

Disagreement, Skewness, and Asset Prices

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We present a frictionless model which bridges two seemingly unrelated empirical anomalies: (1) the negative relationship between dispersion in financial analysts' forecasts and expected returns (Diether et al., 2002) and (2) the negative relationship between idiosyncratic skewness and expected returns (Boyer et al., 2010; Conrad et al., 2013; Amaya et al., 2013; Boyer and Vorkink, 2014). The results obtain because (1) empirically, most stocks have positive expected skewness, (2) positive skewness implies that investors' demand schedules are convex in the relevant price range, and hence, (3) trades due to disagreement do not "cancel out"; asset prices are inflated even without short-selling constraints. Our theory further predicts that skewness and disagreement have an interactive pricing impact. We find support for this prediction in the cross section of stock returns.

KEYWORDS: the disagreement effect; the skewness effect; divergence of opinions.

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1 Introduction

Disagreement (e.g, divergence of opinions) is a ubiquitous feature of our world. In financial markets, Miller (1977) (and many others) argues that disagreement leads to inflated asset prices since short-selling costs/constraints hinder the incorporation of pessimists' beliefs in equilibrium. Short-selling frictions are a common assumption in this literature, because of the traditional view that, without any frictions, disagreement, per se, has no pricing implications since "price equals the average belief." This canonical result, however, generally obtains only when demand schedules are linear (as in the case of a CARA-normal environment or approximate mean-variance preferences). When payoffs are skewed, demand schedules are predictably curved, breaking this irrelevance.

In this paper, we develop a model to analyze the asset pricing implications of disagreement in a completely frictionless framework. In particular, short selling is allowed and is truly the "negative" of buying. In our model, we show that trades due to disagreement typically do not cancel each other out, because investors' demand schedules are generally nonlinear. Our key insight is that positive payoff skewness implies convex demand.¹ Recall that for any risk-avertter, demand is zero when price equals expected value. Combined with convexity, this implies that excess demand is positive when price equals the average belief. Hence, even if the average belief is correct, the equilibrium price must be "biased" upward to clear markets and disagreement has a non-trivial asset pricing effect. Finally, our theory generates joint predictions on the pricing implications of disagreement and skewness.

Our baseline model features two representative investors, a riskfree asset with infinitely elastic supply, and a single risky asset in zero net supply.² We normalize

¹Similarly, negative skewness implies concave demand.

²In Section 3.4, we show our results are robust to non zero supply.

the gross risk-free rate to unity. Trading occurs at time 0, and the risky asset pays a liquidating dividend at time 1. The two investors disagree about the distribution of the dividend at time 1. In our baseline model, the disagreement is modeled as follows: the two investors *disagree* about the mean of the dividend distribution but *agree* about its shape.³ Hence, they agree about all higher-order central moments.⁴

The two investors are price takers, and they have the same utility function, which we assume is strictly increasing, strictly concave, thrice continuously differentiable, and exhibits non-increasing absolute risk aversion (NARA).⁵ Note that the NARA class nests both CARA and CRRA preferences. Arditti (1967) shows that NARA is sufficient to imply a preference for positive skewness. In previous work (Goulding et al., 2021), we show NARA implies that an investor's demand schedule is convex in a neighborhood around her investor's subjective expected payoff if and only if the payoff skewness is positive.

Figure 1 provides a graphical intuition for the relationship between skewness and convexity. Start with the demand function for an asset with an arbitrary distribution, given by the thinner curve.⁶ Now consider an otherwise identical asset with higher skewness, with demand given by the thicker curve.⁷ Since a NARA investor has a preference for skewness, demand for this new asset must be weakly higher at any price. However, when price equals the subjective expected payoff, demand for any asset is zero. As a result, the thicker curve is more "curved" than the thinner curve;

³In Section 3.3, we show our results are robust to alternative structures of disagreement.

⁴Technically, this is not possible unless the distribution has unbounded support.

⁵The assumption of NARA ensures that a risky asset with a positive expected return is not an inferior good, i.e., wealthier investors allocate weakly more money to the risky asset. Arrow (1971) argues these are properties of any reasonable utility function.

⁶As drawn, its second derivative is positive, but the argument works for any value.

⁷We cautiously note that it is generally impossible to change skewness without affecting other moments unless the distribution is unbounded. However, as we show in the proof of Lemma 1, the graphical illustration is correct in a neighborhood around the subjective expected payoff, as the curvature of the demand function locally does not depend on moments higher than the third.

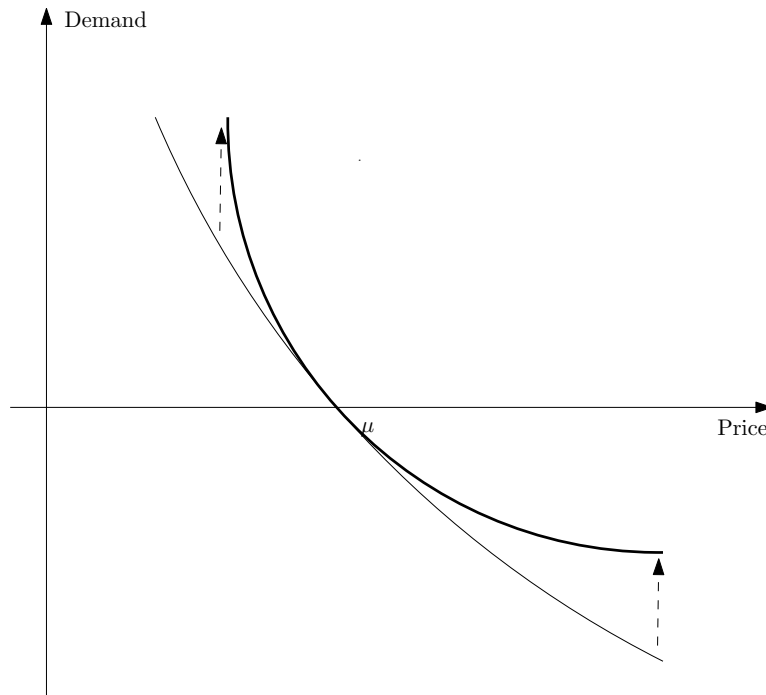


Figure 1: Higher Skewness Leads to a More Convex Demand Curve

This figure illustrates that *ceteris paribus*, higher skewness leads to a more convex demand curve. The thicker curve represents the demand associated with a higher skewness, while the thinner curve represents the demand associated with a lower skewness. Since a NARA investor has a preference for skewness, the thicker curve is weakly above the thinner curve for any price. Furthermore, when price equals the subjective expected payoff, demand is precisely zero, regardless of skewness. Put differently, both curves must cross the price axis at the same point. As a result, the thicker curve is more “curved” than the thinner curve in the neighborhood around the expected payoff.

increasing skewness locally increases the demand function’s convexity.

Returning to our baseline model, trades due to disagreement do not cancel each other out when the investors’ demand schedules are nonlinear. To see this point more concretely, consider a positively-skewed asset.⁸ For expositional convenience, we call the investor whose belief is more optimistic as the positive type and the other as

⁸To be precise, we assume the disagreement is “small”. Recall the two investors agree about the shape of the dividend distribution, implying their demand schedules are parallel. See Section 2.2 for more details. In Section 3.3, we show our results are robust even if investors disagree about higher moments.

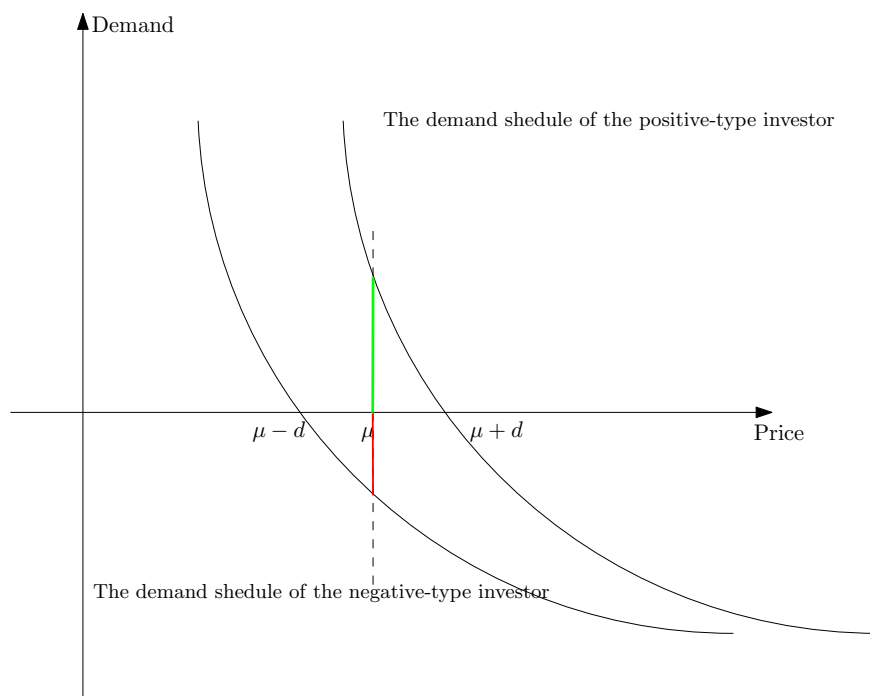


Figure 2: Disagreement Lowers Returns for a Positively-Skewed Asset

This figure illustrates the intuition that disagreement lowers returns for a positive-skewed asset. Because of positive skewness, an investor's demand is convex in the neighbor of the asset's expected payoff under the investor's belief. The two curves represents the demand schedules of a positive-type investor and a negative-type investor respectively. For example, the demand of a positive-type investor crosses zero at price $\mu + d$, which is the risky asset's expected payoff under a positive type's belief. Now suppose the price is equal to μ . The green (upper) line represents the shares that a positive-type investor would buy, while the red (lower) line represents the shares that a negative-type investor would sell. Because of the convexity, the length of the green line is greater than that of the red line. In other words, at price μ , we have excess demand of the asset. To clear the market, the equilibrium price must be higher than μ .

the negative type. Figure 2 plots their demand schedules; each investor's demand schedule crosses zero when price is equal to her subjective expected payoff. Consider a candidate equilibrium price equal to the average belief, $\hat{p} = \mu$. At this price, the positive-type investor perceives the asset as under-priced while the negative-type investor perceives it as over-priced. Hence, the positive type would go long the asset while the negative type would short it. The question is whether the market clears

at this price, μ . We show that the answer is no; at this price there would be excess demand. This is because buying a positively-skewed asset entails more desirable upside risk whereas shorting the asset involves more downside risk. The positive type would want to buy more shares of the asset than those shorted by the negative type. To clear the market, the equilibrium price must be higher than μ . In summary, the equilibrium price will be higher than the expected payoff for a positively-skewed asset.⁹

Thus far, we have shown that trades due to disagreement do not cancel out, and disagreement has a non-trivial impact on asset prices even in a setup without short-selling constraints. We further quantitatively analyze how disagreement affects asset prices. Using a Taylor expansion, we find that when the disagreement is "small",

$$\text{dollar excess return} \approx -\text{skewness} * \text{disagreement}^2, \quad (1)$$

where *disagreement* is the standard deviation of beliefs. Inspection of equation (1) yields the following three effects. First, our theory generates the skewness effect: Expected return is decreasing in skewness. Second, if skewness is positive, expected return is decreasing in disagreement. This is the disagreement effect in the empirical literature. Finally, there is an interaction effect: higher skewness strengthens the disagreement effect, and vice versa.

Our paper makes two main contributions. First, given that most stocks have positive expected idiosyncratic skewness (Boyer et al., 2010), our theory offers an explanation for the documented negative relationship between dispersion in financial analysts' earnings forecasts and expected returns. The mechanism through which disagreement lowers expected returns is novel. Trading is frictionless in our model,

⁹Similarly, the equilibrium price will be lower than the expected payoff for a negatively-skewed asset.

unlike traditional papers which assume short-selling is impossible. This distinction implies that our framework is better suited to explain the “mispricing” due to disagreement for the vast majority of stocks which do not have “special” short rebates (Drechsler and Drechsler, 2021).

Second, the interaction effect between disagreement and skewness is novel to the theoretical asset pricing literature. On a sample of U.S. stocks, our empirical test reveals that a portfolio with is skew and disagreement-neutral but exploits the interaction effect has an average monthly Fama-French alpha of 1.2% with a t -statistic of >4 . To our knowledge, we are the first to demonstrate this phenomenon, both theoretically and empirically.

1.1 Related Literature

A large literature explores the role of disagreement in financial markets.¹⁰ Consistent with empirical findings, these papers predict that disagreement leads to inflated prices and lower expected returns. Short-selling constraints are a common ingredient in these models. Our model differs in that trading is frictionless.

A recent paper by Martin and Papadimitriou (2019) studies disagreement in a binomial tree setting with log investors and no trading frictions. Our paper is different in that we do not specify a particular utility function nor do we impose parametric assumptions on the risky asset’s payoff distribution.

Yan (2010) points out that individual biases may not necessarily cancel out when demand is nonlinear. However, that paper takes non-linearity an assumption. Our

¹⁰See Hong and Stein (2007) and Xiong (2013) for recent surveys. Miller (1977); Jarrow (1980); Diamond and Verrecchia (1987); Chen et al. (2002) explore static models while Harrison and Kreps (1978); Scheinkman and Xiong (2003); Hong et al. (2006) study the dynamics of speculative bubbles. On issues related to trading volumes, price volatility, prices comovement, and informed trading, see Harris and Raviv (1993); Kandel and Pearson (1995); Cao and Ou-Yang (2008); Dumas et al. (2009); Banerjee and Kremer (2010); Ottaviani and Sørensen (2015); Atmaz and Basak (2018); Banerjee et al. (2018); Chabakauri and Han (2020) among others.

paper provides a microfoundation for non-linear demand, relying on only the mild NARA utility assumption. Moreover, our theory leads to joint asset pricing implications of disagreement and skewness which are absent from Yan (2010).

Several recent empirical studies explore the relationship between ex-ante (idiosyncratic) skewness and expected returns. Though they use different predictors of realized skewness, all find evidence that stocks with higher skewness have lower average returns, even after controlling for co-skewness (Harvey and Siddique, 2000) or empirically motivated factor models.

Compared to the theoretical literature on disagreement, there are relatively few models which generate a preference for skewness. Brunnermeier and Parker (2005) and Brunnermeier et al. (2007) show that anticipatory utility and optimally biased expectations generate a preference for skewness. Barberis and Huang (2008) shows that cumulative prospect theory does the same. Mitton and Vorkink (2007) show that if some investors have mean-variance preferences and others have mean-variance-skew preferences (as in Kraus and Litzenberger, 1976), the idiosyncratic skewness phenomenon obtains even without trading frictions or disagreement. In a previous paper (Goulding et al., 2021), we show that NARA utility and the presence of noise traders does the same. In that paper, there is no role for disagreement since all optimizing agents have homogeneous beliefs.

Road Map. The paper is organized as follows. In Section 2, we present a simple baseline model to derive the main results. In Section 3, we show our main results hold in several extensions of the baseline model, including disagreement with arbitrary types (Sec 3.1), heterogeneous preferences (Sec 3.2), disagreement on higher-order moments (Sec 3.3), and non-zero aggregate supply (Sec 3.4). In Section 4, we present the empirical method, data sample, and empirical analyses. We conclude the

main text in Section 5. All proofs are relegated to the appendix.

2 Model

In this section, we present a simple model to understand the main results. We first describe the environment and investor beliefs. We then derive a Taylor expansion for demand curves and solve for equilibrium price and expected return. Finally, we explore comparative statics with respect to skewness, disagreement, and their interaction.

2.1 Model Setup

Consider a two-period financial market, $t \in \{0, 1\}$. There is a risk-free asset, with both its price and payoff normalized to one, and a single risky asset, with time-0 price p and time-1 payoff $\tilde{\theta}$.

There is a continuum of investors in the financial market, indexed by $i \in [-1, 1]$. We assume that all investors have a same utility function $u(w)$ and initial wealth w_0 . The utility function $u(\cdot)$ is strictly increasing, strictly concave, and thrice continuously differentiable. We further assume the utility function exhibits non-increasing absolute risk aversion (NARA). In Section 3.2, we study heterogeneous utility functions.

We allow investors to disagree about the time-1 payoff of the risky asset. Specifically, each investor can be either a positive type or a negative type. For the ease of exposition, we assume investors from $(0, 1]$ are of positive type, and investors from $[-1, 0]$ are of negative type.¹¹ In Section 2, we focus on the following structure of

¹¹Because of the continuum assumption, it does not matter whether we assign investor 0 to be positive or negative.

disagreement. We assume a positive-type investor believes that the time-1 payoff is drawn from CDF F_d while a negative-type investor believes the payoff is drawn from CDF F_{-d} , where the notation F_t means a horizontal shift of a CDF F , i.e., $F_t(x) = F(x - t), \forall x$. When $d = 0$, all investors have the same belief, i.e., there is no disagreement. So we refer to d as the level of disagreement: higher d represents a larger degree of disagreement (i.e., more dispersed beliefs). We remark that this structure of disagreement may not be realistic if the dividend distribution is bounded and investors can write side contracts.¹² We assume this structure in our baseline model to facilitate exposition. In Sections 3.1 and 3.3, we show our results are robust to other structures of disagreement.¹³

Investor i submits a demand schedule $x_i(p)$, which specifies how many shares of the risky asset she would buy or sell at price p . We further assume that the supply of the risky asset is zero, so the market's clearing condition is $\int_{-1}^1 x_i(p) di = 0$, which determines the equilibrium price p . In Section 3.4, we allow for non-zero aggregate supply.

To summarize, an equilibrium consists of $(\{x_i(\cdot) : i \in [-1, 1]\}, p)$ such that the demand schedule $x_i(p)$ solves investor i 's expected utility maximization problem and the price p clears the market.

¹²This is because the upper bound of a positive type's perceived distribution is outside the support of the perceived distribution from a negative type. If side contracts are allowed, investors would bet an infinite amount for a contract that only pays outside the support of their respective perceived distribution. In our baseline model, one way to justify our assumption is that side contracts are not allowed or sufficiently costly to write, which is to say that the market is incomplete. We can justify our assumption by assuming the support of the dividend distribution is the entire real line. Overall, we choose the simple structure in our baseline model to highlight the mechanism through which disagreement affects asset prices. As we show in Sections 3.1 and 3.3, our results do not depend on this particular structure, and our results are robust to general structures of disagreement.

¹³In our model, the financial market is generally incomplete. However, we note that our results hold when the probability space that the random variable θ lives in is a two-state space, in which case the financial market is complete.

2.2 Model Analysis

We start with an investor's problem. Formally, the demand schedule $x_i(p)$ solves the following program:

$$x_i(p) = \arg \max_x E[u(w_0 + x(\tilde{\theta} - p))].$$

where the expectation is taken under investor i 's belief. Since we have two types of beliefs, we write E_+ and E_- to denote the expectation under the belief of a positive type and a negative type, respectively, and $x_+(p)$ and $x_-(p)$ denote the positive-type and negative-type investor's demand, respectively. We work with the positive-type investor's demand first, and the negative-type investor's demand can be derived accordingly.

Suppose $i > 0$ so the investor is a positive type. Since u is strictly concave and thrice continuously differentiable, the necessary and sufficient condition which determines $x_i(p)$ is given by the first-order condition (FOC):

$$E_+[u'(w_0 + x_i(p)(\tilde{\theta} - p))(\tilde{\theta} - p)] = 0. \quad (2)$$

For notational simplicity, under the expectation E_+ , we write the expected payoff as $\mu_d = E_+(\tilde{\theta})$, the variance of payoff as $\sigma_d^2 = E_+(\tilde{\theta} - \mu_d)^2$, and the skewness of payoff as $s_d = \frac{E_+(\tilde{\theta} - \mu_d)^3}{\sigma_d^3}$. Due to the assumption that the CDF F_d is a shifted version of CDF F , it can be shown that the variance and the skewness, σ_d and s_d , do not depend on d . So, we can write $\sigma = \sigma_d$ and $s = s_d$. Furthermore, the expected payoff is given by the following equation.

$$\mu_d = \int x dF_d(x) = \int x dF(x - d) = \int (x - d) dF(x - d) + d = \int x dF(x) + d,$$

where the first equality comes from the definition of μ_d , the second equality comes

from the definition of F_d , the third equality comes from $x = x - d + d$ and $\int dF(\cdot) = 1$, and the last equality comes from relabeling $x - d$ with x . We write $\mu \equiv \int x dF(x)$, i.e., the expected payoff under the belief given by F . Then it follows that $\mu_d = \mu + d$.

Returning to the FOC (2), note that when the price is given by the expected payoff under investor i 's belief: $p = E_+ \tilde{\theta} = \mu_d$, the zero demand $x = 0$ solves the FOC. It follows that $x_+(\mu_d) = 0$, given that the strict concavity of u guarantees a unique solution of the FOC. We next present the results regarding the behavior of $x_+(p)$ when p is close to μ_d . The next lemma, adopted from Goulding et al. (2021), summarizes the result.

Lemma 1. *The demand schedule of a positive-type investor is given by*

$$x_+(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma^2} (p - \mu_d) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s}{\sigma^3} (p - \mu_d)^2 + o(1)(p - \mu_d)^2, \quad (3)$$

where the little- o notation $o(1)$ is an unknown function that converges to 0 as $p \rightarrow \mu_d$.

We obtain Lemma 1 from a Taylor expansion of $x_+(p)$ around $p = \mu_d$. To understand the lemma, We make two remarks. First, as we can see from the lemma, the slope of the demand function at $p = \mu_d$ is given by $x'_+(\mu_d) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma^2}$, which is negative because $u'' < 0$. Consider a special case when the utility function is given by a constant absolute risk aversion (CARA) utility function and the payoff of the underlying risky asset $\tilde{\theta}$ is normally distributed. In this case, skewness is zero, and the quadratic term in equation (3) is zero, consistent with the well-known result that the demand function is linear under CARA and normality.

Second, the quadratic term in equation (3) captures the curvature of the demand function at $p = \mu_d$. It is clear that the sign of $x''_+(\mu_d)$ solely depends on s , since the utility function has a negative second derivative and a positive third derivative. Specifically, if $s > 0$, then $x''_+(\mu_d) > 0$; if $s < 0$, then $x''_+(\mu_d) < 0$. Lemma 1 states that a

NARA investor's demand of a positively- (negatively-) skewed risky asset is strictly convex (concave) at $p = \mu_d$. While the formal proof is in the appendix, Figure 3 offers a graphical intuition for the result. Recall that a NARA investor has a preference for positive skewness. This fact implies that when we increase the skewness of the underlying asset, holding everything else equal, the investor would demand weakly more shares of the risky asset, i.e., the demand function is weakly higher, consistent with the observation that the solid curve is above the dashed line in Figure 3.¹⁴ Furthermore, as we discussed above, the demand has to be zero when $p = \mu_d$ because of the FOC (2). As a result, the "curvature" of the demand function of a positively skewed asset at $p = \mu_d$ has to be greater than the "curvature" of a straight line. Since the curvature of a straight line is 0, the curvature of the solid curve at $p = \mu_d$ has to be positive.

Similar to Lemma 1, the demand schedule of a negative-type investor is given by

$$x_-(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma^2} (p - \mu_{-d}) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s}{\sigma^3} (p - \mu_{-d})^2 + o(1)(p - \mu_{-d})^2, \quad (4)$$

where the little-o notation $o(1)$ is an unknown function that converges to 0 as $p \rightarrow \mu_{-d}$.

To complete the equilibrium analysis, we need to use the market's clearing condition to solve for the equilibrium price. Setting the aggregate demand to be zero, we can derive the price as a solution to the following equation.

$$x_+(p) + x_-(p) = 0.$$

¹⁴We cautiously note that it is generally impossible to increase skewness without affecting other moments. However, as we show in the proof of Lemma 1, the graphical illustration is correct in the neighborhood of $p = \mu_d$, as the curvature of the demand function at $p = \mu_d$ does not depend on moments higher than the third.

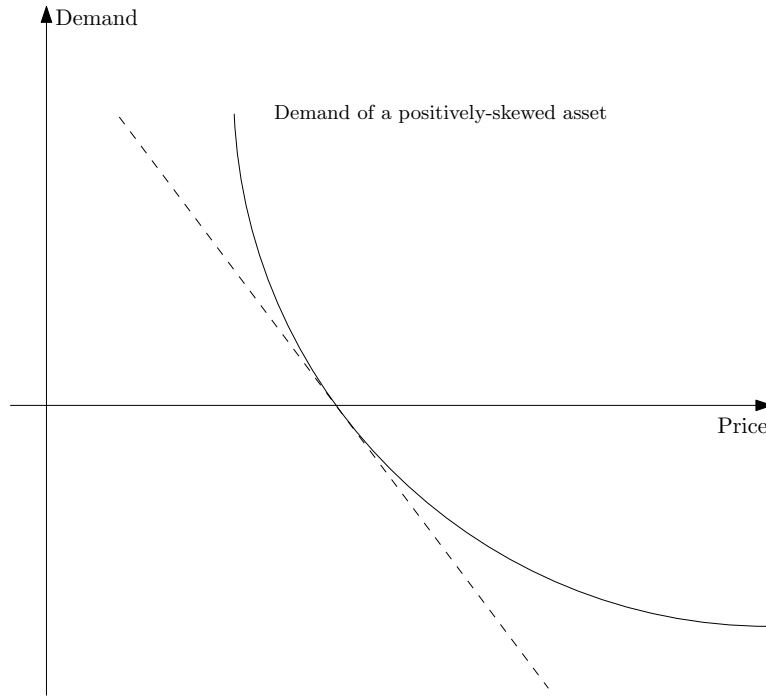


Figure 3: The Demand Schedule of a Positively-Skewed Asset

This figure illustrates the shape of the demand schedule. The dashed line is linear demand, which obtains under CARA and normality. The solid curve is the demand schedule when the underlying risky asset is positively skewed. The solid curve and the dashed line are tangent at the point $(p, x) = (\mu_d, 0)$. Since the solid curve has to be weakly higher than the dashed line, the curvature of the solid curve at $p = \mu_d$ has to be positive, consistent with Lemma 1 that the demand function of a positively skewed asset is convex at $p = \mu_d$.

The following proposition summarizes our main finding.

Proposition 1. *There exists a $\bar{d} > 0$ such that if $d < \bar{d}$, then the equilibrium price is given by the following equation.*

$$p = \mu + \frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2} \frac{s}{\sigma} d^2 + o(1)d^2,$$

where the little- o notation $o(1)$ is an unknown function that converges to 0 as $d \rightarrow 0$.

The term $\frac{u'''(w_0)u'(w_0)}{(u''(w_0))^2}$ is always positive because that the NARA assumption implies $u''' > 0$. Suppose the risky asset has a positive skewness. Proposition 1 states

that the equilibrium price will be higher than μ , which is the price when there is no disagreement, i.e., $d = 0$, or the marginal investor has the average belief. Proposition 1 implies that a higher level of disagreement can make the equilibrium price higher, equivalently, return lower. This result is consistent with a large literature in empirical asset pricing. Moreover, Proposition 1 provides a functional form for the deviation of the equilibrium price from μ .

Figure 2 provides the intuition. Suppose the underlying risky asset is positively-skewed so that an investor's demand is convex in the neighbor of the asset's expected payoff (Lemma 1). The two curves represent the demand schedules of a positive-type investor and a negative-type investor respectively. For example, the demand of a positive-type investor crosses zero at price $\mu + d$, which is the risky asset's expected payoff under a positive type's belief. Now suppose the price is equal to μ . The green line represents the shares that a positive-type investor would buy, while the red line represents the shares that a negative-type investor would sell. Because of the convexity, the length of the green line is greater than that of the red line. Put differently, since buying a positively-skewed asset entails more desirable upside risk whereas shorting the asset involves more downside risk, the positive type would buy more shares of the asset than those shorted by the negative type. So, at a candidate equilibrium price μ , we have excess demand of the asset. To clear the market, the equilibrium price must be higher than μ .

Define the dollar return to be $R := \mu - p$. Immediately from the definition and Proposition 1, we have the following corollary on the dollar return.

Corollary 1. *There exists a $\bar{d} > 0$ such that if $d < \bar{d}$, the dollar return is*

$$R = -\frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2} \frac{s}{\sigma} d^2 + o(1)d^2.$$

where the little- o notation $o(1)$ is an unknown function that converges to 0 as $d \rightarrow 0$.

2.3 Comparative Statics

Corollary 1 allows us to further analyze how various parameters affect the return prediction.

Proposition 2. *There exists a $\bar{d} > 0$ such that if $d < \bar{d}$, then the dollar excess return satisfies the following properties.*

1. *(The skewness effect): The return is decreasing in skewness.*
2. *(The disagreement effect): If skewness is positive, the return is decreasing in d .*
3. *(The interaction effect): Higher skewness strengthens the disagreement effect, and vice versa.*

Proposition 2 offers empirical predictions that we test in Section 4. The proposition offers predictions on two well-known anomalies: (1) the negative relationship between ex-ante return skewness and expected returns and (2) the negative relationship between disagreement and expected returns. Furthermore, to the best of our knowledge, the interaction effect from our theory is novel to the theoretical asset pricing literature.

Given that σ shows up in Corollary 1, the reader might wonder why we do not present predictions related to standard deviation. It may be helpful to consider the following argument. First, the magnitude of the disagreement parameter d is likely affected by the standard deviation σ . Typically, a stock with more uncertain payoffs (i.e., higher σ) may be associated with a larger level of disagreement about its expected payoff. One natural thought is to define disagreement in a unit-free fashion.

Suppose $d = \sigma\delta$, where δ measures investors' disagreement about the asset's expected payoff for every unit of standard deviation. Replacing d with $\sigma\delta$ in Corollary 1, we obtain the dominant term in the dollar return to be $-\frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2}s\sigma\delta^2$. Comparing it with Corollary 1, we note that these two settings offer contradictory predictions on standard deviation. In all, it is important to be clear about what exactly the disagreement proxy we use and what it measures before we can make predictions on the standard deviation. In the meantime, given our flexible framework, we argue that, if we believe that our baseline model is a good approximation of the financial market in practice, it becomes an empirical question to test which relationship between disagreement and standard deviation better explains the data.

3 Discussion

Section 2 presents a simple two-type disagreement model to highlight that disagreement can lower returns for a positively-skewed stock. In addition, our results are derived in a setup without short-selling constraints. In this section, we relax the simplifying assumptions in Section 2, and we show that our main results are robust to several extensions.

3.1 Disagreement with Arbitrary Types

In Section 2, there are two types of investors: the positive type and the negative type. We show that this is not an essential assumption. In this subsection, we assume investor i believes that the time-1 payoff is draw from CDF F_{d_i} , where d_i is drawn from a bounded mean-zero random variable \tilde{d} , whose variance is denoted as $Var(\tilde{d})$.¹⁵ In particular, \tilde{d} does not have to be symmetric. We assume that \tilde{d} is discrete to avoid

¹⁵Because of the bounded assumption, the variance is finite.

technical issues. We note that the model in Section 2 can be rephrased as a special case where the random variable \tilde{d} is binary, i.e., only takes two possible values.

The following proposition summarizes the result.

Proposition 3. *There exists a $\bar{d} > 0$ such that if $\text{Var}(\tilde{d})$ is bounded by \bar{d} , then the equilibrium price is given by the following equation.*

$$p = \mu + \frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2} \frac{s}{\sigma} \text{Var}(\tilde{d}) + o(1)\text{Var}(\tilde{d}),$$

where the little-o notation $o(1)$ is an unknown function that converges to 0 as $\text{Var}(\tilde{d}) \rightarrow 0$.

3.2 Heterogeneous Utility Functions

So far, all our analysis assumes homogeneous utility functions. In this subsection, we show that this assumption does not drive our results. To simplify the analysis, we assume each investor has one of the two utility functions: $u_1(\cdot)$ and $u_2(\cdot)$. We assume that both utility functions exhibit NARA. We return to our baseline model and assume that each investor can be either a positive or a negative type.

First, we consider the case that each investor can be one of the following four types with equal probability: u_1 and positive type; u_2 and positive type; u_1 and negative type; u_2 and negative type. Put differently, utility function being u_1 or u_2 and belief being positive or negative are independent. In this case, it is clear that the equilibrium price is higher than the expected payoff under the average belief, μ . This is because at the price μ , there is excess demand among the two types with the same utility function: u_k and negative, and u_k and positive, for $k = 1, 2$. Summing over k still gives excess aggregate demand. In all, the equilibrium price has to be higher than μ to clear the market.

Second, we consider the case where each investor can be one of the following two types with equal probability: u_1 and positive belief type; u_2 and negative belief type. In other words, investors with positive belief necessarily have the utility function u_1 , while investors with negative belief necessarily have u_2 . We define the benchmark price as the price obtains when investors have the mean-variance preference, in which case the demand schedule is linear. In our context, the benchmark price is denoted as p_0 , which is given by the following equation.

$$p_0 = \frac{\frac{u'_1(w_0)}{-u''_1(w_0)}\mu d + \frac{u'_2(w_0)}{-u''_2(w_0)}\mu - d}{\frac{u'_1(w_0)}{-u''_1(w_0)} + \frac{u'_2(w_0)}{-u''_2(w_0)}}.$$

That is, the benchmark price is the weighted average of the expected payoff under each type's belief, where the weight is given by the reciprocal of each type investor's absolute risk aversion. Intuitively, a less risk-averse investor, trading more aggressively, gets a larger weight in determining the benchmark price.

We are now ready to present the result.

Proposition 4. *There exists a $\bar{d} > 0$ such that if $d \leq \bar{d}$, then the equilibrium price is greater than the benchmark price p_0 if and only if $s > 0$.*

We make a final remark. Section 2 assumes that all investors have same initial wealth w_0 . If investors have different initial wealth, effectively, investors have different risk aversion and prudence. As we can see from the analysis in this subsection, Proposition 4 still holds. So, the same initial wealth assumption is not crucial.¹⁶

¹⁶Our benchmark price p_0 is consistent with Yan (2010), who shows that the idiosyncratic biases do not cancel out because of different initial wealth.

3.3 Disagreement On Other Moments

In Section 2, the two types of investors disagree on the expected payoff but agree on the standard deviation and the skewness of the time-1 payoff. In this subsection, we discuss the possibility of disagreement on other moments. For the ease of exposition, we assume there are two types of investors, type A and type B . The probability of each type is one half. The two types of investors can disagree about the distribution of the time-1 payoff. We assume that the type j investor believes that the expected payoff is μ_j , the standard deviation is σ_j , and the skewness is s_j , for $j = A, B$.

First, we note that if $\mu_A = \mu_B = \mu$, then the equilibrium price is also $p = \mu$. This is because each type investor demands a zero share of the asset so the market is cleared at $p = \mu$, given our zero aggregate supply assumption. Therefore we assume $\mu_A \neq \mu_B$ so that disagreement “matters”.

Now, we can write each type of investors’ demand schedule in a neighborhood around μ_j as the following.

$$x_A(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_A^2} (p - \mu_A) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s_A}{\sigma_A^3} (p - \mu_A)^2 + o(1)(p - \mu_A)^2, \quad (5)$$

$$x_B(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_B^2} (p - \mu_B) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s_B}{\sigma_B^3} (p - \mu_B)^2 + o(1)(p - \mu_B)^2, \quad (6)$$

where $x_j(p)$ denotes type j ’s demand, for $j = A, B$.

We define the benchmark price as the price obtains when investors have the mean-variance preference, in which case the demand schedules are linear. In our context, the benchmark price is given by the following equation.

$$p_0 = \frac{\frac{\mu_A}{\sigma_A^2} + \frac{\mu_B}{\sigma_B^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}}.$$

Intuitively, the benchmark price is the weighted average of the expected payoff under each type's belief, where the weight is given by the reciprocal of each type investor's belief about the variance of the payoff. This is because an investor who believes the variance is smaller would trade more aggressively, implying a higher weight in determining the benchmark price.

We are now ready to present the result.

Proposition 5. *There exists a $\bar{\epsilon} > 0$ such that if $|\mu_A - \mu_B| \leq \bar{\epsilon}$, then the equilibrium price is greater than the benchmark price p_0 if $\sigma_{AS_A} + \sigma_{BS_B} > 0$; the equilibrium price is lower than the benchmark price p_0 if $\sigma_{AS_A} + \sigma_{BS_B} < 0$.*

When $\sigma_{AS_A} + \sigma_{BS_B} = 0$, it is ambiguous whether the equilibrium price is higher or lower than the benchmark price. In some sense, Proposition 5 shows that $\sigma_{AS_A} + \sigma_{BS_B}$ is a "sufficient statistic" that plays the role of skewness in Proposition 1.

3.4 Non-Zero Aggregate Supply

So far all our analysis assumes zero aggregate supply. In this subsection, we discuss the case when the aggregate supply is not zero. Denote the aggregate supply as χ . To be clear, we return to the baseline model in Section 2 and only change the aggregate supply from zero to χ . In particular, we still assume investors have the homogeneous utility function and there are positive type and negative type investors.

It is important to discuss what the benchmark price would be in this case. Ideally, the benchmark price should approach to the non-disagreement price as the level of disagreement approaches to zero. Define $x_0(p)$ as the demand schedule for an investor whose belief is given by $F(\cdot)$. As a result, both $x_+(\cdot)$ and $x_-(\cdot)$ approach to $x_0(\cdot)$ as $d \rightarrow 0$. We define the benchmark price as the price obtains when $d = 0$. That

is, the benchmark price p_0 solves the following equation.¹⁷

$$2x_0(p_0) = \chi. \tag{7}$$

Proposition 6. *Suppose the risky asset's payoff, $\tilde{\theta}$, is bounded under both type investor's belief, and there is non-zero aggregate supply χ . Let p_0 denote the benchmark price as defined in (7). Then, there exists thresholds $\bar{d} > 0$, $\bar{s} > \underline{s}$, such that the following properties hold.*

- *If $s > \bar{s}$ and $d < \bar{d}$, the equilibrium price is greater than p_0 .*
- *If $s < \underline{s}$ and $d < \bar{d}$, the equilibrium price is smaller than p_0 .*

4 Empirical Test

In this section, we test the prediction that skewness and disagreement have an interactive pricing effect, summarized in Section 2.3, on a sample of U.S. stocks.

4.1 Data

Our sample is formed as the intersection of the Center for Research in Security Prices (CRSP), COMPUSTAT, and Institutional Brokers Estimate System (IBES) universes. We obtain monthly stock returns (adjusted for delisting bias¹⁸), prices,

¹⁷We remark that the benchmark price defined here is consistent with the one defined in the previous subsections where we used the mean-variance preference. Specifically, the mean-variance preference implies a linear demand function that pass through the point defined in (7). Despite the disagreement on the first moment, aggregating demand function is still linear line that pass through the point defined in (7). So the equilibrium price associated with the mean-variance preference is given by p_0 .

¹⁸This adjustment is suggested by Shumway (1997). Our results are not sensitive to this adjustment.

volume, and shares outstanding from CRSP; and accounting data from COMPUSTAT. Analysts' forecasts data are taken from the U.S. Unadjusted Detail History dataset of IBES. Our primary proxy for forecast dispersion, DISP, is essentially the same as that used by Diether et al. (2002) and many others: the month-end standard deviation of current-fiscal-year earnings per share (EPS) estimates scaled by the absolute value of the mean forecast across analysts tracked by IBES, from December 1983 to December 2013. Our primary proxy for skewness, SKEW, is the monthly expected total skewness measure for each stock as in Boyer et al. (2010), provided by the authors from July 1969 to December 2011. Therefore, our sample covers all months in the period from December 1983 to December 2011.

We apply standard filters and maintain a complete case for our baseline sample. Our sample includes only common shares traded on NYSE, AMEX, or NASDAQ having price above \$5. Firms must have positive market equity and book equity (for computing their book-to-market ratios), and non-missing book debt (for computing leverage). Accounting data is usually reported within a three-month delay after the fiscal end month, so firms must have a fiscal end date at least three months prior to the current month so that the information was plausibly available to market participants for the current month. Firms must have 12 months of returns (for computing momentum and short-term reversals) and a return for the subsequent month. Monthly variables derived from daily data such as idiosyncratic volatility, illiquidity, or factor loadings require a minimum of 10 days of daily returns in the month.¹⁹ Finally, our measure of skewness implicitly requires at least 250 days of daily returns in the prior 60-month period. Since most firms with 12 months of monthly returns also have 250 days of daily returns, this implicit filter only affects very few stocks.

After applying these filters, there are 13,888 unique firms over about 28 years

¹⁹Our results are robust to minimum day requirements among 5, 10, or 20 days.

(337 months) in the total sample with an average of 3,338 firms per month. In order to compute forecast dispersion, at least two forecasts must be available in a given month. There are 10,404 unique firms with at least two forecasts, for an average of 2,075 firms per month. Our sample consists of 699,364 firm-months with at least two forecasts. Our data sample is comparable to those used in prior studies of the forecast dispersion effect, with the main innovation being the inclusion of a measure of skewness, which does not reduce the sample significantly.

4.1.1 Return Skewness Proxy: SKEW

Expected (or ex-ante) skewness is difficult to measure. As opposed to means, variances and covariances, skewness is not stable over time. Moreover, lagged skewness alone does not adequately forecast skewness [Harvey and Siddique (1999); Boyer et al. (2010)]. Instead, Boyer et al. (2010) (hereafter BMV) use firm-level variables to predict skewness [following the approach of Chen et al. (2001)]. Specifically, BMV develop measures of skewness each month that predict skewness of the return distribution over the next 60 months, based on firm characteristics in the prior 60 months, including lagged skewness, idiosyncratic volatility, momentum, turnover, size, exchange, and industry. BMV point out that although other variables, such as accounting variables, could be useful in predicting skewness, limiting variables to this collection allows the measure to be computed for every stock in CRSP with available history. As a result, using BMV's proxies for skewness maintains a large sample. Moreover, BMV demonstrate that their measures of skewness exhibit a negative cross-sectional relationship with expected returns—the skewness effect.

Thus, our main proxy of skewness at the end of month t for each firm is the

estimated expected total skewness of returns in the subsequent 60 months,

$$\text{SKEW} := \widehat{\mathbf{E}}_t[\text{Total Skewness}_{t+1 \rightarrow t+60}], \quad (\text{BMV}),$$

which is estimated from firm characteristics over the 60 months prior to and including month t .

4.1.2 Forecast Dispersion Proxy: DISP

Our main proxy for forecast dispersion is that most commonly used in the literature, and in particular, by Diether et al. (2002):

$$\text{DISP} := \frac{\text{stdF}_t}{|\text{meanF}_t|},$$

where stdF_t is the month-end standard deviation of current-fiscal-year earnings estimates across analysts tracked by IBES, and meanF_t is the average of those forecasts.

We use IBES *unadjusted* detail files to construct our dispersion proxy. IBES forecast summary files are subject to rounding and staleness problems: the split-adjustment procedure used in the summary files can lead to wrong conclusions,²⁰ and the summary files often include stale forecasts in computing forecast statistics.²¹ We use only the latest forecast by each analyst and no forecasts over 12 months old or for fiscal periods already ended. We adjust for stock splits using CRSP adjustment factors. We exclude firm-months where $\text{mean F}_t = 0$, which is a small number of instances, and we winsorize DISP each month from above at the 97.5% level to handle outliers and any near division-by-zero cases.

²⁰Baber and Kang (2002); Diether et al. (2002); Payne and Thomas (2003).

²¹Morse et al. (1991); Barron and Stuerke (1998).

4.1.3 Some Summary Statistics

Table 2 in Appendix B shows the average monthly means and percentiles for various firm characteristics. Our data sample has similar firm characteristics as other datasets used to study the dispersion effect. Table 2 shows that skewness (SKEW) is positive for the vast majority of stocks, with an average skewness of 0.74 and a median of 0.69. Moreover, less than five percent of firms have negative skewness and average skewness goes up to 1.76 for the 99th percentile, which implies a highly asymmetric return distribution. Forecast dispersion (DISP) is somewhat right-skewed, reflecting the notion that disagreement between analysts is roughly zero for most stocks and then sharply rises for stocks in the upper quartile.

Table 3 in Appendix B shows the average monthly median value of various firm characteristics within groups based on sorting stocks each month into five skewness quintiles and, separately for firms with at least two analysts' forecasts, into five forecast dispersion quintiles. Skewness is somewhat lower for stocks covered by at least two analysts than those covered by zero or one analyst. High skewness firms have lower analyst coverage and tend on average to be smaller, but they are not typically small firms. This association with firm size reflects the fact that analysts usually do not cover small firms and the requirement of at least two analysts in order to calculate DISP. Table 3 also indicates some positive correlation between skewness and forecast dispersion. As SKEW quintiles go from lowest skewness (S1) to highest (S5), there is a gentle increase in average DISP, and vice versa.

4.2 Test of the interaction effect

In this section, we carry out two sorting exercises to assess the presence of the predictions across sorted blocks of stocks. First, we look at high-low return differences

Table 1: Average Monthly Median Returns

This table reports the average monthly median subsequent returns on stocks independently double-sorted each month into 25 blocks of five quintiles by SKEW (S1 “low” to S5 “high”) and five quintiles by DISP (D1 “low” to D5 “high”), where SKEW is the measure of expected total skewness as in Boyer et al. (2010) and DISP is the standard deviation of current-fiscal-year forecasts divided by the absolute mean of the forecasts, winsorized above at the 97.5% level. The data is from December 1983 to December 2011 for common shares traded on NYSE, AMEX, or NASDAQ having price above \$5 and at least two analyst forecasts. Panel A also reports the high-low average median return differences D5-D1 for each SKEW quintile and S5-S1 for each DISP quintile as well as their Fama and French (1993) alphas. Panel B reports the high-high-low-low average median return difference between the highest SKEW and DISP quintiles block and the lowest block as well as its Fama and French (1993) alpha. Averages are reported along with absolute Newey and West (1987) t -statistics (in parentheses). Significance at the 1% and 5% level is indicated by ** and *¹, respectively.

	D1	D2	D3	D4	D5	D5-D1	FF α
S1	1.00	0.91	0.87	0.83	0.70	-0.30 (1.11)	-0.33 (1.12)
S2	0.89	0.91	0.77	0.67	0.33	-0.56** (2.63)	-0.61** (2.80)
S3	0.95	0.90	0.83	0.47	0.16	-0.79** (3.98)	-0.81** (4.08)
S4	1.05	0.94	0.80	0.33	-0.09	-1.14** (4.52)	-1.16** (5.04)
S5	0.68	0.56	0.25	-0.14	-0.82	-1.50** (6.58)	-1.57** (7.19)
S5-S1	-0.32 (1.73)	-0.35 (1.88)	-0.63** (2.87)	-0.97** (3.68)	-1.52** (5.24)	-1.20** (4.21)	
FF α	-0.35 (1.87)	-0.37 (1.87)	-0.66** (3.05)	-0.92** (3.24)	-1.59** (5.16)		-1.24** (4.67)

for various percentile slices of the data on each of the two dimensions of SKEW and DISP. We sort stocks each month into twenty-five blocks (based on an independent double sort) and compute median returns in the subsequent month for the blocks.

Table 1 reports the average monthly median return for each intersection block; the average monthly high-low difference D5–D1 for each SKEW quintile; the average monthly high-low difference S5–S1 for each DISP quintile; and the average monthly high-low Fama and French (1993) (FF) alphas for each high-low difference, respectively.

Table 1 shows that D5–D1 is slightly negative, but is not significant for the lowest skewness quintile, S1. However, D5–D1 widens, becoming more negative, and more significant, as skewness increases from S1 to S5. This evidence is consistent with the interaction effect. Moreover, for the highest skewness quintile, the dispersion effect is economically significant at -1.50% per month, and -1.57% per month after controlling for FF risk factors.

Table 1 also shows that the high-low skewness return, S5–S1, is negative, and mildly significant for the lowest dispersion quintile, D1. In addition, S5–S1 widens, becoming more negative, and more significant, as dispersion increases from D1 to D5. This evidence again suggests the possibility of an interaction effect in which dispersion makes the skewness effect more pronounced. Moreover, for the highest forecast quintile, the skewness effect is economically significant at -1.52% per month, and -1.59% per month after controlling for FF risk factors.

Another notable feature of the data is that the typical stock in each of the three blocks with the highest skewness and forecast dispersion (S4,D5; S5,D4-D5) has negative returns. In particular, the highest block S5D5 has statistically significant negative average returns of -82 basis points per month. Since the risk-free rate is generally positive, this result means that average excess returns are even more negative for these blocks. This fact presents a challenge for standard risk-return explanations since investors are paying a premium to hold these risks under such explanations. In contrast, the theory predicts that for sufficient skewness and forecast dispersion, negative average excess returns should not be atypical.

The bold numbers in the lower right of the table formally test the interaction effect between skewness and dispersion. They are the return (or FF α) on the interaction portfolio, with three equivalent interpretations. The latter two are (9) the difference in skewness effect between high and low dispersion stocks and (10) the dif-

ference in dispersion effect between high and low skewness stocks. For both raw returns and FF α , the interaction portfolio has $\approx -1.2\%$ monthly return with t -statistic > 4 , demonstrating the interaction effect is large, both economically and statistically.

$$\text{Main diagonal - off diagonal} = (S5D5 + S1D1) - (S1D5 + S5D1) \quad (8)$$

$$\text{Right col diff - Left col diff} = (S5D5 - S1D5) - (S5D1 - S1D1) \quad (9)$$

$$\text{Bottom row diff - Top row diff} = (S5D5 - S5D1) - (S1D5 - S1D1) \quad (10)$$

5 Conclusion

We present a frictionless model which bridges two seemingly unrelated empirical anomalies: (1) the negative relationship between dispersion in financial analysts' forecasts and expected returns (Diether et al., 2002) and (2) the negative relationship between idiosyncratic skewness and expected returns.²² The results obtain because (1) empirically, most stocks have positive expected skewness, (2) positive skewness implies that investors' demand schedules are (locally) convex, and hence, (3) trades due to disagreement do not "cancel out"; asset prices are inflated even without short-selling constraints. Our theory further predicts that skewness and disagreement have an interactive pricing impact. We find support for this prediction in the cross section of stock returns.

²²See Boyer et al. (2010); Conrad et al. (2013); Amaya et al. (2013); Boyer and Vorkink (2014).

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Appendix A: Proofs

Proof of Lemma 1. See the proof of Lemma 1 of Goulding et al. (2021).

Proofs of Proposition 1 and Proposition 3. Since Proposition 1 is a special case of Proposition 3 when the random variable \tilde{d} is binary, we proceed to prove the more general case, i.e., Proposition 3.

Since \tilde{d} is discrete, we write the possible realizations as d_i for $i = 1, \dots, n$ with the probability of d_i being q_i . With some abuse of notations, we write $x_{d_i}(p)$ as the demand schedule for the investor with d_i . Following the same argument in Lemma 1, we have the Taylor expansion for $x_{d_i}(p)$ as given by the following equation.

$$x_{d_i}(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma^2} (p - \mu_{d_i}) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s}{\sigma^3} (p - \mu_{d_i})^2 + \epsilon_{d_i}(p)(p - \mu_{d_i})^2, \quad (\text{A1})$$

where $\epsilon_{d_i}(p)$ is a function that converges to 0 as $p \rightarrow \mu_{d_i}$.

The market's clearing condition states that

$$\sum_{i=1}^n q_i x_{d_i}(p) = 0.$$

Substituting $x_{d_i}(p)$ from equation (A1), it leads to the following equation.

$$\begin{aligned} \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma^2} \sum_{i=1}^n q_i (p - \mu_{d_i}) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s}{\sigma^3} \sum_{i=1}^n q_i (p - \mu_{d_i})^2 \\ + \sum_{i=1}^n q_i \epsilon_{d_i}(p)(p - \mu_{d_i})^2 = 0. \end{aligned} \quad (\text{A2})$$

Notice that $\sum_{i=1}^n q_i (p - \mu_{d_i}) = \sum_{i=1}^n q_i (p - \mu - d_i) = p - \mu$, where the last equality uses the fact that \tilde{d} is a mean-zero random variable, and $\sum_{i=1}^n q_i (p - \mu_{d_i})^2 = \sum_{i=1}^n q_i (p - \mu - d_i)^2 = \sum_{i=1}^n q_i [(p - \mu)^2 - 2(p - \mu)d_i + d_i^2] = (p - \mu)^2 + \text{Var}(\tilde{d})$. We write $v^2 := \text{Var}(\tilde{d})$. It

follows that (A2) is equivalent to the following equation.

$$\frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma^2} (p - \mu) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s}{\sigma^3} ((p - \mu)^2 + v^2) + \sum_{i=1}^n q_i \epsilon_{d_i}(p) (p - \mu_{d_i})^2 = 0. \quad (\text{A3})$$

Following Pratt (1964), we normalize the random variable \tilde{d} by its standard deviation v and write $d_i = v d_i^*$. We fix d_i^* and vary v to study the behavior of the equilibrium price when v is small. To emphasize that the price changes when v changes, we write the price as a function of v : $p(v)$. By Taylor expansion, it follows that $p(v) = p(0) + p'(0)v + p''(0)/2v^2 + o(1)v^2$ for small v .

First, $p(0) = \mu$. This can be seen by setting $v = 0$ in (A3). Second, taking derivative with respect to v on both sides in (A3) gives to

$$\frac{\partial(p - \mu)}{\partial v} = \frac{1}{2} \frac{u'''(w_0)u'(w_0)}{(u''(w_0))^2} \frac{s}{\sigma} \left[2(p - \mu) \frac{\partial(p - \mu)}{\partial v} + 2v \right] - \frac{\partial \mathcal{E}(v)}{\partial v}, \quad (\text{A4})$$

where

$$\mathcal{E}(v) = \frac{u''(w_0)\sigma^2}{u'(w_0)} \sum_{i=1}^n q_i \epsilon_{d_i}(p) (p - \mu_{d_i})^2.$$

We claim that $\frac{\partial \mathcal{E}(v)}{\partial v}|_{v=0} = 0$. It is sufficient to show that $\frac{\partial \mathcal{E}_{d_i}(v)}{\partial v}|_{v=0} = 0$, where $\mathcal{E}_{d_i}(v) = \epsilon_{d_i}(p)(p - \mu_{d_i})^2$. Note that $\frac{\partial \mathcal{E}_{d_i}(v)}{\partial v} = \epsilon'_{d_i}(p)p'(v)(p - \mu - v d_i^*)^2 + \epsilon_{d_i}(p)2(p - \mu - v d_i^*)(p'(v) - d_i^*)$, which goes to zero as $v \rightarrow 0$.

Evaluating equation (A4) at $v = 0$ implies that $\frac{\partial(p - \mu)}{\partial v}|_{v=0} = 0$, i.e., $p'(0) = 0$. Taking derivative with respect to v on both sides in (A4) gives to

$$\frac{\partial^2(p - \mu)}{\partial v^2} = \frac{1}{2} \frac{u'''(w_0)u'(w_0)}{(u''(w_0))^2} \frac{s}{\sigma} \left[2 \left(\frac{\partial(p - \mu)}{\partial v} \right)^2 + 2(p - \mu) \frac{\partial^2(p - \mu)}{\partial v^2} + 2 \right] - \frac{\partial^2 \mathcal{E}(v)}{\partial v^2}. \quad (\text{A5})$$

We claim that $\frac{\partial^2 \mathcal{E}(v)}{\partial v^2}|_{v=0} = 0$. It is sufficient to show that $\frac{\partial^2 \mathcal{E}_{d_i}(v)}{\partial v^2}|_{v=0} = 0$. Note that $\frac{\partial^2 \mathcal{E}_{d_i}(v)}{\partial v^2} = \epsilon''_{d_i}(p)p'(v)^2(p - \mu - v d_i^*)^2 + \epsilon'_{d_i}(p)p''(v)(p - \mu - v d_i^*)^2 + \epsilon'_{d_i}(p)p'(v)2(p - \mu - v d_i^*)(p'(v) - d_i^*)$

$d_i^*) + \epsilon'_{d_i}(p)p'(v)2(p - \mu - vd_i^*)(p'(v) - d_i^*) + \epsilon_{d_i}(p)2(p'(v) - d_i^*)(p'(v) - d_i^*) + \epsilon_{d_i}(p)2(p - \mu - vd_i^*)p''(v)$, which goes to zero as $v \rightarrow 0$ because every single term goes to zero as $v \rightarrow 0$. Evaluating (A5) at $v = 0$ implies that

$$\frac{\partial^2(p - \mu)}{\partial v^2} \Big|_{v=0} = \frac{u'''(w_0)u'(w_0) s}{(u''(w_0))^2 \sigma}.$$

Using Taylor expansion, we have

$$p - \mu = \frac{1}{2} \frac{\partial^2(p - \mu)}{\partial v^2} \Big|_{v=0} v^2 + o(1)v^2 = \frac{u'''(w_0)u'(w_0) s}{2(u''(w_0))^2 \sigma} v^2 + o(1)v^2,$$

where the little-o notation $o(1)$ is an unknown function that converges to 0 as $v \rightarrow 0$.

Proof of Proposition 2. Recall that Corollary 1 shows that

$$R = -\frac{u'''(w_0)u'(w_0) s}{2(u''(w_0))^2 \sigma} d^2 + o(1)d^2.$$

Denote the dominant term as $\bar{R} = -\frac{u'''(w_0)u'(w_0) s}{2(u''(w_0))^2 \sigma} d^2$. It is straightforward to see that

$$\frac{\partial \bar{R}}{\partial s} = -\frac{u'''(w_0)u'(w_0) 1}{2(u''(w_0))^2 \sigma} d^2 < 0,$$

$$\frac{\partial \bar{R}}{\partial d} = -\frac{u'''(w_0)u'(w_0) s}{2(u''(w_0))^2 \sigma} 2d < 0 \Leftrightarrow s > 0,$$

and

$$\frac{\partial^2 \bar{R}}{\partial s \partial d} = -\frac{u'''(w_0)u'(w_0) 1}{2(u''(w_0))^2 \sigma} 2d < 0.$$

So there exists a $\bar{d} > 0$ such that if $d < \bar{d}$, the signs of the derivative of the dollar excess return with respect to parameters are exactly given by the signs of the derivative of the dominant term \bar{R} with respect to parameters.

Proof of Proposition 4. Given the two types of investors and Lemma 1, we can write the market's clearing condition as the following equation.

$$\begin{aligned} & \frac{u'_1(w_0)}{u''_1(w_0)} \frac{1}{\sigma^2} (p - \mu_d) - \frac{1}{2} \frac{u'''_1(w_0)}{u''_1(w_0)} \left(\frac{u'_1(w_0)}{u''_1(w_0)} \right)^2 \frac{s}{\sigma^3} (p - \mu_d)^2 + o(1)(p - \mu_d)^2 \\ & + \frac{u'_2(w_0)}{u''_2(w_0)} \frac{1}{\sigma^2} (p - \mu_{-d}) - \frac{1}{2} \frac{u'''_2(w_0)}{u''_2(w_0)} \left(\frac{u'_2(w_0)}{u''_2(w_0)} \right)^2 \frac{s}{\sigma^3} (p - \mu_{-d})^2 + o(1)(p - \mu_{-d})^2 = 0. \end{aligned}$$

With loss, we assume $s > 0$. (When $s < 0$, all analysis carries over). In order to show that the equilibrium price is greater than the benchmark price p_0 , we need to show that at price p_0 , there is excess demand. In other words, we need to show that the left-hand-side of the above market's clearing condition is strictly positive when $p = p_0$.

Note that since

$$p_0 = \frac{\frac{u'_1(w_0)}{-u''_1(w_0)} \mu_d + \frac{u'_2(w_0)}{-u''_2(w_0)} \mu_{-d}}{\frac{u'_1(w_0)}{-u''_1(w_0)} + \frac{u'_2(w_0)}{-u''_2(w_0)}},$$

by algebraic manipulation, it is straightforward to see that

$$\frac{u'_1(w_0)}{u''_1(w_0)} \frac{1}{\sigma^2} (p - \mu_d) + \frac{u'_2(w_0)}{u''_2(w_0)} \frac{1}{\sigma^2} (p - \mu_{-d}) = 0.$$

In order to show there is excess demand p_0 , it is sufficient to show that

$$-\frac{1}{2} \frac{u'''_1(w_0)}{u''_1(w_0)} \left(\frac{u'_1(w_0)}{u''_1(w_0)} \right)^2 \frac{s}{\sigma^3} (p_0 - \mu_d)^2 - \frac{1}{2} \frac{u'''_2(w_0)}{u''_2(w_0)} \left(\frac{u'_2(w_0)}{u''_2(w_0)} \right)^2 \frac{s}{\sigma^3} (p_0 - \mu_{-d})^2 > 0.$$

Plugging into p_0 , note that

$$\begin{aligned} & -\frac{1}{2} \frac{u_1'''(w_0)}{u_1''(w_0)} \left(\frac{u_1'(w_0)}{u_1''(w_0)} \right)^2 \frac{s}{\sigma^3} (p_0 - \mu_d)^2 - \frac{1}{2} \frac{u_2'''(w_0)}{u_2''(w_0)} \left(\frac{u_2'(w_0)}{u_2''(w_0)} \right)^2 \frac{s}{\sigma^3} (p_0 - \mu_{-d})^2 \\ & = \frac{1}{2} \frac{(\mu_d - \mu_{-d})^2}{\left(\frac{u_1'(w_0)}{-u_1''(w_0)} + \frac{u_2'(w_0)}{-u_2''(w_0)} \right)^2} \left[\frac{u_1'''(w_0)}{-u_1''(w_0)} + \frac{u_2'''(w_0)}{-u_2''(w_0)} \right] \left(\frac{u_1'(w_0)}{u_1''(w_0)} \right)^2 \left(\frac{u_2'(w_0)}{u_2''(w_0)} \right)^2 \frac{s}{\sigma^3}, \end{aligned}$$

which is positive as long as $s > 0$. This completes the proof.

Proof of Proposition 5. we can write the market's clearing condition as the following equation.

$$\begin{aligned} & \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_A^2} (p - \mu_A) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s_A}{\sigma_A^3} (p - \mu_A)^2 + o(1)(p - \mu_A)^2 \\ & + \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_B^2} (p - \mu_B) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s_B}{\sigma_B^3} (p - \mu_B)^2 + o(1)(p - \mu_B)^2 = 0. \end{aligned}$$

Similar to the proof of Proposition 4, we need to show that there is excess demand at price p_0 if and only if $\sigma_A s_A + \sigma_B s_B > 0$.

Note that when

$$p_0 = \frac{\frac{\mu_A}{\sigma_A^2} + \frac{\mu_B}{\sigma_B^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}},$$

it follows that

$$\frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma^2} (p_0 - \mu_A) + \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma^2} (p_0 - \mu_B) = 0,$$

and

$$\begin{aligned} & -\frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s_A}{\sigma_A^3} (p_0 - \mu_A)^2 - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{s_B}{\sigma_B^3} (p_0 - \mu_B)^2 \\ & = \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)} \right)^2 \frac{(\mu_A - \mu_B)^2}{\left(\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2} \right)^2} \frac{\sigma_A s_A + \sigma_B s_B}{\sigma_A^4 \sigma_B^4} \end{aligned}$$

which is positive if and only if $\sigma_A s_A + \sigma_B s_B > 0$.

Proof of Proposition 6. The proof follows from the proof of Proposition 3 in Goulding et al. (2021), where they show that the demand function $x(p)$ is convex for a sufficiently large skewness. Similar to that proof, the demand function is concave for a sufficiently small (potentially negative) skewness.

Appendix B: Omitted Tables

Table 2: Distribution of Firm Characteristics

This table reports the average monthly mean and percentiles of firm characteristics from December 1983 to December 2011 for common shares traded on NYSE, AMEX, or NASDAQ having price above \$5 and at least two analyst forecasts. SKEW is the measure of expected total skewness as in Boyer et al. (2010). DISP is the standard deviation of current-fiscal-year forecasts divided by the absolute mean of the forecasts, winsorized above at the 97.5% level. ME is the market equity (in \$1,000). BM is the ratio of book equity to market equity, using book equity from the most recent fiscal year statement at least three months old. MOM is the cumulative return over the last 12 months excluding the most recent month. RET₋₁ is the return in the most recent month. IVOL is the standard deviation of residuals of excess daily returns regressed onto the Fama and French (1993) factors. Factor loadings are regressions of excess monthly returns onto the respective factors using the most recent 36 months of returns: β_{MKT} , β_{SMB} , and β_{HML} are loadings on the three Fama and French (1993) factors. β_{UMD} is the loading on the Carhart (1997) momentum factor; β_{LIQ} is the loading on the Pástor and Stambaugh (2003) traded liquidity factor; β_{CoSkew} is the loading on the squared excess market return following Harvey and Siddique (2000).

	Mean	P01	P05	P10	P25	P50	P75	P90	P95	P99
SKEW	0.74	-0.07	0.14	0.26	0.48	0.69	1.01	1.25	1.45	1.76
DISP (%)	17.1	0.1	1.0	1.5	2.9	6.5	16.7	40.1	77.1	151.9
Number of Analysts	9.4	2.0	2.0	2.2	3.7	6.8	12.8	20.4	25.4	35.5
<i>Valuation, Prior Returns, Idiosyncratic Volatility, and Factor Loadings</i>										
ME (\$ mil.)	3,581	40	76	113	238	646	2,088	6,940	13,926	58,863
BM (%)	59.0	5.4	12.1	17.5	30.1	50.4	76.7	107.2	131.7	207.5
Price	40.3	5.4	6.9	8.6	13.7	22.7	35.3	51.0	63.2	98.0
MOM (%)	18.60	-57.22	-39.08	-28.12	-9.64	10.37	34.33	68.71	101.49	208.66
RET ₋₁ (%)	1.51	-25.86	-15.45	-11.01	-4.91	0.85	7.03	14.50	20.41	36.64
IVOL (%)	2.10	0.54	0.77	0.93	1.26	1.82	2.62	3.59	4.33	6.28
β_{MKT}	1.08	-1.07	-0.12	0.17	0.57	1.00	1.51	2.10	2.56	3.78
β_{SMB}	0.67	-2.06	-0.94	-0.59	-0.08	0.53	1.29	2.15	2.79	4.45
β_{HML}	0.05	-4.20	-2.31	-1.58	-0.62	0.15	0.82	1.53	2.08	3.65
β_{UMD}	-0.08	-2.83	-1.53	-1.09	-0.52	-0.05	0.39	0.86	1.25	2.42
β_{LIQ}	-0.03	-2.34	-1.31	-0.92	-0.42	-0.01	0.39	0.84	1.21	2.20
β_{CoSkew}	0.28	-41.13	-20.49	-14.11	-6.47	-0.01	6.51	14.64	21.80	46.39

Table 3: Firm Characteristics by Skewness Quintiles and by Forecast Dispersion Quintiles

This table reports the average monthly median characteristics on stocks sorted each month into either five quintiles by SKEW (S1 “low” to S5 “high”) or five quintiles by DISP (D1 “low” to D5 “high”), where SKEW is the measure of expected total skewness as in Boyer et al. (2010) and DISP is the standard deviation of current-fiscal-year forecasts divided by the absolute mean of the forecasts, winsorized above at the 97.5% level. The data is from December 1983 to December 2011 for common shares traded on NYSE, AMEX, or NASDAQ having price above \$5. ME is the market equity. BM is the ratio of book equity to market equity, using book equity from the most recent fiscal year statement at least three months old. MOM is the cumulative return over the last 12 months excluding the most recent month. RET₋₁ is the return in the most recent month. IVOL is the standard deviation of residuals of excess daily returns regressed onto the Fama and French (1993) factors. Factor loadings are regressions of excess monthly returns onto the respective factors using the most recent 36 months of returns: β_{MKT} , β_{SMB} , and β_{HML} are loadings on the three Fama and French (1993) factors; β_{UMD} is the loading on the Carhart (1997) momentum factor; β_{LIQ} is the loading on the Pástor and Stambaugh (2003) traded liquidity factor; β_{CoSkew} is the loading on the squared excess market return following Harvey and Siddique (2000).

	Baseline Dataset: Number of Analysts ≥ 2										Number of Analysts ≤ 1				
	S1	S2	S3	S4	S5	D1	D2	D3	D4	D5	S1	S2	S3	S4	S5
SKEW	0.26	0.52	0.69	0.95	1.25	0.65	0.64	0.68	0.72	0.82	0.58	0.93	1.26	1.49	1.75
DISP (%)	5.6	5.9	6.1	7.0	9.8	1.5	3.4	6.5	13.5	40.1	—	—	—	—	—
Number of Analysts	10.0	9.2	8.4	5.7	3.8	5.9	7.5	7.3	7.1	6.6	—	—	—	—	—
<i>Valuation, Prior Returns, Idiosyncratic Volatility, and Factor Loadings</i>															
ME (\$ mil.)	1,463	1,302	1,085	401	162	952	922	704	534	379	405	165	78	50	35
BM (%)	47.8	44.3	48.5	53.7	60.2	43.0	47.2	50.7	53.6	61.0	51.9	59.9	65.5	73.0	78.3
Price	30.0	28.8	26.3	19.6	12.1	29.5	27.4	23.5	19.1	14.1	21.8	15.9	11.8	11.0	8.6
MOM (%)	23.01	13.95	10.98	7.41	-3.31	15.52	13.77	10.80	7.56	-1.04	22.63	15.16	10.96	8.32	3.76
RET ₋₁ (%)	1.21	1.05	0.88	0.85	0.17	1.12	1.03	0.92	0.65	0.29	1.03	0.81	0.50	0.31	0.60
IVOL (%)	1.64	1.67	1.65	1.89	2.42	1.48	1.56	1.77	2.04	2.36	1.86	2.01	2.29	2.29	2.62
β_{MKT}	0.95	1.03	1.04	0.98	0.98	0.87	0.93	1.01	1.09	1.16	0.82	0.76	0.69	0.61	0.57
β_{SMB}	0.28	0.44	0.46	0.67	0.92	0.37	0.41	0.52	0.66	0.79	0.59	0.71	0.71	0.65	0.68
β_{HML}	0.18	0.07	0.14	0.18	0.15	0.14	0.15	0.16	0.15	0.14	0.22	0.25	0.23	0.25	0.25
β_{UMD}	0.05	-0.05	-0.07	-0.07	-0.11	0.03	-0.01	-0.06	-0.09	-0.16	-0.02	-0.02	-0.06	-0.07	-0.09
β_{LIQ}	0.00	-0.03	-0.01	0.00	-0.03	-0.01	-0.01	-0.02	0.00	-0.03	0.01	0.01	0.02	0.04	0.02
β_{CoSkew}	0.08	0.10	0.24	-0.20	-0.34	-0.08	0.00	0.04	0.08	-0.07	-0.41	-0.93	-1.14	-0.61	-0.89
<i>Subsequent Returns</i>															
RET (%)	0.87	0.74	0.72	0.64	0.01	0.91	0.85	0.71	0.41	-0.05	0.51	0.41	0.04	-0.01	-0.40