Indices are unmanaged and one cannot invest directly in an index.
Source: BlackRock, with data from Bloomberg as of March 31, 2023.

Allen, Qian, Shan, and Zhu (2022) refer to this as the "disconnect" puzzle, which they attribute to institutional structure and weak corporate governance. Nevertheless, we show below that an investor would optimally allocate to China in certain regimes.[4]

---

[3] See https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=CN
[4] We obtain similar results using the MSCI China A Onshore Net Return Index. The MSCI China index and the MSCI China A Onshore Net Return index have similar means and standard deviations over their common sample periods. The parameters of the regime-switching model (see Section 2.2) are similar when using the MSCI China A Onshore Net Return index.

In the 1990s, China constituted a small proportion of the EM index, but more recently it has reached upwards of 40% in September 2020 and is currently around 35%.[5] Thus, China's weight in the EM index means we may overstate the diversification benefits of investing in China. (The correlation of China returns with EM returns is 0.72 over the sample.) In our model, this is not a concern as we work with a given target allocation to China to infer risk aversion; different starting weights in China will result in different risk aversion levels. On the other hand, we can interpret the investor's opportunity set as two funds, one tracking the MSCI China index and one tracking the MSCI EM index and we solve for the optimal weights in each strategy.

2.2 Markov Model

We estimate the following regime-switching model on continuously compounded returns, $r_t = (r_{1t}\ r_{2t})$, where r_{1t} and r_{2t} are the continuously compounded returns of China and the EM index, respectively:

$$r_t = N(\mu(s_t), \Sigma(s_t)),\qquad (1)$$

where there are different Normal distributions depending on the regime s_t, with the mean, $\mu(s_t)$, and covariance, $\Sigma(s_t)$, dependent on regime s_t.

There are two Markov regimes s_t= 1, 2 with the following transition probability matrix:

$$\begin{bmatrix} P & 1-P \\ 1-Q & Q \end{bmatrix},\qquad (2)$$

where $P = Pr\ (s_{t+1} = 1 | s_t = 1)$ and $Q = Pr\ (s_{t+1} = 2 | s_t = 2)$. This is a regime-switching model of Hamilton (1989) which has been used by many researchers to capture non-linearities of equity returns (see the literature review by Ang and Timmermann, 2012).

---

[5] See https://www.bloomberg.com/news/articles/2023-01-06/china-set-to-reclaim-third-of-global-em-stock-gauge-on-xi-pivot#xj4y7vzkg

Exhibit 2 reports the estimates, where the state-dependent means, $\mu(s_t)$, and the state-dependent volatilities, $\sigma(s_t)$, have been annualized by multiplying by 12 and $\sqrt{12}$, respectively. In the first regime, $s_t = 1$, China and EM have approximately the same mean at 8% per year. In the second regime, $s_t = 2$, China exhibits a steeper drawdown, at -19.5% compares to EM at -5.4%. Thus, the first regime corresponds to a "bull" market state for both China and EM returns, whereas the second regime corresponds to "bear" market conditions for both China and EM, with China drawing down four times as much as the EM index. Both regimes are persistent, with the probability of staying in regime 1 (2) equal to 0.98 (0.96) conditional on the first (second) regime. Important for our calibration, there is a close match of the model-implied arithmetic moments to the data. For example, the implied arithmetic mean returns from the regime-switching model are 3.75% and 6.32% per year for China and EM, respectively, which are close to the arithmetic returns in data at 3.63% and 6.18%, respectively.

Exhibit 2: Estimated Markov Switching Parameters

	$s_t = 1$	$s_t = 2$
μ China	0.085	-0.195
μ EM	0.087	-0.054
σ China	0.206	0.451
σ EM	0.172	0.294
Correlation	0.822	0.672
	P	Q
Transition probabilities	0.980	0.963

The figures shown relate to past performance. The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account. Past performance is not a reliable indicator of current or future results.
Source: BlackRock, with data from Bloomberg as of March 31, 2023

Exhibit 3: Smoothed Probabilities of Regime $s_t = 2$
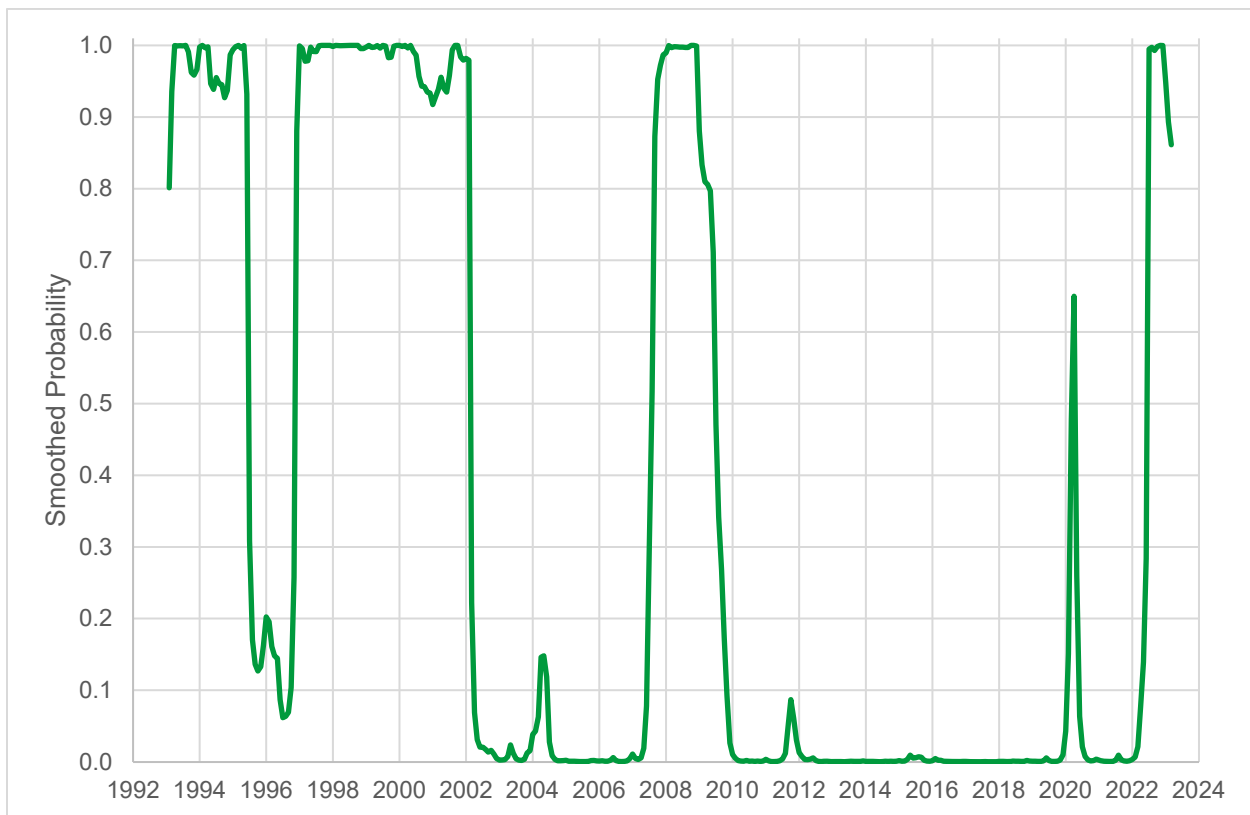
The figures shown relate to past performance. The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account. Past performance is not a reliable indicator of current or future results.
Source: BlackRock, with data from Bloomberg as of March 31, 2023.

Exhibit 3 graphs the smoothed probabilities of the estimation, which is the probability of the bear market $s_t = 2$ conditioning on full sample information. The second bear market regime occurs during the emerging markets defaults of the late 1990s (see also Exhibit 1), during the Financial Crisis, and more recently ticks upward during the corona virus period of 2020 and at the end of 2022.

3. Markov Chain Model with Decoupling

To introduce the probability of China becoming decoupled, we introduce one more state, $s_t = 3$, and an additional transition probability, ψ, from regime $s_t = 2$ to regime $s_{t+1} = 3$. The estimation of the two-state Markov chain in the previous section is conditional on China

9

not becoming decoupled (that is, $\psi = 0$). The three-state Markov chain transition probabilities are given by:

$$\begin{bmatrix} P & 1-P & 0 \\ 1-Q-\psi & Q & \psi \\ 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

where $P = Pr\,(s_{t+1} = 1|s_t = 1)$, $Q = Pr\,(s_{t+1} = 2|s_t = 2)$, and $\psi = Pr\,(s_{t+1} = 3|s_t = 2)$. Once the system enters the third Decoupled regime, the system remains in that regime, $Pr(s_{t+1} = 3|s_t = 3) = 1$, so $s_t = 3$ is an absorbing state. In the Decoupled regime, we assume investors lose all their capital in the first asset.

We take the parameters of the first regime, $s_t = 1$, from the first regime of the two-state model in Exhibit 2. Since in the transition probability matrix in equation (3), there is no transition to becoming decoupled over the next month, $s_t = 1$ is the "Fully Investable" regime.

In the second "Potentially Decouple" regime, $s_t = 2$, there is a possibility of China becoming decoupled next period with probability ψ. As a baseline case, we set ψ to correspond to an annual probability of 0.05, so $\psi = 1 - \exp\,(\log(0.05/12)) = 0.0043$ per month. (Below, we examine sensitivity of results to different ψ.) The parameters of the two-state Markov model in Exhibit 2 correspond to $\psi = 0$. When we introduce non-zero ψ, the moments of the regime-switching model change. In the second regime of the two-state Markov model, the gross return for the first asset, China, is $\bar{R}_1(s_t = 2) = \exp\left(\mu_1(s_t = 2) + \frac{1}{2}\sigma_1^2(s_t = 2)\right) = 0.9923$ per month, where $\mu_2(s_t = 2)$ and $\sigma_1(s_t = 2)$ are the per month values given from the regime $s_t = 2$ parameters for China in Exhibit 2. We keep the gross return conditional on $s_t = 2$ the same, so for a given value of ψ, we solve for $\mu_1(s_t = 2)$ such that $\bar{R}_1(s_t = 2)$ is kept constant:

$$\bar{R}_1(s_t = 2) = (1 - Q - \psi) \exp\left(\mu_1(s_t = 1) + \frac{1}{2}\sigma_1^2(s_t = 1)\right)$$
$$+ Q\exp\left(\mu_1(s_t = 2) + \frac{1}{2}\sigma_1^2(s_t = 2)\right) + \psi \times 0. \tag{4}$$

The first term on the right-hand side (RHS) of equation (4) is the probability of transitioning from the Potentially Decouple regime back to the Fully Investable regime multiplied by the gross return of the Fully Investable regime. The second term on the RHS of equation (4) is the probability of remaining in the Potentially Decouple regime multiplied by the gross return of the Potentially Decouple regime—in this term, we implicitly solve for $\mu_1(s_t = 2)$ to yield $\bar{R}_1(s_t = 2)$ on the left-hand side. The last term is included for pedagogy: the probability of transitioning to the Decoupled regime is ψ, and the gross return of the first asset in this regime is zero. It is worth noting that while we are setting the parameters to hold the mean arithmetic return constant, changing $\mu_1(s_t = 2)$ does change the variance of returns—which is precisely the point of introducing the Decoupled regime, where continuously compounded returns for the first asset can jump to $-\infty$ (or the gross return jumps to zero).

4. Asset Allocation with an Absorbing State

In this section, we set out the investor's asset allocation problem.

The investor receives utility, $\mathcal{U}$, over intermediate consumption from months $t + 1$ to T and rebalances their portfolio at each time s:

$$\mathcal{U}(W_t, s_t, T) = \sum_{s=1}^{T} \beta^s \, U(c_{t+s}W_{t+s}), \tag{5}$$

where we set the monthly time discount $\beta = (0.98)^{1/12}$ (see Cohen, Ericson, Laibson, and White, 2020). The terminal condition is $c_{t+T} = 1$. We express consumption as a rate, c_t, of

11

current wealth, W_t. A fraction, $c_t W_t$, of the portfolio is consumed at each time t. We will exogenously set a constant consumption rate $c_t = \bar{c}$.[6]

We assume Constant Relative Risk Aversion (CRRA) utility:

$$U(c_t W_t) = \frac{(c_t W_t)^{1-\gamma}}{1-\gamma} \tag{6}$$

where γ is the investor's coefficient of risk aversion.

The investor's wealth dynamics are given by:

$$W_{t+1} = (1 - c_t)W_t(\alpha_t R_{1t+1} + (1 - \alpha_t)R_{2t+1}), \tag{7}$$

where α_t is the fraction of the investor's portfolio held in the first asset chosen at the beginning of the period t. The gross return of the i-th asset is $R_{it+1} = R_{it+1}(s_{t+1})$ over time t to $t+1$ and depends on the regime prevailing at time $t+1$, s_{t+1}. The probabilities of the regime at time $t+1$, s_{t+1}, depend on the current regime at time t, s_t. If the system is in the Potentially Decouple regime $s_t = 2$ at time t, it may transition to being decoupled at time $t+1$. If the first asset has decoupled at time t (and thus is in regime $s_t = 3$), it remains decoupled at time $t+1$, and the regime $s_{t+1} = 3$ with probability one. In the Decoupled regime, the gross return of the first asset is equal to zero.

While the academic literature usually solves the portfolio weights, $\{\alpha_t\}$, and consumption stream, $\{c_t\}$, simultaneously (see, for example, Garlappi and Skoudakis, 2010), we take the consumption rate as given and in our baseline calibrations set $\bar{c} = c_t = 0.05/12$ at the monthly frequency. This is consistent with industry practice where payout rates tend to be fixed or liabilities are exogenous. For example, endowments tend to pay out at least 5% per year to preserve their tax-exempt status (see Brown and Tiu, 2013), financial advisors recommend a fixed spending rule, often 4%, for a retired person (see Bengen, 1994), and

---

[6] See the Appendix for details on the solution of the model in equations (5)-(7) and the definition of the certainty equivalent in the context of the model.

income funds often target fixed payout rates like 7% or more which cater to individuals' demands for stable cashflows (see Harris, Hartzmark, and Solomon, 2015).

There are three regimes in the Markov chain: Fully Investable, Potentially Decouple, and Decoupled, so at each horizon, T, there are three optimal portfolio weights corresponding to each regime, that is, $\alpha_t = \alpha_t(T, s_t)$. In the Decoupled regime, $s_t = 3$, it is only possible to allocate to the EM index.

To calibrate the investor's risk aversion, we assume a target China holding of 35% for the first Fully Investable regime, $s_t = 1$, for a myopic horizon of $T = 1$ month. This corresponds to a risk aversion of $\gamma = 0.662$. Below, we also conduct exercises varying the investor's risk aversion.[7]

Using the asset allocation framework, we compute the certainty equivalent required to compensate an investor for not being able to invest in China, where the restricted portfolio consists only of the EM asset. The certainty equivalent depends on the prevailing regime, s_t, and the investor's horizon, T. In our calculations, we also compute solutions limiting the amount of leverage.

5. Empirical Results

In this section, we report the empirical results. Section 5.1 reports the optimal allocations to the decoupling asset with and without leverage constraints. Section 5.2 reports certainty equivalents of not investing in the decoupling market. In Sections 5.3 to 5.5, we consider various comparative statics exercises changing the probability of decoupling, the consumption payout ratio, and maximum leverage, respectively.

---

[7] Unconstrained mean-variance or CRRA utility using the data first and second moments (see Exhibit 1) produces short positions in China for all positive levels of risk aversion because the returns of China are lower and more volatile than the EM index with correlations that are too high for any mean-variance diversification benefit.

5.1 Optimal Allocations

Exhibit 4 reports the optimal holdings for China in the Fully Investable regime and the Potentially Decouple regime in Panels A and B, respectively. These are produced under our baseline parameters: risk aversion of $= 0.662$, 5% annual payout ratio, and a one-year probability of becoming decoupled of 0.05. We consider two cases: an unconstrained case that allows shorting and a no-leverage case.

Exhibit 4: Optimal Holdings of China

Panel A: Fully Investable Regime $s_t = 1$

The figures shown relate to past performance and are calculated under the model equations (1)-(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0.662$, $\bar{c} = 0.05/12$, and an annual one-year decoupling probability of 0.05. The model parameters have inherent limitations. See "Hypothetical Example" section of the disclosures at the end of this material. The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account. Past performance is not indicative of future results.
Source: BlackRock, with data from Bloomberg as of March 31, 2023.

Exhibit 4: Optimal Holdings of China

Panel B: Potentially Decoupled Regime $s_t = 2$

The figures shown relate to past performance and are calculated under the model equations (1)-(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0.662$, $\bar{c} = 0.05/12$, and an annual one-year decoupling probability of 0.05. The model parameters have inherent limitations. See "Hypothetical Example" section of the disclosures at the end of this material. The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account. Past performance is not indicative of future results.
Source: BlackRock, with data from Bloomberg as of March 31, 2023.

In both the unconstrained and no-leverage cases, the starting myopic portfolio holding of 35.0% in China is by construction (as this is the target weight to calibrate the risk aversion parameter). As the horizon increases, the investor seeks to hold a larger amount of China—indicating there are intertemporal hedging demands. At the 5-year horizon, the unconstrained investor optimally holds 35.4% in China and at the 10-year horizon, the unconstrained investor holds 36.6%. It may seem unintuitive that the longer the horizon, the higher the holding of an asset that may become decoupled! But these results are consistent with predictability arising from the regime persistence (see Exhibit 2 for the two-state Markov model, for example). The longer the horizon, the more consumption is foregone from not investing in the first asset. Further, the longer the horizon, the more the investor can take advantage of the persistent regimes.

15

In Exhibit 4, Panel A, the effect is smaller for the no-leverage case as horizon increases because the optimizer wishes to short China in the second Potentially Decouple regime, shown in Panel B. In this regime, $s_t = 2$, the investor takes a short position because China may decouple; as a reminder, we have calibrated $\mu_1(s_t = 2)$ so that the arithmetic return of the first asset in the second regime remains the same as the two-state Markov model without decoupling risk, so this short position is caused by the investor anticipating a downward jump to zero on the probability ψ of China becoming decoupled. By construction in the no-leverage case, the weight in China is zero (not graphed), and in the third regime when China becomes decoupled, the investor holds $\alpha = 0$ and 100% EM.

5.2 Certainty Equivalent Costs

Exhibit 5 reports the certainty equivalent costs of not investing in China in the Fully Investable ($s_t = 1$) and Potentially Decouple ($s_t = 2$) regimes. (Since China has decoupled in the $s_t = 3$ regime, the certainty equivalent cost is zero.) Exhibit 5 shows that the certainty equivalent costs are very high in the case of allowing leverage—0.23 cents per initial dollar of initial wealth for a one-year horizon increasing to an extremely large 5.49 cents at the 10-year horizon. In the Potentially Decouple regime, the costs are even higher—at 2.26 and 10.74 cents per dollar of wealth for the 1- and 10-year horizons, respectively. This is because the investor takes short positions in China in preparation for a possible decoupling event in the Potentially Decouple regime.

The last two columns of Exhibit 5 list certainty equivalents for the no-leverage case. Not surprisingly, the certainty equivalents are now much smaller than the unconstrained case. In the no-leverage case, the investor does not hold any China exposure in the Potentially Decouple regime (see Section 5.1 and Exhibit 4), but Exhibit 5 reports the certainty equivalent costs are non-zero in that regime. This is because there is a probability $(1 - Q - \psi)$ of transitioning back to the Fully Investable regime, which is anticipated and taken into account in the certainty equivalent calculation. The one-year and 10-year certainty equivalent costs in the Fully Investable regime are 3.8 bp and 24 bp, respectively.

Exhibit 5: Certainty Equivalent Costs of Not Investing in China

|	Unconstrained		No-Leverage	
	Fully Investable Regime	Potentially Decouple Regime	Fully Investable Regime	Potentially Decouple Regime
1 month	0.0046	0.2893	0.0046	0.0000
2 months	0.0136	0.5412	0.0086	0.0001
6 months	0.0790	1.3420	0.0220	0.0015
1 year	0.2311	2.2550	0.0383	0.0053
3 years	1.1564	4.8247	0.0911	0.0296
5 years	2.3142	6.7547	0.1369	0.0604
10 years	5.4910	10.7410	0.2383	0.1422

The figures shown relate to past performance and are calculated under the model equations (1)-(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0.662$, $\bar{c} = 0.05/12$, and an annual one-year decoupling probability of 0.05. The model parameters have inherent limitations. See "Hypothetical Example" section of the disclosures at the end of this material. The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account. Past performance is not indicative of future results.
Source: BlackRock, with data from Bloomberg as of March 31, 2023.

In the next sections 5.3 to 5.5, we vary the baseline parameters to investigate how the certainty equivalent costs change as we change other parameters.

5.3 Changing the Probability of Decoupling

Exhibit 6 plots the certainty equivalents in cents per initial dollar of wealth as a function of horizon, T, for the unconstrained leverage case as we vary the probability of becoming decoupled. We plot the certainty equivalents of not investing in China in the Fully Investable regime. The units on the x-axis are the probability of decoupling over the next year (which are then converted to monthly ψ units in the model). As a reminder, the baseline probability of decoupling used for Exhibits 4 and 5 is 0.05 per year. The other parameters are held at baseline.

Interestingly, in Panel A the certainty equivalent costs for the one-year horizon are a non-linear function of horizon for the unconstrained leverage case. There are two offsetting

effects: at low probabilities of decoupling, the investor seeks to hold the additional source of return in China with little probability of loss, so the certainty equivalent is relatively high. At high probabilities of decoupling, the investor can take advantage of becoming decoupled by taking short positions in the Potentially Decoupled regime, again increasing the certainty equivalent. In the no-leverage case, there is no benefit for the short position and so the certainty equivalents are lower and monotonically increasing.

Why does the certainty equivalent generally increase in both the unconstrained and constrained leverage cases at the one-year horizon with an *increasing* probability of decoupling? This is caused by two reasons. First, the Fully Investable regime is persistent and the one-year horizon is relatively short. We require two Markov chain transitions—from the Fully Investable to the Potentially Decouple regimes, and then the Potentially Decouple to the Decoupled regimes—and the investor is more likely not to experience decoupling starting from the Fully Investable regime. Second, we have held the gross return of the Potentially Decouple regime constant (we solve for $\mu_1(s_t = 2)$ as detailed in Section 4 and equation (3)), and at relatively low levels of risk aversion, the effect of the higher moments of the jump to zero in decoupling is second to the mean effect for CRRA utility (see Pratt, 1964).

When the horizon is long enough in Exhibit 6, Panel B, the certainty equivalents do decrease for the unconstrained case. In the case of no leverage, however, the certainty equivalents are still an increasing function of the probability of decoupling. In this case, the optimal holdings of China in the second and third regime are already zero. These constrained positions, combined with the two necessary regime changes to become decoupled (Fully Investable to Potentially Decouple to Decoupled) mean that the decoupling asset is a relatively attractive asset in the Fully Investable regime.

18

Exhibit 6: Certainty Equivalent Costs Varying Probabilities of Decoupling in the Fully Investable Regime

Panel A: One-Year Horizon
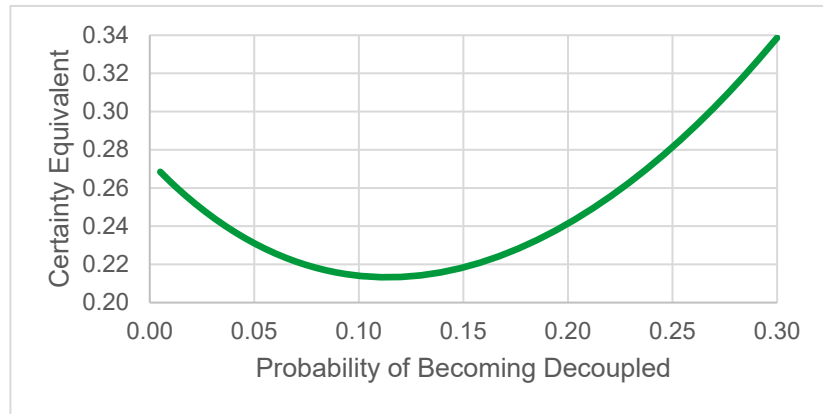
Unconstrained Case   No-Leverage Case
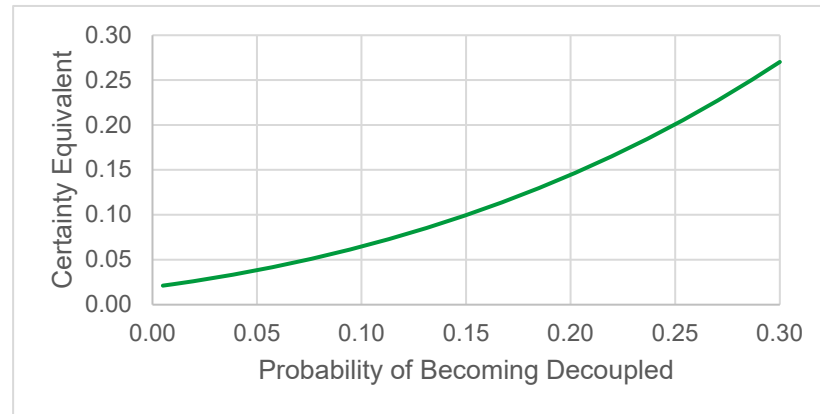
Panel B: One-Year, Five-Year, and 10-Year Horizons
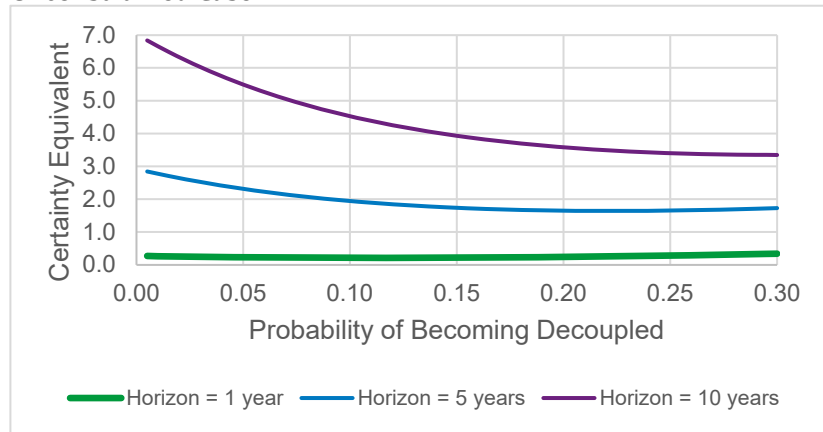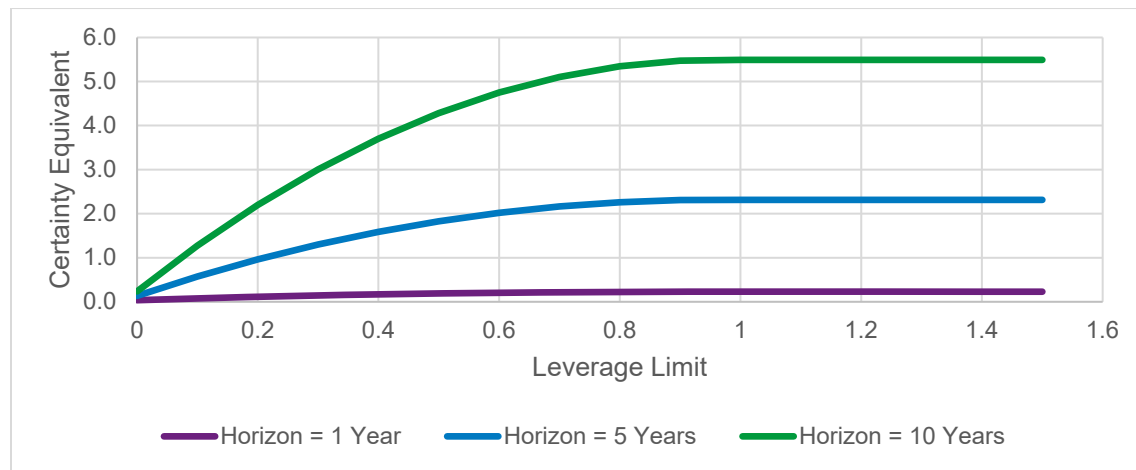
Unconstrained Case   No-Leverage Case
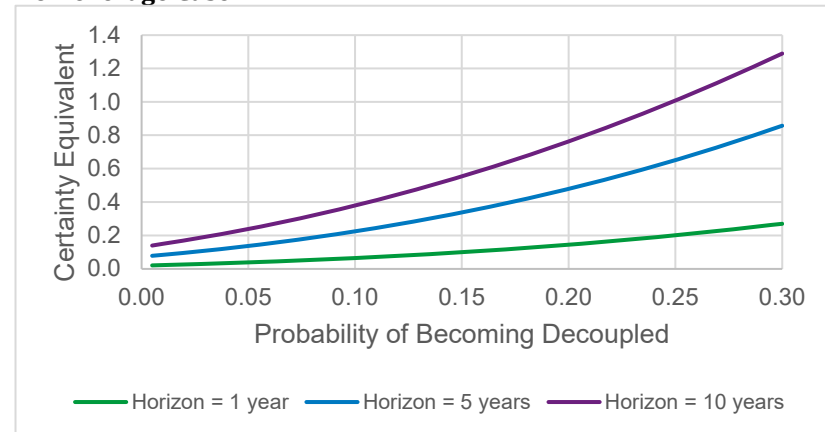
The figures shown relate to past performance and are calculated under the model equations (1)-(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0.662$, and $\bar{c} = 0.05/12$. The model parameters have inherent limitations. See "Hypothetical Example" section of the disclosures at the end of this material. The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account. Past performance is not indicative of future results. The figures shown relate to past performance. The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account. Past performance is not a reliable indicator of current or future results.
Source: BlackRock, with data from Bloomberg as of March 31, 2023.

5.4 Changing the Payout Ratio

In Exhibit 7, we vary the consumption rate, $c_t = \bar{c}$, or payout ratio for the unconstrained case holding other parameters fixed at baseline. The certainty equivalents become smaller as the payout ratio increases: for example, at the 10-year horizon, the certainty equivalent of not investing in the decoupling asset is over 5c per dollar of initial wealth for a 1% consumption rate decreasing to 0.67c at a 20% consumption rate. As Wachter (2002) notes, introducing consumption into a CRRA portfolio choice problem effectively shortens the effective horizon, or decreases duration. Lower payout ratios imply higher duration, and although the cumulative probability of China becoming decoupled increases with longer horizons, there is a greater time for the agent to accrue consumption—translating into higher certainty equivalents for lower payout ratios.

Exhibit 7: Certainty Equivalent Costs Varying Payout Ratio
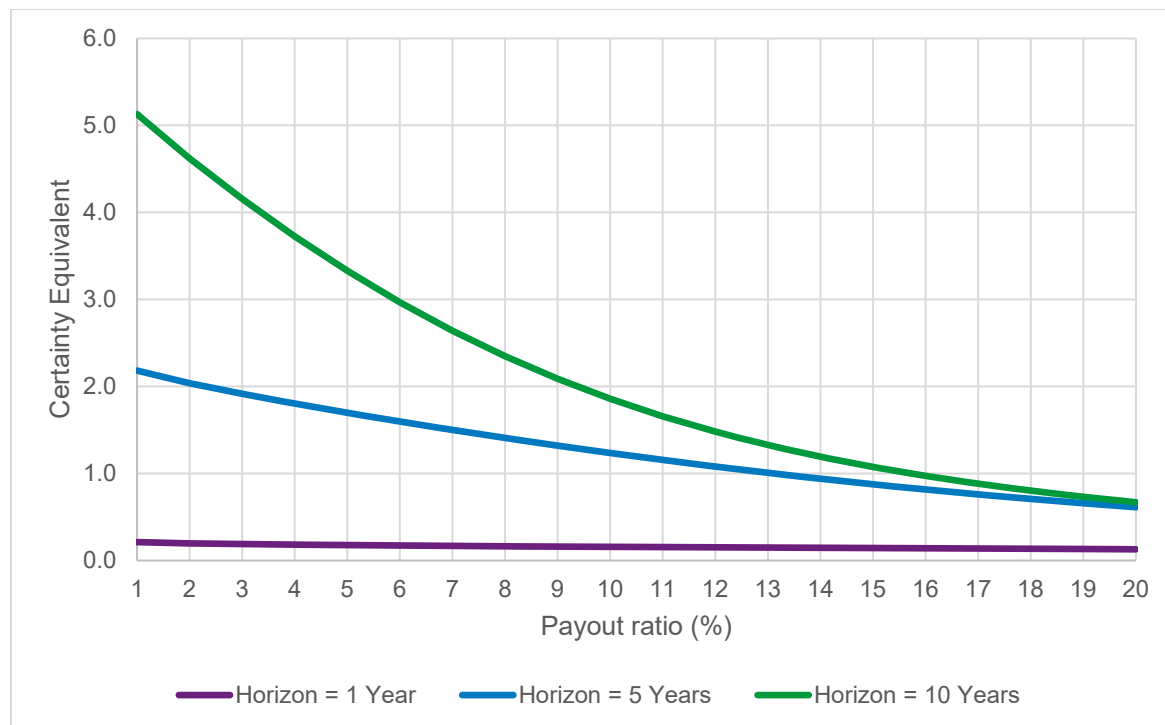
The figures shown relate to past performance and are calculated under the model equations (1)-(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0.662$ and an annual one-year decoupling probability of 0.05. The model parameters have inherent limitations. See "Hypothetical Example" section of the disclosures at the end of this material. The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account. Past performance is not indicative of future results.
Source: BlackRock, with data from Bloomberg as of March 31, 2023.

5.5 Changing the Leverage Limit

Our final comparative statics exercise is to change the leverage limit in Exhibit 8. The maximum short position for the baseline parameters is approximately -0.9, which is taken in the Potentially Decouple regime, $s_t = 2$. Thus, the certainty equivalents flatline at this leverage as there are no more benefits to increasing the leverage limit after this value. Before this, the leverage limit is binding and the utility, and thus certainty equivalents, increase up until this value.

Exhibit 8: Certainty Equivalent Costs Varying the Leverage Limit

The figures shown relate to past performance and are calculated under the model equations (1)-(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0.662$, $\bar{c} = 0.05/12$, and an annual one-year decoupling probability of 0.05. The model parameters have inherent limitations. See "Hypothetical Example" section of the disclosures at the end of this material. The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account. Past performance is not indicative of future results.
Source: BlackRock, with data from Bloomberg as of March 31, 2023.

6. Calibration with Negative Hedging Demands

In this section, we examine optimal asset allocation to MSCI Russia and the MSCI EM indexes.

Russia decoupled in March 2022, so these hypothetical exercises can be interpreted as applying prior to this date. Exhibit 9 plots cumulated returns of Russia and EM from January 1995 to February 2022. Prior to Russia decoupling, Russia experienced steep

negative returns at the beginning of 2022—suggestive of the Potentially Decoupled regime, and then Russia moved to the Decoupled regime in March 2022. The returns since Russia liberalized to when it decoupled were high—at 16.5% per year compared to 5.7% per year for the EM index. Russia returns were also approximately twice as volatile, at 48.4%, as the EM index, at 21.9%.

Exhibit 9: Cumulated Returns of MSCI Russia and MSCI Emerging Markets Indexes
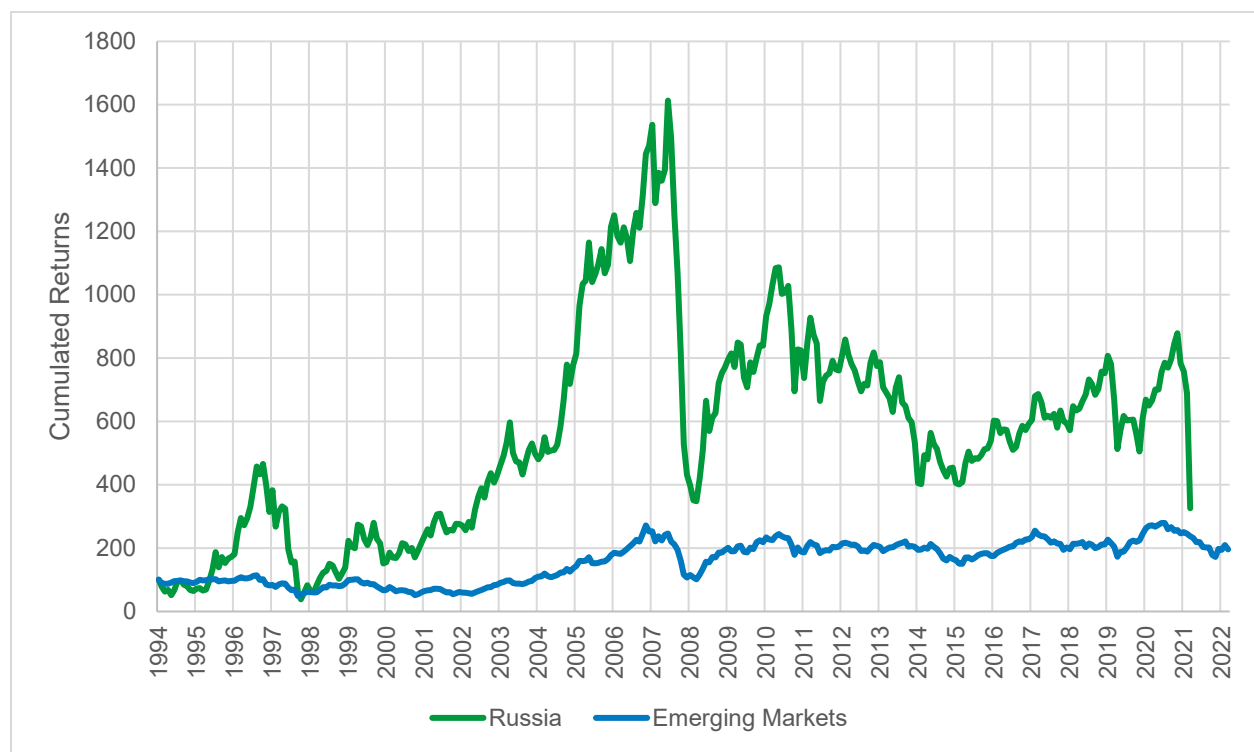
The figures shown relate to past performance. Past performance is not a reliable indicator of current or future results. Index performance returns do not reflect any management fees, transaction costs or expenses. Indices are unmanaged and one cannot invest directly in an index.
Source: BlackRock, with data from Bloomberg as of March 31, 2023.

We follow the same procedure as the previous section, where we model China as an asset that may potentially decouple. We fit a two-state Markov chain model to Russia and EM, and then introduce a third absorbing state where Russia becomes decoupled. The parameters of the regime-switching model have the same qualitative characteristics as the model estimated on China and EM in Section 2—in the second regime Potentially Decouple, the

returns of Russia are worse than EM and are also more volatile. This leads to an investor wanting to take modest short positions in Russia in the Potentially Decouple regime.

We calibrate the risk aversion to a target allocation of Russia of 5% in the Fully Investable regime, which corresponds to $\gamma = 10.0$—this is much larger than the calibrated risk aversion in the China-EM system ($\gamma = 0.67$) because the historical returns of Russia are much higher than China (up until Russia became decoupled). We hold the other parameters the same as the China case.

In Exhibit 10, we report portfolio weights and certainty equivalent costs for not investing in Russia. The portfolio weight starts, by construction, at 5.0% and the optimal portfolio weights decrease; this is unlike the China case because the probability of decoupling is high. The portfolio weights asymptote at -2.8%. The short positions are higher in the Potentially Decouple regime, as expected, as the optimizer seeks to take advantage of the possibility that Russia can transition to being decoupled. The certainty equivalents are negligible at the one-month horizon and rise to 6.8c per dollar of initial wealth at the 10-year horizon. The certainty equivalents are approximately the same across the Fully Investable regime and the Potentially Decouple regime because the probability of decoupling is relatively high.

Exhibit 10: Certainty Equivalent Costs of Not Investing in Russia

	Portfolio Weights		Certainty Equivalent Costs	
	Fully Investable Regime	Potentially Decouple Regime	Fully Investable Regime	Potentially Decouple Regime
1 month	0.0500	-0.0772	0.0000	0.0000
2 months	0.0500	-0.0772	0.0028	0.1020
6 months	0.0260	-0.0697	0.0441	0.4960
1 year	-0.0102	-0.0606	0.3847	0.9963
3 years	-0.0276	-0.0504	2.2383	2.4117
5 years	-0.0276	-0.0498	3.5256	3.6962
10 years	-0.0276	-0.0498	6.7914	6.9673

The figures shown relate to past performance and are calculated under the model equations (1)-(7) with the solution method in the Appendix with model parameters calibrated to the historical returns in Exhibit 9, $\gamma = 10.0$, $\bar{c} = 0.05/12$, and an annual one-year decoupling probability of 0.05. The model parameters have inherent limitations. See "Hypothetical Example" section of the disclosures at the end of this material. The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account. Past performance is not indicative of future results.
Source: BlackRock, with data from Bloomberg as of March 31, 2023.

7. Conclusion

We model optimal allocations to markets that may become decoupled. The investor receives utility over intermediate consumption, which is funded by liquidating the investment portfolio. While the market is still investable, holdings of the market that may potentially decouple represent a source of returns and diversification to partially fund consumption. We model time-varying investment returns with a Markov switching model with three Markov regimes—a Fully Investable regime where there is no possibility of the market becoming decoupled, a Potentially Decouple regime where the market can transition to the third regime, which is an absorbing state, the Decoupled regime where the market experiences a -100% return. The regimes are persistent, which induces hedging demands, or where long-horizon portfolio weights are different from short-horizon portfolio weights.

Using the model, we compute certainty equivalents—the sure amount of money required to compensate the investor for not being able to invest in the market that may

become decoupled—and find these are large and increase with horizon when calibrating the model with an emerging market and the MSCI Emerging Market index. In particular, we find that hedging demands are positive: optimal holdings of the potentially decoupling market can to increase with horizon in the Fully Investable regime, and certainty equivalents even increase when the probability of becoming decoupled increases.

Because there are two non-decoupled regimes, the probability of decoupling already varies through time. However, it is likely that the probability of decoupling may be a function of macro variables. An interesting and serious extension of the model is to introduce time-varying transition probabilities as a function of exogenous predictors along the lines of Diebold, Lee, and Weinbach (1999) or Bazzi et al. (2016). In this case, there would be additional state variables in the asset allocation problem making the dynamic programming solution much more involved. Finally, it is a straightforward extension to allowing the potentially decoupling market to re-integrate into world capital markets along the lines of Bekaert and Harvey (1995). In this case, we would assume some potential loss in the Decoupling regime, which could be stochastic, and the Decoupling regime would no longer be an absorbing state.

Appendix

We solve the dynamic asset allocation problem using dynamic programming. The Bellman equation for the system in equations (5)-(7) can be written as

$$\frac{\vartheta(T, s_t)}{1-\gamma} = \max_{\alpha_t} \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \beta E_t \left[\frac{\left((1-c_t)W_t(\alpha_t R_1 + (1-\alpha_t)R_2)\right)^{1-\gamma}}{1-\gamma} \vartheta(T-1, s_{t+1}) \right] \right], \quad \text{(A.1)}$$

where $\vartheta(T, s_t)$ is a function of horizon T and state s_t and $R_i = R_i(s_{t+1})$ is the return for asset i which is a function of the regime prevailing at time $t + 1$, s_{t+1}. This is a simpler problem than the standard CRRA intermediate consumption problem because we take the consumption rate, c_t, as given. However, as Brandt (2010) notes, with CRRA utility the value function is homothetic in wealth and the consumption choice is solved independently of the portfolio weight, α_t. That is, even with endogenous consumption, the portfolio choice weight at time t, α_t, is solved ignoring the consumption rate, and then the optimal consumption rate is solved taken as given the optimal portfolio weight.

The first-order conditions (FOCs) are given by:

$$E_t[(\alpha_t R_1 + (1-\alpha_t)R_2)^{-\gamma} \vartheta(T, s_{t+1})(R_1 - R_2)] = 0. \quad \text{(A.2)}$$

In equation (A.2), $(R_1 - R_2)$ is the return of the first asset relative to the second asset. There are three regimes at time $t + 1$, $s_{t+1} = 1, 2,$ or 3, whose probabilities depend on the current regime at time t, s_t, and hence there are three sets of FOCs. Likewise the value function, $\vartheta(\tau, s_t)$ takes on three values for each $\tau = T \dots 1$, one for each state, s_t. The boundary conditions are $\vartheta(T, s_t) = 1$ for all s_t.

The FOC conditional on $s_t = 1$ is given by:

$$(1-P)E_t\left[W_{t+1}^{-\gamma}(s_{t+1})\vartheta(s_{t+1}, T-1)\left(R_1(s_{t+1}) - R_2(s_{t+1})\right)|s_{t+1} = 1\right]$$
$$+ PE_t\left[W_{t+1}^{-\gamma}(s_{t+1})\vartheta(s_{t+1}, T-1)\left(R_1(s_{t+1}) - R_2(s_{t+1})\right)|s_{t+1} = 2\right] = 0, \quad \text{(A.2)}$$

where $W_{t+1}^{-\gamma}(s_{t+1}) = (\alpha R_1(s_{t+1}) + (1-\alpha)R_2(s_{t+1}))^{-\gamma}$ and $P = Pr(s_{t+1} = 1|s_t = 1)$ in the probability transition matrix in equation (3). We solve the non-linear root for equation (A.2) to find $\alpha_t(s_t = 1)$.

The FOC conditional on $s_t = 2$ is given by:

$$
\begin{aligned}
(1 - Q - \psi)&E_t\big[W_{t+1}^{-\gamma}(s_{t+1})\vartheta(s_{t+1}, T-1)(R_1(s_{t+1}) - R_2(s_{t+1}))|s_{t+1} = 1\big] \\
&+ QE_t\big[W_{t+1}^{-\gamma}(s_{t+1})\vartheta(s_{t+1}, T-1)(R_1(s_{t+1}) - R_2(s_{t+1}))|s_{t+1} = 2\big] \quad \text{(A.3)} \\
&+ \psi E_t\big[W_{t+1}^{-\gamma}(s_{t+1})\vartheta(s_{t+1}, T-1)(-R_2(s_{t+1}))|s_{t+1} = 3\big] = 0,
\end{aligned}
$$

where $R_1(s_{t+1} = 3) = 0$ is the decoupled first asset which has no return in regime $s_{t+1} = 3$ and $W_{t+1}^{-\gamma}(s_{t+1} = 3) = (R_2(s_{t+1} = 3))^{-\gamma}$ as only the second asset exists in regime $s_{t+1} = 3$. The probability $Q = Pr(s_{t+1} = 2|s_t = 2)$ and $\psi = Pr(s_{t+1} = 3|s_t = 2)$. Solving equation (A.3) yields $\alpha_t(s_t = 2)$.

The only solution for the third regime $s_t = 3$ is for $\alpha_t(s_t = 3) = 0$. Stated another way, only the second asset is held when the first asset has become decoupled.

We evaluate the integrals in the FOCs numerically by quadrature, which is very accurate for as few as four or five quadrature points (see Kochenderfer and Wheeler, 2019).

We compute the certainty equivalent after solving for the value function and optimal portfolio weights with and without holding the first asset. The latter corresponds only to the expected utility for holding only EM. We denote the value function with optimal first asset holdings as $\vartheta^*(s_t, T)$ and the value function without the first asset as $\vartheta^\dagger(s_t, T)$. The certain amount of wealth, $\widetilde{w}$, required to compensate an investor for not being able to invest in the first asset is given by

$$
\widetilde{w}(s_t, T) = \left(\frac{\vartheta^*(s_t, T)}{\vartheta^\dagger(s_t, T)}\right)^{1/(1-\gamma)}. \tag{A.4}
$$

Note that the certainty equivalent depends on the current regime, s_t, and the horizon, T. We express the compensation required in cents per dollar of initial wealth, $100 \times (\widetilde{w} - 1)$.

References

Allen, F., J. Qian, C. Shan, and J. Zhu. 2022. Dissecting the Long-Term Performance of the Chinese Stock Market. SSRN Working Paper.

Ang, A., and G. Bekaert. 2002. International Asset Allocation with Regime Shifts. *Review of Financial Studies*, Vol. 15, No. 4, 1137–1187.

Ang, A., and G. Bekaert. 2004. How do Regimes Affect Asset Allocation. *Financial Analysts Journal*, *Vol.* 60, No. 2, 86–99.

Ang, A., and A. Timmermann. 2012. Regime Changes and Financial Markets. *Annual Review of Financial Economics*, Vol. 4, 313-337.

Bazzi, M., F. Blasques, S. J. Koopman, and A. Lucas. 2016. Time-Varying Transition Probabilities for Markov Regime Switching Models. *Journal of Time Series Analysis*, Vol. 38, No. 3, 458-478.

Bekaert, G., and C. R. Harvey. 1995. Time-Varying World Market Integration. *Journal of Finance*, Vol. 50, No. 2, 403-444.

Bengen, W. P. 1994. Determining Withdrawal Rates Using Historical Data. *Journal of Financial Planning*, Vol. 7, No. 4, 14-24.

Brandt, M. W. 2010. Portfolio Choice Problems. In Ait-Sahalia, Y., and L. P. Hansen, eds. *Handbook of Financial Econometrics: Tools and Techniques*, North Holland, pp. 269-336.

Brown, K. C., and C. I. Tiu. 2013. The Interaction of Spending Policies, Asset Allocation Strategies, and Investment Performance at University Endowment Funds, in Brown, K. C., ed. *How the Financial Crisis and Great Recession Affected Higher Education*. University of Chicago Press, Chicago, IL, pp. 43-98.

Brown, S. J., W. N. Goetzmann, and S. A. Ross. 1985. Survival. *Journal of Finance*, Vol. 50, No. 3, 853-873.

Campbell, J. Y., Y. L. Chan, and L. M. Viceira. 2003. A Multivariate Model of Strategic Asset Allocation. *Journal of Financial Economics*, Vol. 67, No. 1, 41–80.

Campbell, J. Y., and L. M. Viceira. 1999. Consumption and Portfolio Decisions When Expected Returns Are Time Varying. *Quarterly Journal of* Economics, Vol. 114, No. 3, 433–495.

Cohen, J., K. M. Ericson, D. Laibson, and J. M. White. 2020. Measuring Time Preferences. *Journal of Economic Literature*, Vol. 58, No. 2, 299-347.

Das, S., D. Ostrov, A. Casanova, A. Radhakrishnan, and D. Srivastav. 2022. Optimal Goals-Based Investment Strategies for Switching between Bull and Bear Markets. *Journal of Wealth Management*, Vol. 24, No. 4, 8-36.

Das, S. R., and R. Uppal. 2004. Systemic Risk and International Portfolio Choice. *Journal of Finance*, Vol. 59, No. 6, 2809–2834.

Diebold, F. X., J.-H. Lee, and G. Weinbach. 1999. Regime Switching with Time-Varying Probabilities, in Diebold, F. X., and G. D. Rudebusch, eds., *Business Cycles: Durations, Dynamics, and Forecasting*, Princeton University Press, pp. 144-166.

Garlappi, L., and G. Skoulakis. 2010. Solving Consumption and Portfolio Choice Problems: The State Variable Decomposition Method. *Review of Financial Studies*, Vol. 23, No. 9, 3346-3400.

Guidolin, M., and A. Timmermann. 2008. International Asset Allocation under Regime Switching, Skew, and Kurtosis Preferences. *Review of Financial Studies*, Vol. 21, No. 2, 889–935.

Hamilton, J. D. 1989. A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, Vol. 57, No. 2, 357-384.]

Harris, L. E., S. M. Hartzmark, and D. H. Solomon. 2015. Juicing the Dividend Yield: Mutual Funds and the Demand for Dividends. *Journal of Financial Economics*, Vol. 116, No. 3, 433-451.

He, R., S. Siwinski, and C. Yu, 2022. Public Pensions in a World of Low Rates and Low Risk Premiums. *Journal of Investment Consulting*, Vol. 21, No. 1, 30-44.

Kandel. S., and R. F. Stambaugh. 1996. On the Predictability of Stock Returns: An Asset Allocation Perspective. *Journal of Finance*, Vol. 51, No. 2, 385-424.

Kochenderfer, M. J., and T. A. Wheeler. 2019. *Algorithms for Optimization*. Cambridge, MA: MIT Press.

Lewin, M., and Campani, C. H. 2022. Constrained Portfolio Strategies in a Regime-Switching Economy. *Financial Markets and Portfolio Management*, https://doi.org/10.1007/s11408-022-00414-x

Liu, J., F. A. Longstaff, and J. Pan. 2003. Dynamic Asset Allocation with Event Risk. *Journal of Finance*, Vol. 58, No. 1, 231-259.

Pratt, J. W. 1964. Risk Aversion in the Small and in the Large. *Econometrica*, Vol. 32, No. 1/2, 122-36.

Samuelson, P. A., 1969. Lifetime Portfolio Selection by Dynamic Stochastic Programming. *Review of Economics and Statistics*, Vol. 51, No. 3., 239-246.

Tu, J. 2010. Is Regime Switching in Stock Returns Important in Portfolio Decisions? *Management Science*, Vol. 56, No. 7, 1198–1215.

Wachter, J. A. 2002. Portfolio and Consumption Decisions Under Mean-Reverting Returns: An Explicit Solution for Complete Markets. *Journal of Financial and Quantitative Analysis*, Vol. 37, No. 1, 63-91.

Risk warnings

Capital at risk. The value of investments and the income from them can fall as well as rise and are not guaranteed. The investor may not get back the amount originally invested. All investments involve risks and may lose value. Past performance is not a reliable indicator of current or future results and should not be the sole factor of consideration when selecting a product or strategy.

Changes in the rates of exchange between currencies may cause the value of investments to diminish or increase. Fluctuation may be particularly marked in the case of a higher volatility product or strategy and the value of an investment may fall suddenly and substantially. Levels and basis of taxation may change from time to time.

Important information

This material is provided for educational purposes only and is not intended to be relied upon as a forecast, research or investment advice, and is not a recommendation, offer or solicitation to buy or sell any securities or to adopt any investment strategy. The opinions expressed are subject to change. References to specific securities, asset classes and financial markets are for illustrative purposes only and are not intended to be and should not be interpreted as recommendations. Reliance upon information in this material is at the sole risk and discretion of the reader. The material was prepared without regard to specific objectives, financial situation or needs of any investor.

This material may contain "forward-looking" information that is not purely historical in nature. Such information may include, among other things, projections, forecasts and estimates of yields or returns. No representation is made that any performance presented will be achieved by any BlackRock Funds, or that every assumption made in achieving, calculating or presenting either the forward-looking information or any historical performance information herein has been considered or stated in preparing this material. Any changes to assumptions that may have been made in preparing this material could have a material impact on the investment returns that are presented herein. Past performance is not a reliable indicator of current or future results and should not be the sole factor of consideration when selecting a product or strategy.

The information and opinions contained in this material are derived from proprietary and nonproprietary sources deemed by BlackRock to be reliable, are not necessarily all-inclusive and are not guaranteed as to accuracy.

Hypothetical example

The model performance shown is hypothetical and for illustrative purposes only and does not represent the performance of a specific investment product or any client account and as such is not an investible product. Past performance is not indicative of future results. The aggregate performance of the model is hypothetical and the model is formulated with benefit of hindsight.

The displayed hypothetical returns are subject to a number of significant limitations. They are illustrative of a product or strategy that does not exist, and therefore do not reflect the deduction of any fees or expenses, including advisory, management and performance fees, as well as brokerage fees, commissions and other expenses that might normally apply. In addition, the allocation decisions reflected in the hypothetical returns were not made under actual market conditions and cannot completely account for the impact of financial risk in actual portfolio management. Any securities used within the hypothetical performance were selected with the full benefit of hindsight, after their performance over the period shown was known. Past hypothetical performance results are not indicative of future returns.

There are frequently sharp differences between a hypothetical performance record and the actual record subsequently achieved. Therefore, hypothetical performance records invariably show positive rates of return. Another inherent limitation of these results is that the allocation decisions reflected in the performance record were not made under actual market conditions and, therefore, cannot completely account for the impact of financial risk in actual portfolio management.

FOR PROFESSIONAL, INSTITUTIONAL, QUALIFIED, WHOLESALE INVESTORS AND PERMITTED, PROFESSIONAL AND QUALIFIED CLIENT USE ONLY – NOT FOR PUBLIC DISTRIBUTION (PLEASE READ IMPORTANT DISCLOSURES)

In Argentina, only for use with Qualified Investors under the definition as set by the Comisión Nacional de Valores (CNV).

In Brazil, this private offer does not constitute a public offer, and is not registered with the Brazilian Securities and Exchange Commission, for use only with professional investors as such term is defined by the *Comissão de Valores Mobiliários*.

In Chile, the offer of each security not registered with the Comisión para el Mercado Financiero ("CMF") is subject to General Rule No. 336 issued by the SVS (now the CMF). The subject matter of this offer may include securities not registered with the CMF; therefore, such securities are not subject to the supervision of the CMF. Since the securities are not registered in Chile, there is no obligation of the issuer to make publicly available information about the securities in Chile. The securities shall not be subject to public offering in Chile unless registered with the relevant registry of the CMF.

In Colombia, the offer of each Fund is addressed to less than one hundred specifically identified investors, and such Fund may not be promoted or marketed in Colombia or to Colombian residents unless such promotion and marketing is made in compliance with Decree 2555 of 2010 and other applicable rules and regulations related to the promotion of foreign financial and/or securities related products or services in Colombia.

IN MEXICO, FOR INSTITUTIONAL AND QUALIFIED INVESTORS USE ONLY. INVESTING INVOLVES RISK, INCLUDING POSSIBLE LOSS OF PRINCIPAL. THIS MATERIAL IS PROVIDED FOR EDUCATIONAL AND INFORMATIONAL PURPOSES ONLY AND DOES NOT CONSTITUTE AN OFFER OR SOLICITATION TO SELL OR A SOLICITATION OF AN OFFER TO BUY ANY SHARES OF ANY FUND OR SECURITY.

This information does not consider the investment objectives, risk tolerance or the financial circumstances of any specific investor. This information does not replace the obligation of financial advisor to apply his/her best judgment in making investment decisions or investment recommendations. It is your responsibility to inform yourself of, and to observe, all applicable laws and regulations of Mexico. If any funds, securities or investment strategies are mentioned or inferred in this material, such funds, securities or strategies have not been registered with the Mexican National Banking and Securities Commission (Comisión Nacional Bancaria y de Valores, the "CNBV") and thus, may not be publicly offered in Mexico. The CNBV has not confirmed the accuracy of any information contained herein. The provision of investment management and investment advisory services ("Investment Services") is a regulated activity in Mexico, subject to strict rules, and performed under the supervision of the CNBV. These materials are shared for information purposes only, do not constitute investment advice, and are being shared in the understanding that the addressee is an Institutional or Qualified investor as defined under Mexican Securities (Ley del Mercado de Valores). Each potential investor shall make its own investment decision based on their own analysis of the available information. Please note that by receiving these materials, it shall be construed as a representation by the receiver that it is an Institutional or Qualified investor as defined under Mexican law. BlackRock México Operadora, S.A. de C.V., Sociedad Operadora de Fondos de Inversión ("BlackRock México Operadora") is a Mexican subsidiary of BlackRock, Inc., authorized by the CNBV as a Mutual Fund Manager (Operadora de Fondos), and as such, authorized to manage Mexican mutual funds, ETFs and provide Investment Advisory Services. For more information on the Investment Services offered by BlackRock Mexico, please review our Investment Services Guide available in www.blackrock.com/mx. This material represents an assessment at a specific time and its information should not be relied upon by the you as research or investment advice regarding the funds, any security or investment strategy in particular. Reliance upon information in this material is at your sole discretion. BlackRock México is not authorized to receive deposits, carry out intermediation activities, or act as a broker dealer, or bank in Mexico. For more information on BlackRock México, please visit: www.blackrock.com/mx. BlackRock receives revenue in the form of advisory fees for our advisory services and management fees for our mutual funds, exchange traded funds and collective investment trusts. Any modification, change, distribution or inadequate use of information of this document is not responsibility of BlackRock or any of its affiliates. Pursuant to the Mexican Data Privacy Law (Ley Federal de Protección de Datos Personales en Posesión de Particulares), to register your personal data you must confirm that you have read and understood the Privacy Notice of BlackRock México Operadora. For the full disclosure, please visit www.blackrock.com/mx and accept that your personal information will be managed according with the terms and conditions set forth therein.

In Peru, this private offer does not constitute a public offer, and is not registered with the Securities Market Public Registry of the Peruvian Securities Market Commission, for use only with institutional investors as such term is defined by the *Superintendencia de Banca, Seguros y AFP*.

In Uruguay, the Securities are not and will not be registered with the Central Bank of Uruguay. The Securities are not and will not be offered publicly in or from Uruguay and are not and will not be traded on any Uruguayan stock exchange. This offer has not been and will not be announced to the public and offering materials will not be made available to the general public except in circumstances which do not constitute a public offering of securities in Uruguay, in compliance with the requirements of the Uruguayan Securities Market Law (Law Nº 18.627 and Decree 322/011).

For investors in Central America, these securities have not been registered before the Securities Superintendence of the Republic of Panama, nor did the offer, sale or their trading procedures. The registration exemption has made according to numeral 3 of Article 129 of the Consolidated Text containing of the Decree-Law No. 1 of July 8, 1999 (institutional investors). Consequently, the tax treatment set forth in Articles 334 to 336 of the Unified Text containing Decree-Law No. 1 of July 8, 1999, does not apply to them. These securities are not under the supervision of the Securities Superintendence of the Republic of Panama. The information contained herein does not describe any product that is supervised or regulated by the National Banking and Insurance Commission (CNBS) in Honduras. Therefore any investment described herein is done at the investor's own risk. This is an individual and private offer which is made in Costa Rica upon reliance on an exemption from registration before the General Superintendence of Securities ("SUGEVAL"), pursuant to articles 7 and 8 of the Regulations on the Public Offering of Securities ("Reglamento sobre Oferta Pública de Valores"). This information is confidential, and is not to be reproduced or distributed to third parties as this is NOT a public offering of securities in Costa Rica. The product being offered is not intended for the Costa Rican public or market and neither is registered or will be registered before the SUGEVAL, nor can be traded in the secondary market. If any recipient of this documentation receives this document in El Salvador, such recipient acknowledges that the same has been delivered upon their request and instructions, and on a private placement basis.

In Singapore, this document is provided by BlackRock (Singapore) Limited (company registration number:200010143N) for use only with institutional as defined in Section 4A of the Securities and Futures Act, Chapter 289 of Singapore. This advertisement or publication has not been reviewed by the Monetary Authority of Singapore.

In Hong Kong, this material is issued by BlackRock Asset Management North Asia Limited and has not been reviewed by the Securities and Futures Commission of Hong Kong. This material is for distribution to "Professional Investors" (as defined in the Securities and Futures Ordinance (Cap.571 of the laws of Hong Kong) and any rules made under that ordinance.) and should not be relied upon by any other persons or redistributed to retail clients in Hong Kong.

FOR PROFESSIONAL, INSTITUTIONAL, QUALIFIED, WHOLESALE INVESTORS AND PERMITTED, PROFESSIONAL AND QUALIFIED CLIENT USE ONLY – NOT FOR PUBLIC DISTRIBUTION (PLEASE READ IMPORTANT DISCLOSURES)

In South Korea, this information is issued by BlackRock Investment (Korea) Limited. This material is for distribution to the Qualified Professional Investors (as defined in the Financial Investment Services and Capital Market Act and its sub-regulations) and for information or educational purposes only, and does not constitute investment advice or an offer or solicitation to purchase or sells in any securities or any investment strategies.

In Taiwan, independently operated by BlackRock Investment Management (Taiwan) Limited. Address: 28F., No. 100, Songren Rd., Xinyi Dist., Taipei City 110, Taiwan. Tel: (02)23261600.

In Australia & New Zealand, issued by BlackRock Investment Management (Australia) Limited ABN 13 006 165 975, AFSL 230 523 (BIMAL) for the exclusive use of the recipient, who warrants by receipt of this material that they are a wholesale client as defined under the Australian Corporations Act 2001 (Cth) and the New Zealand Financial Advisers Act 2008 respectively.

This material provides general information only and does not take into account your individual objectives, financial situation, needs or circumstances. Before making any investment decision, you should therefore assess whether the material is appropriate for you and obtain financial advice tailored to you having regard to your individual objectives, financial situation, needs and circumstances. Refer to BIMAL's Financial Services Guide on its website for more information. This material is not a financial product recommendation or an offer or solicitation with respect to the purchase or sale of any financial product in any jurisdiction.

This material is not intended for distribution to, or use by, any person or entity in any jurisdiction or country where such distribution or use would be contrary to local law or regulation. BIMAL is a part of the global BlackRock Group which comprises of financial product issuers and investment managers around the world. BIMAL is the issuer of financial products and acts as an investment manager in Australia. BIMAL does not offer financial products to persons in New Zealand who are retail investors (as that term is defined in the Financial Markets Conduct Act

2013 (FMCA)). This material does not constitute or relate to such an offer. To the extent that this material does constitute or relate to such an offer of financial products, the offer is only made to, and capable of acceptance by, persons in New Zealand who are wholesale investors (as that term is defined in the FMCA). BIMAL, its officers, employees and agents believe that the information in this material and the sources on which it is based (which may be sourced from third parties) are correct as at the date of publication. While every care has been taken in the preparation of this material, no warranty of accuracy or reliability is given and no responsibility for the information is accepted by BIMAL, its officers, employees or agents. Except where contrary to law, BIMAL excludes all liability for this information.

In China, This material may not be distributed to individuals resident in the People's Republic of China ("PRC", for such purposes, not applicable to Hong Kong, Macau and Taiwan) or entities registered in the PRC unless such parties have received all the required PRC government approvals to participate in any investment or receive any investment advisory or investment management services.

The information provided here is not intended to constitute financial, tax, legal or accounting advice. You should consult your own advisers on such matters. BlackRock does not guarantee the suitability or potential value of any particular investment. Investment involves risk including possible loss of principal. International investing involves risks, including risks related to foreign currency, limited liquidity, less government regulation, and the possibility of substantial volatility due to adverse political, economic or other developments. These risks are often heightened for investments in emerging/developing markets or smaller capital markets.

Any research in this document has been procured and may have been acted on by BlackRock for its own purpose. The results of such research are being made available only incidentally. The views expressed do not constitute investment or any other advice and are subject to change. They do not necessarily reflect the views of any company in the BlackRock Group or any part thereof and no assurances are made as to their accuracy.

This document is for information purposes only and does not constitute an offer or invitation to anyone to invest in any BlackRock funds and has not been prepared in connection with any such offer.