A Joint Factor Model for Bonds, Stocks, and Options

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Motivation

Integration across markets

Stocks, Bonds, and Options markets are partially integrated.

- Stocks and Bonds are claims on the assets of the firm.
  - $\Rightarrow$ Stocks and Bonds are options on firm value.
- Options are derivatives on stocks.
  - $\Rightarrow$ Options are compound options on firm value, Geske (1979).

Models of stochastic volatility (Du, Elkhami, and Ericsson, 2019; Doshi, Ericsson, Fournier, and Seo, 2022) provide a formal justification for why corporate securities share linkages but are not as tightly linked as in Merton (1974).

Some extant evidence

Perhaps no surprise that:

- Stock characteristics predict stock returns (obviously; it is 2023)
- Bond characteristics predict bond returns (Houweling and van Zundert, 2017; Chordia, Goyal, Nozawa, Subrahmanyam, and Tong, 2017)
- Option characteristics predict option returns (Goyal and Saretto, 2009)

But also:

- Stock characteristics predict bond returns (Bektić, Wenzler, Wegener, Schiereck, and Spielmann, 2019)
- Stock characteristics predict option returns (Bali, Beckmeyer, Moerke, and Weigert, 2023)
- Option characteristics predict stock returns (An, Ang, Bali, and Cakici, 2014)
- Option characteristics predict bond returns (Cao, Goyal, Xiao, and Zhan, 2023)

This Paper

- Existing factor models (Fama and French (2015) for stocks; Kelly, Palhares, and Pruitt (2023) for bonds) create factors by looking at own asset class characteristics.
  - → Value factor in stocks from book/market of stocks.
  - → Latent bond factors from bond characteristics.
- We therefore ignore information from other asset classes.
  - Remember: the asset classes should be linked in theory (and their are in practice).

What is missing?

- A systematic study of which characteristics (from stocks, bonds, and/or options) matter for which asset class (stocks, bonds, and/or options)
- A characterization of the *joint* risk-and-return trade-off through a joint factor model that, given theoretical justification, can handle cross-asset predictability we see empirically.

Data and Methodology

Data

U.S. firms for which we can collect stock, bond, and option information:

$\Rightarrow$ In total, 5,958 unique firms.

We start with 254 firm characteristics:

- 153 stock characteristics (such as size, value, momentum, etc.).
- 38 bond characteristics (such as age, credit rating, issuance size, etc.).
- 63 option characteristics (such as IV, open interest, greeks, etc.).

All characteristics are cross-sectionally rank standardized to lie between -0.5 and 0.5.

Sample period is Aug 2002 to Aug 2022 (limited by availability of bond data through TRACE).

Representative Bond & Option

Per firm, we potentially have access to multiple bonds and up to hundreds of options:

- To make the sample comparable across stocks, bonds, and options, we choose a representative bond and option for each firm in each month.
- For bonds, we follow Dieck-Nielsen et al. (2024) and calculate the value-weighted bond return for each firm in the sample.
- For options, we follow standard practice (Cao and Han (2013)) and use the at-the-money call contract that expires on the third Friday of the month after the next.

Return calculation

- Stock returns R^S_{t+1} as usual.

- Bond returns:

$$R^B_{t+1} = \frac{P_{t+1} + A_{t+1} + C_{t+1}}{P_t + A_t} - 1.$$

- Daily delta-hedged option returns:

$$R^O_{t+1} = \frac{1}{|\Delta_t S_t - O_t|}\left[O_{t+1} - O_t - \sum_{n=0}^{N-1} \Delta_{t_n}\left(S_{t_{n+1}} - S_{t_n}\right) - \sum_{n=0}^{N-1} \frac{R_{ft_n}}{365}\left(O_{t_n} - \Delta_{t_n} S_{t_n}\right)\right].$$

Introducing the **characteristic-managed portfolio**:

$$X_{t+1} = Z_t' R_{t+1} / N_{t+1}$$

CMPs with highest Sharpe ratios

	Bonds	Options	Stocks
Panel A: Top Bond CMPs			
S_ret_1_0	1.55***	−0.09	−0.15
O_S_ivrv	−1.20***	−2.38***	−0.21
S_iskew	1.11***	−0.98***	0.56*
Panel B: Top Option CMPs			
O_civpiv	−0.02	−3.65***	1.30***
O_rnk	−0.29	3.47***	0.00
O_ivud	−0.24	3.35***	−0.11
Panel C: Top Stock CMPs			
O_shrtfee	0.21	2.64***	−1.51***
O_fric	−0.17	−2.53***	1.51***
O_civpiv	−0.03	−3.29***	1.45***

⇒ Information from one asset class is potentially informative about other asset classes.

A Joint Factor Model

IPCA

How do we describe the joint risk-return trade-off?

$\Rightarrow$ *Instrumented Principal Component Analysis (IPCA)*:

$$R_{it+1}^{AC} = \beta_{it}^{AC\,\prime} F_{t+1} + \varepsilon_{it+1}, \qquad \beta_{it}^{AC} = z_{it}' \Gamma_{\beta}^{AC}$$

- Fs are latent factors (same for all assets and all asset classes).
- z_{it} are observable combination of stock, bond, option characteristics (same for all assets and all asset classes).
- β_{it}^{AC} are asset specific factor loadings.
- Γ_{β}^{AC} defines the mapping from characteristics to factor exposures (asset-class specific).

A Simple Example

Say that we have *three characteristics* per firm, BM (from the stock), Yld (from bonds), and IV (from options).

Then, βs for firm i are:

$$\begin{aligned}
\text{Stock } \beta_{it}^{S} &= \gamma_1^S BM_{it} + \gamma_2^S Yld_{it} + \gamma_3^S IV_{it} \\
\text{Bond } \beta_{it}^{B} &= \gamma_1^B BM_{it} + \gamma_2^B Yld_{it} + \gamma_3^B IV_{it} \\
\text{Option } \beta_{it}^{O} &= \gamma_1^O BM_{it} + \gamma_2^O Yld_{it} + \gamma_3^O IV_{it}.
\end{aligned}$$

γs are asset class specific; characteristics are the same.

$\rightarrow$ βs to a *joint set of factors* can differ per asset class.

Estimation

Estimation is simple with an alternating least squares approach:

$$\widehat{F}_{t+1} = \left(\widehat{\beta}_t' \widehat{\beta}_t\right)^{-1} \widehat{\beta}_t' R_{t+1},$$

$$\text{vec}\left(\widehat{\Gamma}_\beta\right) = \left(\sum_{t=1}^{T-1} \mathcal{Z}_t' \mathcal{Z}_t \otimes \widehat{F}_{t+1} \widehat{F}_{t+1}'\right)^{-1} \left(\sum_{t=1}^{T-1} \left[\mathcal{Z}_t \otimes \widehat{F}_{t+1}'\right]' R_{t+1}\right),$$

- $\widehat{F}_{t+1}$ are obtained through month-by-month cross-sectional regressions of excess returns on βs.
- Γ_β are coefficients of regressing individual returns on the latent factors F_{t+1} interacted with stacked characteristics $\mathcal{Z}_t$.

Pricing Performance

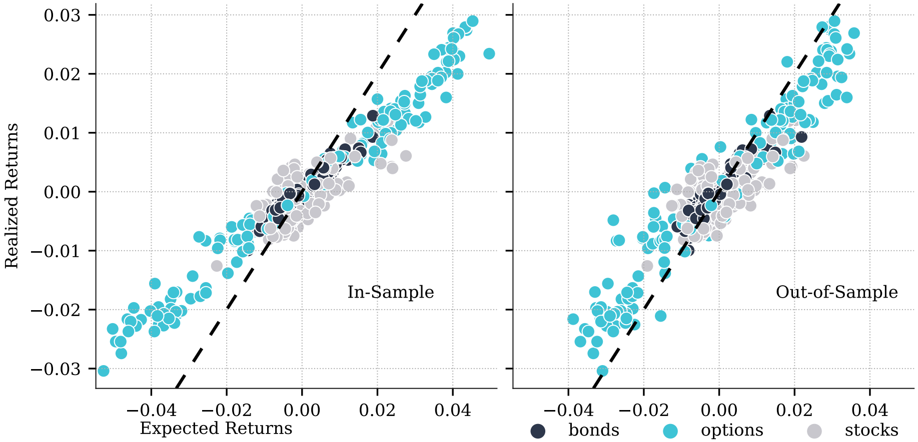
Pricing performance: In-sample and Out-of-sample

$K \rightarrow$	1		2		3		4		5	
	IS	OOS	IS	OOS	IS	OOS	IS	OOS	IS	OOS
Total	0.12	0.11	0.14	0.12	0.15	0.13	0.16	0.14	0.17	0.15
Bonds	0.25	0.25	0.26	0.27	0.27	0.28	0.29	0.32	0.30	0.34
Options	0.04	0.03	0.08	0.06	0.10	0.08	0.12	0.10	0.12	0.10
Stocks	0.12	0.12	0.15	0.14	0.15	0.14	0.16	0.14	0.16	0.14

$$\text{Total } R^2 = 1 - \frac{\sum_{it}\left(R_{it+1} - Z'_{it}\hat{\Gamma}_\beta \hat{F}_{t+1}\right)^2}{\sum_{it} R^2_{it+1}}$$

- Five factors are sufficient to explain between 12% (options) and 30% (bonds) of the return variation.
- Model fit is remarkably stable, with little deterioration out-of-sample.

Joint IPCA (with 5 factors) explains returns well



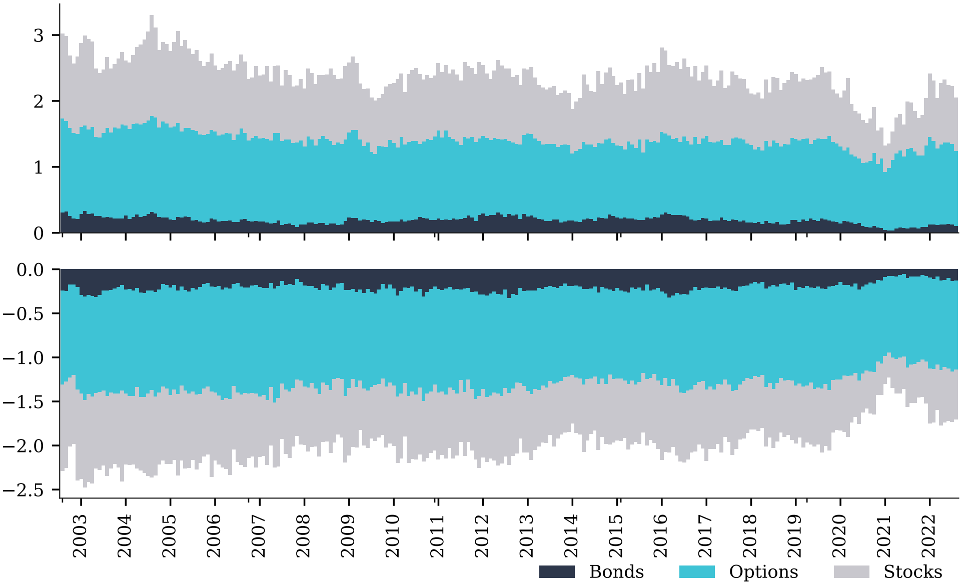
Average expected and realized returns of characteristic-managed portfolios of bonds, options, and stocks.

Joint Tangency portfolio

	Joint	Bonds	Options	Stocks
		Panel A: Returns		
Return	1.75	0.13	1.38	0.24
Std	0.88	0.27	0.82	0.38
SR	**6.88**	1.60	5.83	2.22
Skew	0.86	3.47	1.30	−0.81
Kurt	2.69	19.95	4.22	1.21
MDD	−1.44	−1.40	−1.22	−3.34
TO	1.03	0.40	2.38	1.12
TC	1.18	0.12	0.72	0.34
		Panel B: Correlation		
Bonds	0.25			
Options	0.90	**0.09**		
Stocks	0.19	**-0.32**	**-0.14**	

Adjusting for realistic-to-high levels of transaction costs still generates profitable investment advice.

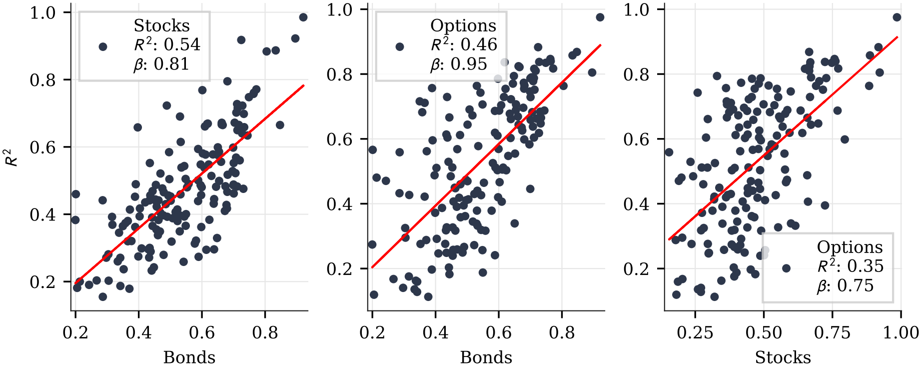
Tangency Portfolio Weights by Asset Class



Commonality

Commonality in predictability

If CMP of stocks managed by market cap is predictable, is the CMP of bonds managed by the same variable also predictable?



$\rightarrow$ Yes! Markets do seem (partially) integrated.

Influence of Latent Factors

Measure the influence of each latent factor by setting its realizations to zero and recording the drop in R^2, rescaled to sum to one:

K	Factor Influence				Cumulative Factor Influence			
$\downarrow$	Bonds	Options	Stocks	Total	Bonds	Options	Stocks	Total
1	**-0.27**	**-0.30**	**-0.22**	-0.25	-0.27	-0.30	-0.22	-0.25
2	**-0.58**	-0.21	**-0.53**	-0.48	-0.86	-0.51	-0.75	-0.72
3	-0.01	**-0.27**	**-0.08**	-0.11	-0.87	-0.79	-0.83	-0.83
4	-0.03	**-0.11**	**-0.12**	-0.10	-0.90	-0.91	-0.96	-0.94
5	**-0.12**	**-0.09**	-0.04	-0.06	-1.00	-1.00	-1.00	-1.00

- “Market factor” F1.
- “Bond-stock factor” F2.
- “Option-stock factors” F3/F4.
- “Bond-option factor” F5.

Top characteristics per asset class

	Bonds		Options		Stocks	
	Imp.	Sens.	Imp.	Sens.	Imp.	Sens.
Panel A: Top Characteristics for Bonds						
B_rating	1	0.29	57	0.11	57	−0.06
const	2	0.22	12	−0.10	1	0.12
S_ret_1_0	3	0.45	43	−0.15	23	−0.09
B_DURATION	4	0.00	106	−0.04	87	0.02
Panel B: Top Characteristics for Options						
O_C_embedlev	23	0.01	1	1.04	40	−0.08
O_C_theta	82	−0.01	2	0.73	67	0.01
O_S_rnv_30	41	−0.11	3	0.67	37	−0.09
O_S_rns_30	19	0.08	4	−0.62	71	0.02
Panel C: Top Characteristics for Stocks						
const	2	0.22	12	−0.10	1	0.12
S_zero_trades_126d	39	0.13	16	−0.35	2	0.34
O_S_demand_roll252D	17	0.04	56	−0.02	3	0.26
S_dolvol_126d	8	0.09	24	−0.20	4	0.07

Comparison

Comparison with single IPCA (1)

Joint tangency portfolio outperforms individual IPCA models:

	Joint	Single IPCA			
	IPCA	5 Bond	5 Option	5 Stock	$2+2+2$
Sharpe Ratio	6.88	1.37	5.90	2.30	5.99
Outperformance		4.59	0.82	3.81	0.74
		(12.92)	(6.76)	(8.00)	(6.06)

Outperformance measured by:

$$r_{t,\sigma=10\%}^{\mathit{JIPCA}} - r_{t,\sigma=10\%}^{\mathit{AC}} = \boldsymbol{\alpha} + \varepsilon_t$$

Comparison with single IPCA (2)

Joint IPCA leaves only 11 out of 219 CMP αs unexplained:

	Average returns	Unconditional alphas				
		Joint IPCA	Single IPCA			
			5 Bond	5 Option	5 Stock	$2+2+2$
Bonds	37	1	23	6	17	10
Options	142	7	139	20	129	14
Stocks	40	3	31	8	27	7
Σ	219	11	193	34	173	31

Benchmark factors for stocks, bonds, and options fail to explain αs:

		Unconditional alphas				
	Average	Joint	Benchmark Factors			
	returns	IPCA	MKTB	CS	FF6	Comb.
Bonds	37	1	32	21	33	21
Options	142	7	138	125	142	128
Stocks	40	3	36	25	35	33
Σ	219	11	206	171	210	182

Regressions on macroeconomic indicators

We regress each joint latent factor on several macroeconomic indicators:

	F1	F2	F3	F4	F5
const.	2.69^{***}	0.88^{***}	0.16	0.11	0.03
CFNAI	0.03	-0.01	-0.03	0.10	-0.03
UNC	-0.07	0.29^{***}	-0.25^{**}	-0.28^{**}	0.43^{***}
ICR	0.89^{***}	-1.91^{***}	0.18	-0.38^{**}	0.21
Adj.R^2	0.23	0.53	0.06	0.04	0.09

- Evidence that latent factors are exposed to (F1, ICR; F3, UNC) or hedge (F2, UNC/ICR; F5, UNC) macroeconomic risks.
- F4 is exposed to overall macroeconomic uncertainty but hedges deterioration in capital ratio of intermediaries.

What explains the latent factors?

We regress each latent factor on a constant and....

- ...the three macroeconomic indicators (CFNAI, UNC, ICR)
- ...or the benchmark factors (CS; bond MKT; FF6)

$$F_{t,k} = \alpha_k + \boldsymbol{\beta}_k \boldsymbol{X}_t + \varepsilon_{t,k}$$

We replace each factor with the fitted value from this regression:

$$\hat{F}_{t,k} = \alpha_k + \boldsymbol{\beta}_k \boldsymbol{X}_t,$$

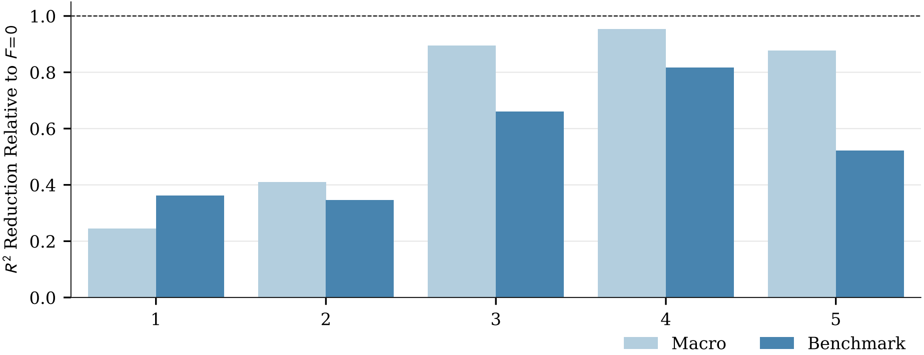
and record the resulting Total R^2.

Finally, we record the reduction in the model's Total R^2 relative to setting the realizations of the kth factor to zero:

$$\text{Relative Reduction (RR)} = \frac{R_X^2 - R^2}{R_{\text{zero}}^2 - R^2}.$$

⇒ RR=0: replaced factors explain return variation as well as true factors.

What explains the latent factors?



Macroeconomic information and benchmark factors explain some but not all of the explanatory power captured by joint IPCA.

$\Rightarrow$ Joint estimation uncovers valuable information for the risk-return trade-off across asset classes.

Conclusion

Conclusion

- A joint factor structure with asset class-specific betas explains the variation in stock, bond, and option returns well.
- Joint tangency portfolio has a SR higher than six and survives realistic-to-high levels of transaction costs.
- Joint IPCA factor model leaves fewer alphas than (i) single IPCA models; or (ii) observable factor models.
- All three markets are partially integrated.