Disagreement, Skewness, and Asset Prices

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Overview

- Disagreement (e.g., divergence of opinions) is a ubiquitous feature
- This paper: static asset pricing with disagreement in **frictionless** setup
- Non-increasing absolute risk aversion (NARA)
  - Wealthier investors allocate more $$$$ to risky asset
  - NARA $\Rightarrow u''' > 0$, implies preference for positive skewness
  - Nests CARA & CRRA
- No parametric assumptions on utility functions or payoff distributions
- Trades due to disagreement bias price upward relative to fundamental value
  - Does not rely short-selling constraints or market frictions
  - Skewness matters
  - Buying a positively-skewed asset entails more desirable upside risk whereas shorting involves more downside risk
- We generate and test unique predictions

How does skewness impact prices?

Consider a simple lottery:

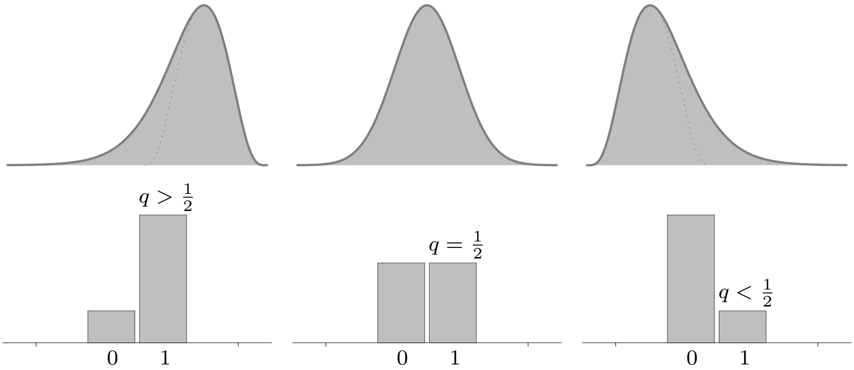
$$\widetilde{\theta} = \begin{cases} 1, & \text{with probability } q, \\ 0, & \text{with probability } 1-q, \end{cases} \qquad q \in (0,1).$$

Questions of interest:

Q1: At what prices would an agent be willing to take different sides of this bet? (i.e., buying vs. insuring the lottery)

Q2: How do prices for each side of the bet relate to the expected (or fundamental) value of the lottery (q)?

Skewness in continuous and discrete distributions



Negative Skewness

No Skewness

Positive Skewness

Opposite sides of a skewed bet, example

Example:

$$\widetilde{\theta} = \begin{cases} \$1\text{M} & \text{with probability } q = 0.20, \\ 0 & \text{with probability } 1 - q = 0.80, \end{cases}$$

Suppose **I offer you** the above random payoff for price B.
How much would you be **willing to pay** me to play this lottery?

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Suppose **I offer you** the above random payoff for price B.
How much would you be **willing to pay** me to play this lottery?

- If you are risk averse, you would pay something **below** the fundamental (expected) value of \$200,000.

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How much would you be **willing to pay** me to play this lottery?

- If you are risk averse, you would pay something **below** the fundamental (expected) value of $200,000.

Suppose **you offer me** the above random payoff for price S.
How much would you **require me to pay** to play this lottery?

Opposite sides of a skewed bet, example

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How much would you be **willing to pay** me to play this lottery?

- If you are risk averse, you would pay something **below** the fundamental (expected) value of \$200,000.

Suppose **you offer me** the above random payoff for price S.
How much would you **require me to pay** to play this lottery?

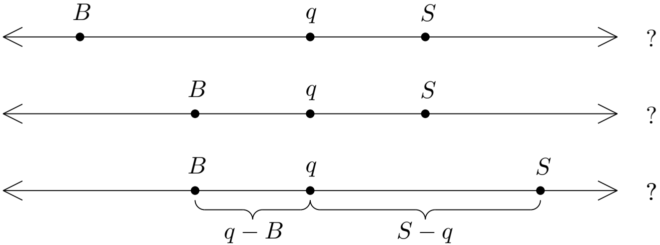
- If you are risk averse, you would require something **above** the fundamental (expected) value of \$200,000.

Opposite sides of a skewed bet

Q1: Direct application of Jensen's rule to risk averse utility gives the widely-known result:

$$B < q < S.$$

Q2: But, how do prices for each side of the bet relate to the expected value of the lottery (q)?

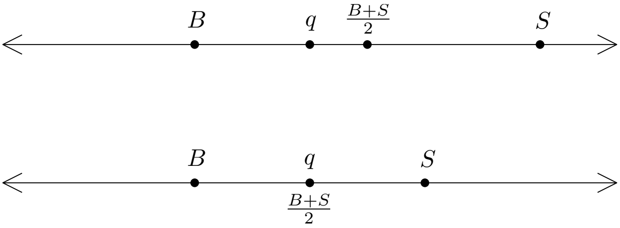


Opposite sides of a skewed bet

$q < \frac{1}{2}$
positive skewness

$q = \frac{1}{2}$
no skewness

$q > \frac{1}{2}$
negative skewness



Intuition: Upside vs. downside risk

Opposite sides of a skewed bet ... market implications?

In order to clear a market with random liquidity demand:

- sell, 50% of the time
- buy, 50% of the time

the price must induce us to

- buy, 50% of the time: $P = B$
- sell, 50% of the time: $P = S$

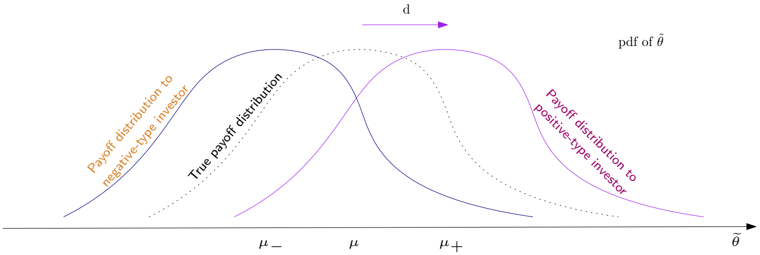
Then, the average price of such a market will have these properties:

$$\text{average } P = \begin{cases} \frac{B+S}{2} > q, & \text{for } q < \frac{1}{2} \quad \text{(positive skewness)}, \\ \frac{B+S}{2} = q, & \text{for } q = \frac{1}{2}, \\ \frac{B+S}{2} < q, & \text{for } q > \frac{1}{2} \quad \text{(negative skewness)}. \end{cases}$$

Baseline Model

A two-period financial market with one risk-free asset and one risky asset

- Net risk-free rate $= 0$
- Risky asset's time-1 payoff denoted by $\tilde{\theta}$
- Investors have NARA utility $u(w)$ and initial wealth w_0
- Zero supply of risky asset [relaxed later]
- Disagreement: positive and negative investors
  - They disagree about the mean of $\tilde{\theta}$, but agree about the shape [relaxed later]
  - d: the level of disagreement or dispersion



Equilibrium

Equilibrium. An equilibrium consists of a tuple $(x_+(p), x_-(p), p)$ such that

- Demand schedule solves each type of investor's expected utility maximization problem conditional on his/her subjective belief
- p clears the market

Notation: $\mu_+ = E_+(\tilde{\theta})$, $\mu_- = E_-(\tilde{\theta})$, $\sigma^2 = E(\tilde{\theta} - E[\tilde{\theta}])^2$, and $s = \frac{E(\tilde{\theta} - E[\tilde{\theta}])^3}{\sigma^3}$.

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Lemma

A positive investor's demand schedule is given by

$$x_+(p) = \frac{u'(w_0)}{-u''(w_0)} \frac{\mu_+ - p}{\sigma^2} + \frac{1}{2} \frac{u'''(w_0)}{-u''(w_0)} \left(\frac{u'(w_0)}{-u''(w_0)} \right)^2 \frac{s}{\sigma^3} (p - \mu_+)^2 + o(1)(p - \mu_+)^2,$$

where the little-o notation $o(1)$ is an unknown function that converges to 0 when $p \to \mu_+$.

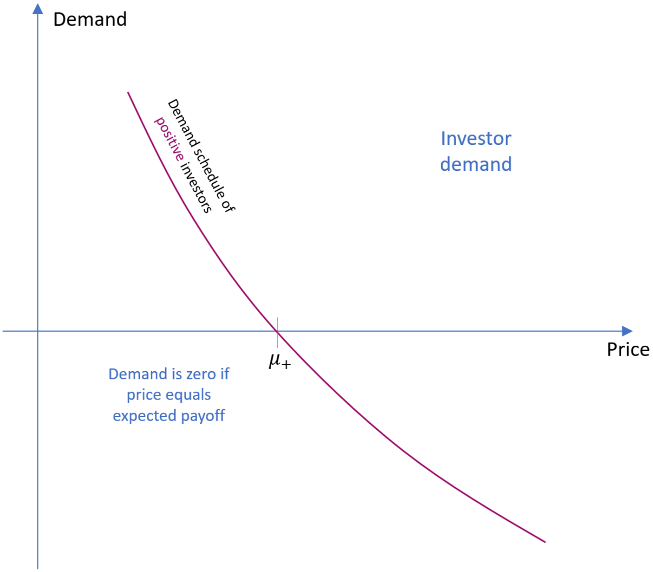
Proof of Lemma

- *CARA+Normal $\Rightarrow$ First term, i.e., $x_+(p) = \frac{\mu_+ - p}{\gamma \sigma^2}$*

Investor demand

Positive-type NARA investor:

- Optimistic about expected payoff
- Prefers more to less
- Risk averse
- Prefers skewness



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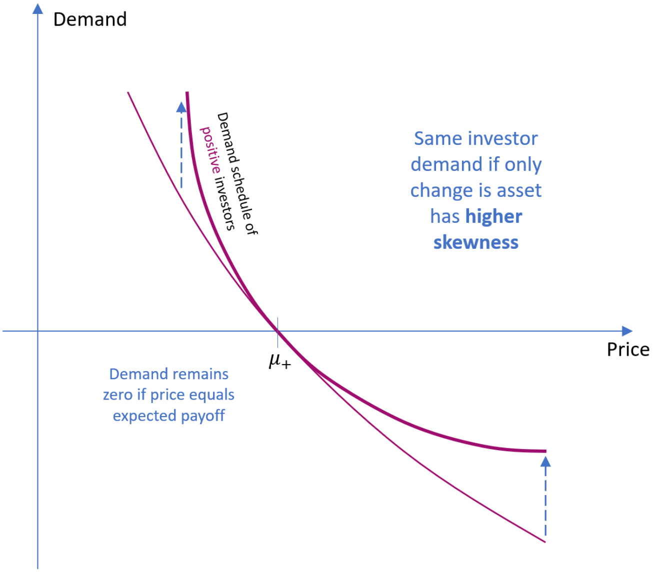
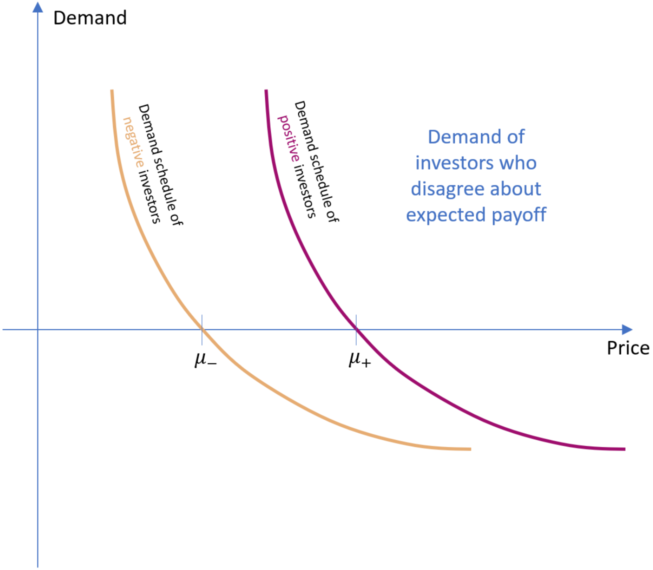


Equilibrium price

- Market clearing price:
  $x_+(p) + x_-(p) = 0$
- Positive skew $(s > 0)$
  $\Rightarrow$ non-linear demand
- Disagreement matters because of non-linear demand
- p must be greater than μ to clear the market



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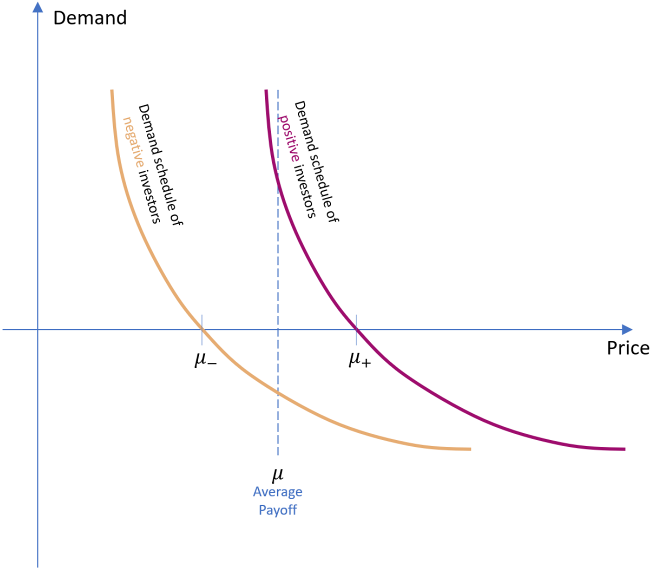

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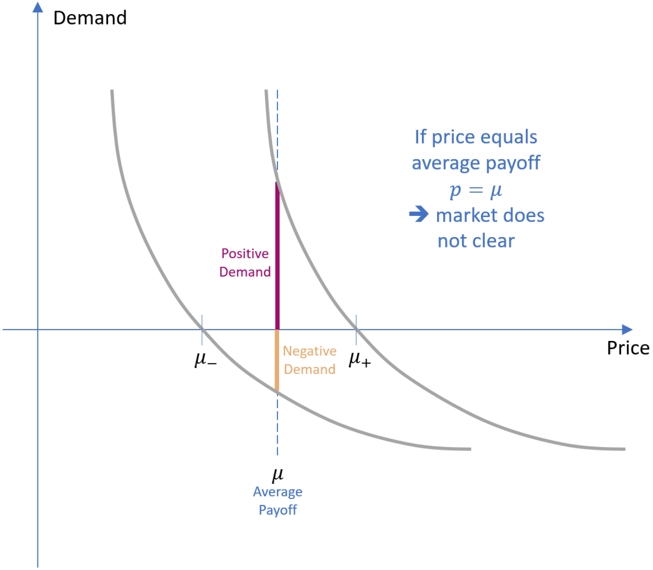

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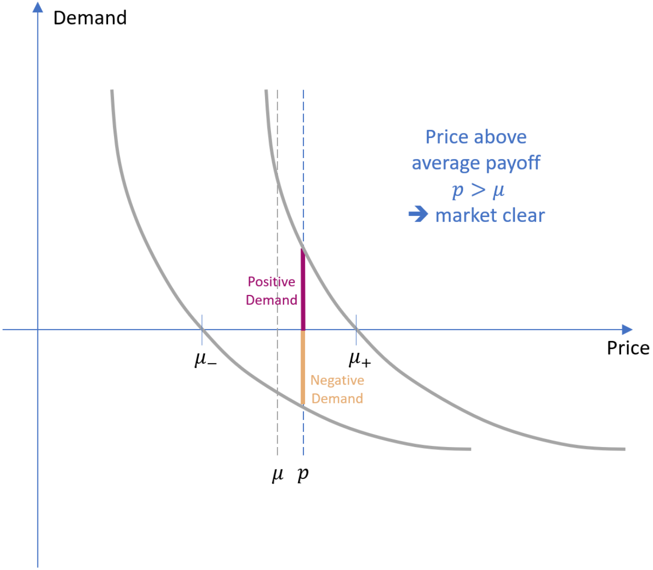

Main result

$$\textbf{return} \propto -\textbf{skewness} * \textbf{dispersion}^2$$

Proposition

There exists a $\bar{d} > 0$ *such that if* $d < \bar{d}$, *then the equilibrium price is given by the following equation.*

$$\mu - p = -\frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2}\frac{s}{\sigma}d^2 + o(1)d^2,$$

where the little-o notation $o(1)$ *is an unknown function that converges to 0 as* $d \to 0$.

The dispersion effect

$$\textbf{return} \propto -\textbf{skewness} * \textbf{dispersion}^2$$

Consistent with the documented **negative** relationship between dispersion in financial analysts' earnings forecasts and expected returns [Diether, Malloy, and Scherbina (2002)]

- Most stock have positive ex-ante skew [Boyer, Mitton, and Vorkink (2010)]
- In our model, investors are not restricted from short selling
- When short selling is less an issue (such as futures, broad stock market indexes, etc), our model provides a unique prediction

The skewness effect

$$\textbf{return} \propto -\textbf{skewness} * \textbf{dispersion}^2$$

Consistent with the documented **negative** relationship between ex-ante skewness and expected returns [Boyer, Mitton, and Vorkink (2010)]

- Co-skewness: Kraus and Litzenberger (1976, 1983); Harvey and Siddique (2000)
- Behavioral finance: Brunnermeier and Parker (2005); Brunnermeier, Gollier, and Parker (2007); Barberis and Huang (2008)
- Nonlinear demand and noise trading: Goulding, Santosh, and Zhang (2021)

New predictions

$$\textbf{return} \propto -\textbf{skewness} * \textbf{dispersion}^2$$

P1 Skewness effect and forecast dispersion effect *interact* and *amplify* each other

P2 Forecast dispersion effect *requires* ex-ante skewness

P3 *Negative* average excess returns with high enough ex-ante skewness and investor disagreement

A simple linear specification can capture the model's insights

Approximation of the model's expected return function:

$$\text{Expected Return}_{t+1} = \gamma_0 + \gamma_1 \cdot \text{Skewness}_t + \gamma_2 \cdot \text{Forecast Dispersion}_t + \gamma_3 \cdot \text{Skewness}_t \times \text{Forecast Dispersion}_t$$

We follow the cross-sectional literature by implementing tests using standard Fama and MacBeth (1973) regressions.

Hypotheses formation

$$\text{Expected Return}_{t+1} = \gamma_0 + \gamma_1 \cdot \text{Skewness}_t + \gamma_2 \cdot \text{Forecast Dispersion}_t + \gamma_3 \cdot \text{Skewness}_t \times \text{Forecast Dispersion}_t$$

H1 **Interaction effect**: The coefficient on the interaction term between skewness and forecast dispersion proxies (γ_3) should be negative: H1: $\gamma_3 < 0$

Hypotheses formation

$$\text{Expected Return}_{t+1} = \gamma_0 + \gamma_1 \cdot \text{Skewness}_t + \gamma_2 \cdot \text{Forecast Dispersion}_t + \gamma_3 \cdot \text{Skewness}_t \times \text{Forecast Dispersion}_t$$

H2 **Dispersion effect requires skewness**: The coefficient on forecast dispersion (γ_2) should be insignificant H2: $\gamma_2 = 0$ when interaction is included.

The marginal effect of forecast dispersion as a composite coefficient estimate is:

$$\frac{\partial\,\text{Expected Return}_{t+1}}{\partial\,\text{Forecast Dispersion}_t} = \gamma_2 + \gamma_3 \cdot \text{Skewness}_t.$$

Hypotheses formation

$$\text{Expected Return}_{t+1} = \gamma_0 + \gamma_1 \cdot \text{Skewness}_t^{P_{high}} + \gamma_2 \cdot \text{Forecast Dispersion}_t^{P_{high}} + \gamma_3 \cdot \text{Skewness}_t^{P_{high}} \times \text{Forecast Dispersion}_t^{P_{high}} < 0$$

H3 **Negative average excess returns**: The typical stock in the upper percentiles by skewness and forecast dispersion should exhibit negative average excess returns.

Data

Sample from December 1983 to December 2011

- SKEW: ex-ante skewness from Boyer, Mitton, and Vorkink (2010)
- DISP: analyst forecast dispersion from Diether, Malloy, and Scherbina (2002): $\text{DISP} := \frac{\text{stdF}_t}{|\text{meanF}_t|}$.
- Similar characteristics to other datasets used to study dispersion effect

	Mean	P01	P05	P10	P25	P50	P75	P90	P95	P99
SKEW	0.74	−0.07	0.14	0.26	0.48	0.69	1.01	1.25	1.45	1.76
DISP (%)	17.1	0.1	1.0	1.5	2.9	6.5	16.7	40.1	77.1	151.9
Analysts	9.4	2.0	2.0	2.2	3.7	6.8	12.8	20.4	25.4	35.5

- Skewness is positive for the vast majority of stocks (Avg=0.74, Med=0.69)
  - Our model predicts a negative interaction effect when skewness is positive

More data details...

Average monthly median returns by SKEW/DISP block

	D1	D2	D3	D4	D5	D5-D1	FF α
S1	1.00	0.91	0.87	0.83	0.70	-0.30	-0.33
						(-1.11)	(-1.12)
S2	0.89	0.91	0.77	0.67	0.33	-0.56^{***}	-0.61^{***}
						(-2.63)	(-2.80)
S3	0.95	0.90	0.83	0.47	0.16	-0.79^{***}	-0.81^{***}
						(-3.98)	(-4.08)
S4	1.05	0.94	0.80	0.33	-0.09	-1.14^{***}	-1.16^{***}
						(-4.52)	(-5.04)
S5	0.68	0.56	0.25	-0.14	-0.82	-1.50^{***}	-1.57^{***}
						(-6.58)	(-7.19)
S5-S1	-0.32^{*}	-0.35^{*}	-0.63^{***}	-0.97^{***}	-1.52^{***}		
	(-1.73)	(-1.88)	(-2.87)	(-3.68)	(-5.24)		
FF α	-0.35^{*}	-0.37^{*}	-0.66^{***}	-0.92^{***}	-1.59^{***}		
	(-1.87)	(-1.87)	(-3.05)	(-3.24)	(-5.16)		

Newey-West (1987) t-statistics; ***, **, * are 1%, 5%, and 10% significance levels, respectively.

Each month, double-sort (indep.) stocks in 25 blocks:

- 5 quintiles by SKEW: S1 $\nearrow$ S5
- 5 quintiles by DISP: D1 $\nearrow$ D5
- Compute median subsequent return for each block.

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Dispersion effect:

- D5−D1 not significant for lowest skewness quintile S1
- D5−D1 widens (negative) and becomes more significant as S1 $\nearrow$ S5
- Consistent with Hypothesis H1 (interaction), Hypothesis H2 (no skewness $\Rightarrow$ no dispersion effect)

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Skewness effect:

- S5−S1 weakly significant even for lowest forecast dispersion D1 quintile
- S5−S1 also widens (negative) and becomes more significant as D1 $\nearrow$ D5
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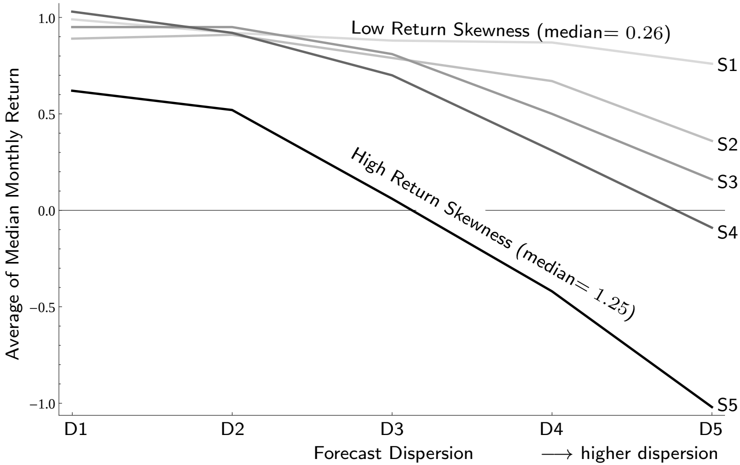
- Negative monthly average median returns! (S4,D5; S5,D4-D5) (Hypothesis H3 ✓)
- S5D5−S1D1 $\approx 1.8\%$ per month, $\nearrow \approx 1.9\%$ per month FF-α

S5D5-S1D1	-1.81^{***}	FF α	-1.92^{***}
	(-6.41)		(-7.18)

Average monthly median returns by SKEW/DISP block: graph



Hypotheses:

- All hypotheses, Hypothesis H1, Hypothesis H2, Hypothesis H3, are exhibited.

Fama-MacBeth (1973) return regressions

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1 \,\text{SKEW} + \gamma_2 \,\text{DISP} + \gamma_3 \,\text{SKEW} \times \text{DISP} + \varepsilon,$$

	I	II	III	IV
SKEW	-0.634^{***}		-0.550^{***}	-0.415^{**}
	(-2.96)		(-2.65)	(-1.98)
DISP		-0.829^{***}	-0.743^{***}	0.001
		(-3.75)	(-3.58)	(0.00)
SKEW×DISP				-0.956^{***}
				(-3.38)

Newey-West (1987) t-statistics; ***, **, * are 1%, 5%, and 10% significance levels, respectively.

Separate effects of SKEW and DISP:

Model I: Skewness effect is strongly exhibited in our sample.

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Separate effects of SKEW and DISP:

Model II: Dispersion effect is strongly exhibited in our sample.

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Newey-West (1987) t-statistics; ***, **, * are 1%, 5%, and 10% significance levels, respectively.

Separate effects of SKEW and DISP:

Model III: Skewness and forecast dispersion pick up different properties of average returns. Mild positive association between SKEW and DISP (0.14)

Fama-MacBeth (1973) return regressions

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1 \,\text{SKEW} + \gamma_2 \,\text{DISP} + \gamma_3 \,\text{SKEW} \times \text{DISP} + \varepsilon,$$

	I	II	III	IV
SKEW	-0.634^{***}		-0.550^{***}	-0.415^{**}
	(-2.96)		(-2.65)	(-1.98)
DISP		-0.829^{***}	-0.743^{***}	0.001
		(-3.75)	(-3.58)	(0.00)
SKEW×DISP				-0.956^{***}
				(-3.38)

Newey-West (1987) t-statistics; ***, **, * are 1%, 5%, and 10% significance levels, respectively.

Main predictions of the model are well supported:

Model IV: The interaction coefficient is negative, highly significant (Hypothesis H1 ✓).

Fama-MacBeth (1973) return regressions

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1\,\text{SKEW} + \gamma_2\,\text{DISP} + \gamma_3\,\text{SKEW} \times \text{DISP} + \varepsilon,$$

	I	II	III	IV
SKEW	-0.634^{***}		-0.550^{***}	-0.415^{**}
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		(-3.75)	(-3.58)	(0.00)
SKEW×DISP				-0.956^{***}
				(-3.38)

Newey-West (1987) t-statistics; ***, **, * are 1%, 5%, and 10% significance levels, respectively.

Main predictions of the model are well supported:

Model IV: Marginal effect of DISP as a composite coefficient estimate:

$$\frac{\partial\,\text{RET}}{\partial\,\text{DISP}} = \gamma_2 + \gamma_3 \cdot \text{SKEW}.$$

Hypothesis H2: SKEW $= 0 \Rightarrow$ DISP insignificant. ✓

Fama-MacBeth (1973) return regressions

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1\,\text{SKEW} + \gamma_2\,\text{DISP} + \gamma_3\,\text{SKEW} \times \text{DISP} + \varepsilon,$$

	I	II	III	IV
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SKEW×DISP				-0.956^{***}
				(-3.38)

Newey-West (1987) t-statistics; ***, **, * are 1%, 5%, and 10% significance levels, respectively.

Economic significance:

low dispersion firm (≈ 0.01, P05) with low skewness (≈ 0.14, P05)

outperforms

high dispersion firm (≈ 0.771, P95) with high skewness (≈ 1.45, P95)

by $\approx 1.61\%$ per month (19.3% annualized)

FM regressions: controls for cross-sectional variation

However, the results so far are unconditional...

- How are these initial results sensitive to cross-sectional variation?
- Are SKEW and DISP just picking up other omitted determinants of returns?
- How do results hold up/compare to other explanations?

Market Frictions Controls:

- Turnover (TURN), Illiquidity (ILLIQ), and Instituitional Ownership (IO).

Financial Distress Controls:

- Leverage (LEV) and probability of near-term failure (OSCORE).

Other Common Controls:

- Valuation and prior returns: log(ME), log(BM), MOM, RET_{-1}
- Idiosyncratic volatility: IVOL
- Factor Loadings: β_{MKT}, β_{SMB}, β_{HML}, β_{UMD}, β_{LIQ}, β_{CoSkew}

FM regressions: Market Frictions controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1\,\text{SKEW} + \gamma_2\,\text{DISP} + \gamma_3\,\text{SKEW} \times \text{DISP} + \phi'\,\mathbf{Z} + \varepsilon$$

	V	VI	VII	VIII	IX	X	XI	XII
SKEW				−0.375**	−0.354	−0.378*		−0.280
				(−1.97)	(−1.64)	(−1.81)		(−1.46)
DISP				0.042	−0.038	−0.014		0.013
				(0.14)	(−0.11)	(−0.04)		(0.05)
SKEW				−0.964***	−0.899***	−0.931***		−0.906***
×DISP				(−3.75)	(−3.24)	(−3.35)		(−3.58)
TURN	−0.263			−0.397			−0.517	−0.541
	(−0.43)			(−0.66)			(−0.84)	(−0.91)
ILLIQ		−1.152**			−0.301		−1.186***	−0.471
		(−2.08)			(−0.38)		(−2.77)	(−0.78)
IO			0.473***			0.332**	0.411***	0.300**
			(3.28)			(2.41)	(2.93)	(2.27)

Proxies for short-sale, transactions costs, and barriers to trade:

- Turnover (TURN): Monthly volume per shares outstanding (D'Avolio (2002))
- Illiquidity (ILLIQ): Monthly average daily abs. return per $-volume (Amihud (2002))
- Instituitional Ownership (IO): Thompson's 13F filings data. Shares held by institutions per total shares outstanding (D'Avolio (2002))

FM regressions: Market Frictions controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1 \,\text{SKEW} + \gamma_2 \,\text{DISP} + \gamma_3 \,\text{SKEW} \times \text{DISP} + \phi' \mathbf{Z} + \varepsilon$$

	V	VI	VII	VIII	IX	X	XI	XII
SKEW				-0.375^{**}	-0.354	-0.378^{*}		-0.280
				(-1.97)	(-1.64)	(-1.81)		(-1.46)
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$\times$DISP				(-3.75)	(-3.24)	(-3.35)		(-3.58)
TURN	-0.263			-0.397			-0.517	-0.541
	(-0.43)			(-0.66)			(-0.84)	(-0.91)
ILLIQ		-1.152^{**}			-0.301		-1.186^{***}	-0.471
		(-2.08)			(-0.38)		(-2.77)	(-0.78)
IO			0.473^{***}			0.332^{**}	0.411^{***}	0.300^{**}
			(3.28)			(2.41)	(2.93)	(2.27)

Results: TURN

- TURN negative (cf. Chordia, Subrahmanyam, & Anshuman (2001)).
- But not significant in our sample

FM regressions: Market Frictions controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1 \, \text{SKEW} + \gamma_2 \, \text{DISP} + \gamma_3 \, \text{SKEW} \times \text{DISP} + \phi' \mathbf{Z} + \varepsilon$$

	V	VI	VII	VIII	IX	X	XI	XII
SKEW				−0.375**	−0.354	−0.378*		−0.280
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×DISP				(−3.75)	(−3.24)	(−3.35)		(−3.58)
TURN	−0.263			−0.397			−0.517	−0.541
	(−0.43)			(−0.66)			(−0.84)	(−0.91)
ILLIQ		−1.152**			−0.301		−1.186***	−0.471
		(−2.08)			(−0.38)		(−2.77)	(−0.78)
IO			0.473***			0.332**	0.411***	0.300**
			(3.28)			(2.41)	(2.93)	(2.27)

Results: ILLIQ

- ILLIQ negative, significant alone or with other market frictions (consistent with prior dispersion effect studies)
- But not significant with SKEW, DISP, and SKEW×DISP
- Suggests SKEW/DISP already contain whatever information was relevant in ILLIQ

FM regressions: Market Frictions controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1 \,\text{SKEW} + \gamma_2 \,\text{DISP} + \gamma_3 \,\text{SKEW} \times \text{DISP} + \phi' \mathbf{Z} + \varepsilon$$

	V	VI	VII	VIII	IX	X	XI	XII
SKEW				−0.375**	−0.354	−0.378*		−0.280
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SKEW ×DISP				−0.964***	−0.899***	−0.931***		−0.906***
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TURN	−0.263			−0.397			−0.517	−0.541
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		(−2.08)			(−0.38)		(−2.77)	(−0.78)
IO			0.473***			0.332**	0.411***	0.300**
			(3.28)			(2.41)	(2.93)	(2.27)

Results: IO

- IO is positive, significant in all cases (cf. Gompers & Metrick (2001))
- But does not alter the picture from unconditional results

FM regressions: Market Frictions controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1\,\text{SKEW} + \gamma_2\,\text{DISP} + \gamma_3\,\text{SKEW} \times \text{DISP} + \phi'\,\mathbf{Z} + \varepsilon$$

	V	VI	VII	VIII	IX	X	XI	XII
SKEW				−0.375**	−0.354	−0.378*		−0.280
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IO			0.473***			0.332**	0.411***	0.300**
			(3.28)			(2.41)	(2.93)	(2.27)

Results: Hypotheses

- SKEW×DISP remains negative, significant in all models (H1✓)
- DISP remains insignificant when SKEW is zero in all models (H2✓)

FM regressions: Market Frictions controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1\,\text{SKEW} + \gamma_2\,\text{DISP} + \gamma_3\,\text{SKEW} \times \text{DISP} + \phi'\,\mathbf{Z} + \varepsilon$$

	V	VI	VII	VIII	IX	X	XI	XII
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IO			0.473***			0.332**	0.411***	0.300**
			(3.28)			(2.41)	(2.93)	(2.27)

Note:

- Supports expectation that other channels can operate separately
- SKEW/DISP a layer underneath other determinants of average returns?

FM regressions: Financial Distress controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1\,\text{SKEW} + \gamma_2\,\text{DISP} + \gamma_3\,\text{SKEW} \times \text{DISP} + \phi'\,\mathbf{Z} + \varepsilon$$

	XIII	XIV	XV	XVI	XVII	XVIII
SKEW			−0.316	−0.397*		−0.293
			(−1.64)	(−1.92)		(−1.56)
DISP			0.056	0.030		0.096
			(0.16)	(0.09)		(0.30)
SKEW×DISP			−1.006***	−0.984***		−1.029***
			(−3.57)	(−3.51)		(−3.69)
LEV	−0.001		0.109		0.080	0.124
	(−0.00)		(0.24)		(0.16)	(0.26)
OSCORE		−0.270**		−0.070	−0.251*	−0.079
		(−2.14)		(−0.70)	(−1.76)	(−0.70)

Proxies for financial distress:

- Leverage (LEV): Ratio of book debt to book debt + market equity. (Johnson 2004).
- OSCORE: Probability of near-term failure, $\frac{e^O}{1+e^O}$, where O is Ohlson's (1980) measure of the probability of financial distress (model 1). Details...

FM regressions: Financial Distress controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1 \,\text{SKEW} + \gamma_2 \,\text{DISP} + \gamma_3 \,\text{SKEW} \times \text{DISP} + \phi' \mathbf{Z} + \varepsilon$$

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LEV	-0.001		0.109		0.080	0.124
	(-0.00)		(0.24)		(0.16)	(0.26)
OSCORE		-0.270^{**}		-0.070	-0.251^{*}	-0.079
		(-2.14)		(-0.70)	(-1.76)	(-0.70)

Results: LEV

- LEV is not significant in our sample.

FM regressions: Financial Distress controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1 \text{ SKEW} + \gamma_2 \text{ DISP} + \gamma_3 \text{ SKEW} \times \text{DISP} + \phi' \mathbf{Z} + \varepsilon$$

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OSCORE		−0.270**		−0.070	−0.251*	−0.079
		(−2.14)		(−0.70)	(−1.76)	(−0.70)

Results: OSCORE

- OSCORE is negative, significant alone in our sample (cf. Dichev (1998); Griffin & Lemmon (2002))
- Not significant with SKEW, DISP, & SKEW×DISP
- Distressed firms tend to be positively skewed (e.g., Campbell, Hilscher & Szilagyi (2008))

FM regressions: Financial Distress controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1\, \text{SKEW} + \gamma_2\, \text{DISP} + \gamma_3\, \text{SKEW} \times \text{DISP} + \phi'\,\mathbf{Z} + \varepsilon$$

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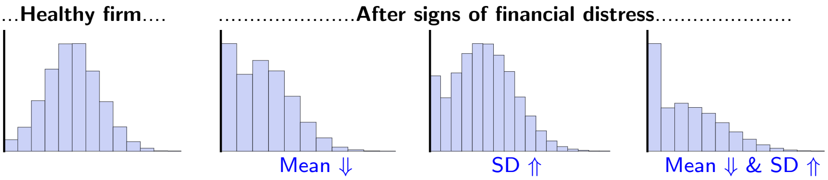
	(-0.00)		(0.24)		(0.16)	(0.26)
OSCORE		-0.270^{**}		-0.070	-0.251^{*}	-0.079
		(-2.14)		(-0.70)	(-1.76)	(-0.70)



Volatile stocks likely positively skewed, limited liability nature of equity (Conine & Tamarkin (1981)).

FM regressions: Financial Distress controls

$$\text{RET}_{(t+1)} = \gamma_0 + \gamma_1\,\text{SKEW} + \gamma_2\,\text{DISP} + \gamma_3\,\text{SKEW} \times \text{DISP} + \phi'\,\mathbf{Z} + \varepsilon$$

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OSCORE		−0.270**		−0.070	−0.251*	−0.079
		(−2.14)		(−0.70)	(−1.76)	(−0.70)

Results: Hypotheses

- SKEW×DISP remains negative, significant in all models (H1✓)
- DISP remains insignificant when SKEW is zero in all models (H2✓)

FM regressions: other cross-sectional controls

	XIX	XX	XXI	XXII	XXIII	XXIV	XXV	XXVI
SKEW				-0.214	-0.250	-0.259	-0.137	-0.026
				(-1.10)	(-1.40)	(-1.61)	(-1.11)	(-0.20)
DISP				-0.080	0.219	-0.057	-0.049	-0.013
				(-0.27)	(0.72)	(-0.20)	(-0.19)	(-0.05)
SKEW$\times$DISP				-0.741^{***}	-1.014^{***}	-0.801^{***}	-0.593^{**}	-0.615^{***}
				(-2.71)	(-3.66)	(-3.12)	(-2.46)	(-2.71)
log(ME)	0.004			-0.045			-0.093^{**}	-0.098^{***}
	(0.08)			(-1.06)			(-2.56)	(-2.63)
log(BM)	0.220^{*}			0.225^{**}			0.121	0.183^{***}
	(1.95)			(2.05)			(1.54)	(2.98)
MOM	0.007^{***}			0.006^{***}			0.007^{***}	0.007^{***}
	(2.94)			(2.67)			(4.10)	(4.06)
RET_{-1}	-0.018^{***}			-0.019^{***}			-0.025^{***}	-0.026^{***}
	(-3.75)			(-4.01)			(-5.03)	(-5.29)
IVOL		-25.461^{***}			-21.799^{***}		-21.484^{***}	-19.838^{***}
		(-3.37)			(-3.26)		(-5.63)	(-5.36)
β_{HML}			0.149			0.160^{*}	0.070	0.090
			(1.50)			(1.67)	(0.90)	(1.28)
β_{CoSkew}			-0.024^{*}			-0.023^{*}	-0.019^{*}	-0.016^{*}
			(-1.90)			(-1.91)	(-1.88)	(-1.76)
$\beta_{\text{MKT}}, \beta_{\text{SMB}}, \beta_{\text{UMD}}, \beta_{\text{LIQ}}$			Y			Y	Y	Y
Friction/Distress Controls								Y

Standard controls:

- Valuation and prior returns: log(ME), log(BM), MOM, RET_{-1}
- Idiosyncratic volatility: IVOL (Ang et al 2006)
- Factor Loadings: β_{MKT}, β_{SMB}, β_{HML}, β_{UMD}, β_{LIQ}, β_{CoSkew}

FM regressions: other cross-sectional controls

	XIX	XX	XXI	XXII	XXIII	XXIV	XXV	XXVI
SKEW				-0.214	-0.250	-0.259	-0.137	-0.026
				(-1.10)	(-1.40)	(-1.61)	(-1.11)	(-0.20)
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SKEW$\times$DISP				-0.741^{***}	-1.014^{***}	-0.801^{***}	-0.593^{**}	-0.615^{***}
				(-2.71)	(-3.66)	(-3.12)	(-2.46)	(-2.71)
$\log(\text{ME})$	0.004			-0.045			-0.093^{**}	-0.098^{***}
	(0.08)			(-1.06)			(-2.56)	(-2.63)
$\log(\text{BM})$	0.220^{*}			0.225^{**}			0.121	0.183^{***}
	(1.95)			(2.05)			(1.54)	(2.98)
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β_{CoSkew}			-0.024^{*}			-0.023^{*}	-0.019^{*}	-0.016^{*}
			(-1.90)			(-1.91)	(-1.88)	(-1.76)
$\beta_{\text{MKT}}, \beta_{\text{SMB}}, \beta_{\text{UMD}}, \beta_{\text{LIQ}}$			Y			Y	Y	Y
Friction/Distress Controls								Y

Results:

- Size is significant, negative, given all other controls
- MOM, short-term reversals RET_{-1}, IVOL all significant in our sample
- Factor loadings not significant, except β_{CoSkew} which is weakly significant, negative

FM regressions: other cross-sectional controls

	XIX	XX	XXI	XXII	XXIII	XXIV	XXV	XXVI
SKEW				-0.214	-0.250	-0.259	-0.137	-0.026
				(-1.10)	(-1.40)	(-1.61)	(-1.11)	(-0.20)
DISP				-0.080	0.219	-0.057	-0.049	-0.013
				(-0.27)	(0.72)	(-0.20)	(-0.19)	(-0.05)
SKEW$\times$DISP				-0.741^{***}	-1.014^{***}	-0.801^{***}	-0.593^{**}	-0.615^{***}
				(-2.71)	(-3.66)	(-3.12)	(-2.46)	(-2.71)
$\log$(ME)	0.004			-0.045			-0.093^{**}	-0.098^{***}
	(0.08)			(-1.06)			(-2.56)	(-2.63)
$\log$(BM)	0.220^{*}			0.225^{**}			0.121	0.183^{***}
	(1.95)			(2.05)			(1.54)	(2.98)
MOM	0.007^{***}			0.006^{***}			0.007^{***}	0.007^{***}
	(2.94)			(2.67)			(4.10)	(4.06)
RET_{-1}	-0.018^{***}			-0.019^{***}			-0.025^{***}	-0.026^{***}
	(-3.75)			(-4.01)			(-5.03)	(-5.29)
IVOL		-25.461^{***}			-21.799^{***}		-21.484^{***}	-19.838^{***}
		(-3.37)			(-3.26)		(-5.63)	(-5.36)
β_{HML}			0.149			0.160^{*}	0.070	0.090
			(1.50)			(1.67)	(0.90)	(1.28)
β_{CoSkew}			-0.024^{*}			-0.023^{*}	-0.019^{*}	-0.016^{*}
			(-1.90)			(-1.91)	(-1.88)	(-1.76)
$\beta_{\text{MKT}}, \beta_{\text{SMB}}, \beta_{\text{UMD}}, \beta_{\text{LIQ}}$			Y			Y	Y	Y
Friction/Distress Controls								Y

Results: Hypotheses

- SKEW$\times$DISP remains negative, significant in all models (H1✓)
- DISP remains insignificant when SKEW is zero in all models (H2✓)

Summary

Minimal theory of equilibrium prices with disagreement

$\Rightarrow$ new explanation of skewness and forecast dispersion effects

$+$ new predictions supported by the data

- Interaction term is negative, significant (H1)
- No significant dispersion effect independent of skewness (H2)
- High skewness and dispersion yield negative average returns (H3)
  - Joint effects economically significant (over 1.6%/month)
- Robust to market frictions, financial distress, or standard determinants

Appendix

Proof of Lemma

- Taylor expand demand, $x_+(p)$, at $p = \mu_+$

$$x_+(p) = x_+(\mu_+) + x'_+(\mu_+)(p - \mu_+) + \frac{x''_+(\mu_+)}{2}(p - \mu_+)^2 + o(1)(p - \mu_+)^2.$$

- $x_+(\mu) = 0$ (because risk aversion $\rightarrow$ no exposure to a mean-zero lottery)
- FOC: $E_+[u'(w_0 + x_+(p)(\tilde{\theta} - p)](\tilde{\theta} - p) = 0.$
- Differentiate FOC wrt p. Plug in $p = \mu_+$ and $x_+(\mu_+) = 0$

$$x'_+(\mu_+) = \frac{u'(w_0)}{u''(w_0)\sigma^2}$$

- Differentiate FOC twice, plug in $p = \mu_+$

$$x''_+(\mu_+) = -\frac{u'''(w_0)E_+(\tilde{\theta} - \mu_+)^3 x'_+(\mu)^2}{u''(w_0)\sigma^2} = -\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}$$

Back

What about σ?

Dimensional analysis: $d \approx \sigma\delta$, implies

$$\mu - p \approx -\frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2} s\sigma\delta^2.$$

If skewness is positive, the return is decreasing in volatility!

- Consistent with the idio vol puzzle [Ang, Hodrick, Xing, and Zhang (2006)]

Caveat: What is the relationship between d and σ? Ultimately, an empirical question.

Disagreement with Arbitrary Types

Suppose investor i believes that the time-1 payoff is draw from CDF F_{d_i}, where d_i is drawn from a **bounded mean-zero** random variable $\tilde{d}$.

Proposition

There exists a $\overline{d} > 0$ such that if $Var(\tilde{d})$ is bounded by $\overline{d}$, then the equilibrium price is given by the following equation.

$$\mu - p = -\frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2}\frac{s}{\sigma}Var(\tilde{d}) + o(1)Var(\tilde{d}),$$

where the little-o notation $o(1)$ is an unknown function that converges to 0 as $Var(\tilde{d}) \to 0$.

General Structures of Disagreement

- Two types of investors, type A and type B.
- Type j's belief: (μ_j, σ_j, s_j), for $j = A, B$.
- The mean-variance (or, linear-demand) benchmark price is

$$p_0 = \frac{\frac{\mu_A}{\sigma_A^2} + \frac{\mu_B}{\sigma_B^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}}.$$

Proposition

Suppose $\mu_A \neq \mu_B$. There exists a $\bar{\epsilon} > 0$ such that if $|\mu_A - \mu_B| \leq \bar{\epsilon}$, then the equilibrium price is greater than the benchmark price p_0 if $\sigma_A s_A + \sigma_B s_B > 0$; the equilibrium price is lower than the benchmark price p_0 if $\sigma_A s_A + \sigma_B s_B < 0$.

Heterogeneous Investor Preferences (1)

- Suppose all investors' utility functions can be either u_1 or u_2.
- If $s > 0$,
  - the total demand from positive investors is "locally" convex
  - the total demand from negative investors is "locally" convex
- Sum of convex functions is convex!

Heterogeneous Investor Preferences (2)

- What if positive investors have u_1 and negative investors have u_2?
- The linear-demand benchmark price

$$p_0 = \frac{\frac{u_1'(w_0)}{-u_1''(w_0)}\mu_+ + \frac{u_2'(w_0)}{-u_2''(w_0)}\mu_-}{\frac{u_1'(w_0)}{-u_1''(w_0)} + \frac{u_2'(w_0)}{-u_2''(w_0)}}.$$

This price is consistent with Lintner (1969).

Proposition

There exists a $\overline{d} > 0$ such that if $d \leq \overline{d}$, then the equilibrium price is greater than the benchmark price p_0 if and only if $s > 0$.

Non-Zero Aggregate Supply

Proposition

Suppose the risky asset's payoff, $\tilde{\theta}$, is bounded under both type investor's belief, and there is non-zero aggregate supply. Let p_0 denote the mean-variance benchmark price. Then, there exists thresholds $\overline{d} > 0$, $\overline{s} > \underline{s}$, such that the following properties hold.

- *If $s > \overline{s}$ and $d < \overline{d}$, the equilibrium price is greater than p_0.*
- *If $s < \underline{s}$ and $d < \overline{d}$, the equilibrium price is smaller than p_0.*

Key: demand schedule becomes **convex** over the relevant price range when skewness is sufficiently **large**

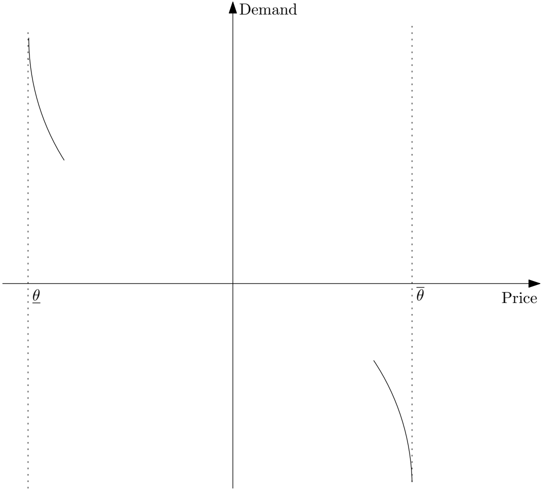
- This is true regardless of the moments higher than the fourth

“Small” Disagreement

- Generally no global convexity/concavity Appendix
- The convex/concave region depends on kurtosis.
  - For CARA, suppose $s = 1$ and $k = 10$, then $\frac{x''(\mu)}{x'''(\mu)}$ is about $\frac{\sigma}{4}$.
  - For CRRA, suppose $\gamma = 2, s = 1$, and $k = 10$, then $\frac{x''(\mu)}{x'''(\mu)}$ is $\frac{3}{14}\sigma$.
- Impose parametric assumptions
  - Harris and Raviv (1993): CARA+binary
  - Martin and Papadimitriou (2019): log+binary

Global Convexity/Concavity?

In general, the demand function can not be globally convex/concave



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Data (1/2)

Intersection of the CRSP, COMPUSTAT, and IBES universes, and firms with expected skewness measures (Dec 1983-Dec 2011):

- CRSP: monthly stock returns (adjusted for delisting bias), prices, volume, and shares outstanding.
- COMPUSTAT: accounting data merged with CRSP database.
- IBES: analysts forecasts from Institutional Brokers Estimate System (I/B/E/S) U.S. Unadjusted Detail History data set.
- Our primary proxy for expected skewness, SKEW: expected total skewness measure (Boyer, Mitton and Vorkink (2010)) provided by the authors from Jul 1969–Dec 2011.
- Our primary proxy for forecast dispersion, DISP: essentially same as that used by Diether, Malloy and Scherbina (2002) and many others: month-end standard deviation of current-fiscal-year EPS estimates scaled by the mean across analysts tracked by IBES, from Dec 1983–Dec 2013.

Data (2/2)

Filters:

- Common shares traded on NYSE, AMEX, or NASDAQ having price $> \$5$.
- Positive ME and BE (for log(BM)), and non-missing book debt (for LEV), with recent accounting data at least 3 months old.
- 12 months of returns (MOM and RET$_{-1}$) & return for subsequent month (RET).
- Non-missing IVOL, ILLIQ, & Factor Loadings (≥ 10 days of daily returns in month, and ≥ 12 months of monthly returns in prior 36-month period).
- Non-missing expected skewness measure ($\geq$ 250 days of daily returns in prior 60-month period).

Baseline dataset:

- 13,888 unique firms over about 28 years (337 months) in the total sample. Average of 3,338 firms per month.
- 10,404 unique firms with ≥ 2 forecasts. Average of 2,075 firms per month with ≥ 2 forecasts. 699,364 firm-months with ≥ 2 forecasts.

Expected skewness proxy: SKEW

Expected (ex-ante) skewness is difficult to measure.

- Lagged skewness alone does not adequately forecast skewness.
- As opposed to variances and covariances, skewness is not stable over time (Harvey and Siddique 1999).

Boyer, Mitton and Vorkink (2010) use firm-level variables to predict skewness:

- BMV's measures of skewness each month predict skewness of return distribution over the next 60 months using firm characteristics in prior 60 months.
  - Firm characteristics: lagged skewness, idiosyncratic volatility, momentum, turnover, size, exchange, and industry
  - Available for every CRSP stock with sufficient history to estimate it.
- Strong negative cross-sectional relationship between skewness and average returns. (Skewness effect)

$$\text{SKEW} := \widehat{\mathbf{E}}_t[\text{Total Skewness}_{t+1\to t+60}] \quad \text{(BMV)}$$

Forecast dispersion proxy: DISP

Most commonly used measure of analysts' forecast dispersion:

$$\text{DISP} := \frac{\text{stdF}^*}{|\text{meanF}|}$$

- IBES forecast summary files (rounding and staleness problems):
  - split-adjustment procedure can lead to wrong conclusions (Diether, Malloy and Scherbina (2002), Payne and Thomas (2003), Baber and Kang (2002))
  - often includes stale forecasts in computing forecast statistics (Morse, Stephan, and Stice (1991), Brown and Han (1992), Stickel (1996), Barron and Stuerke (1998))
- We use IBES **unadjusted** detail files:
  - Only latest forecast by each analyst
  - No forecasts over 12 months old or for fiscal periods already ended.
  - Adjust for stock splits using CRSP adjustment factors
  - Exclude firm-months where meanF $= 0$ (small number of instances)
  - We winsorize each month above at 97.5% level to handle outliers and any near division-by-zero observations.

Back to data overview...

*stdF = month-end std. deviation of current-fiscal-year earnings estimates across analysts tracked by IBES.

OSCORE

The variable O is defined as a specific weighted sum of nine accounting factors (or ratios):

$$\begin{aligned} O = -1.32 - 0.407O_1 + 6.03O_2 - 1.43O_3 \\ + 0.076O_4 - 1.72O_5 - 2.37O_6 \\ - 1.83O_7 + 0.285O_8 - 9.521O_9, \end{aligned}$$

where $O_1 := \log(\text{total assets})$; $O_2 := \frac{\text{total liabilities}}{\text{total assets}}$; $O_3 := \frac{\text{working capital}}{\text{total assets}}$; $O_4 := \frac{\text{current liabilities}}{\text{current assets}}$; $O_5 := 1$ if total liabilities $>$ total assets, 0 otherwise; $O_6 := \frac{\text{net income}}{\text{total assets}}$; $O_7 := \frac{\text{funds from operations}}{\text{total liabilities}}$; $O_8 := 1$ if a net loss for the last two years, 0 otherwise; and $O_9 := \frac{\text{net income}_t - \text{net income}_{t-1}}{|\text{net income}_t| + |\text{net income}_{t+1}|}$.

- Factors not always available in COMPUSTAT for every firm, but most factors available for most firms.
- Instead of throwing out observations with some missing factors, we replace any missing factor with a conservative representative value for all COMPUSTAT firms that have a non-missing value for that factor in the same year, to maximize total observations in the complete case.
- Missing factors that would increase the probability of failure measure (O_2, O_4, and O_8) are replaced by the 30th percentile across COMPUSTAT firms for the current year while missing factors that would decrease the probability of failure measure (O_1, O_3, O_5, O_6, O_7, and O_9) are replaced by the 70th percentile.
- OSCORE is the probability transformed version of the O measure: OSCORE $:= \frac{e^O}{1+e^O}$.

Our results are robust to using the untransformed O measure or to dropping firms with missing observations for any of the nine factors.

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Glossary

- TURN: Monthly share volume per monthly shares outstanding.
- ILLIQ: Monthly average of (absolute daily return (in %) per $1,000 daily trading volume).
- IO: Thompson's 13F filings data. Use CRSP share factors to adjust for stock splits. Ratio of the sum of most recent reported shares held by institutions to the total shares outstanding. If no 13f shares held is reported for a firm, reported shares are set to zero. Source: CDA/Spectrum files maintained by Thomson Financial.
- LEV: Book value of debt over the sum of book value of debt and market value of equity as of the most recent statement at least three months old.
- OSCORE: $\frac{e^O}{1+e^O}$ (interpreted as a probability of near-term failure), where O is Ohlson's (1980) measure of the probability of financial distress (model 1).
- ME: Market value of equity (in thousands). This variable measures the market value of the firm at the end of the fiscal year in the most current annual financial statement reported prior to month t
- BM: Ratio of book equity to market equity, using book equity from most recent statement at least three months old.
- MOM: Cumulative return over last 12 months excluding the most recent month.
- RET_{-1}: Return in most recent month.
- IVOL: Standard deviation of residuals of excess daily returns regressed onto daily FF factors. At least 10 days of returns within the month required, otherwise set to missing for that month.
- β's: Factor loadings are based on rolling regressions of excess monthly returns on all factors using the most recent 36 months of returns data. At least 12 months of returns are required. Factors: FF (Mktrf SMB HML), UMD, Pastor-Stambaugh value-weighted traded factor, Mktrf^2.

Summary statistics: average monthly distribution of firm characteristics

	Mean	P01	P05	P10	P25	P50	P75	P90	P95	P99
SKEW	0.74	−0.07	0.14	0.26	0.48	0.69	1.01	1.25	1.45	1.76
DISP (%)	17.1	0.1	1.0	1.5	2.9	6.5	16.7	40.1	77.1	151.9
Analysts	9.4	2.0	2.0	2.2	3.7	6.8	12.8	20.4	25.4	35.5
Market Frictions and Financial Distress Proxies										
TURN (%)	14.6	0.9	2.1	3.2	5.7	9.9	17.5	30.5	42.3	78.2
ILLIQ ($\times 1,000$)	20.4	0.0	0.0	0.1	0.3	1.4	7.1	30.8	73.0	315.8
IO (%)	44.2	0.0	0.1	3.6	22.0	47.7	65.5	77.0	82.1	89.2
LEV (%)	23.4	0.0	0.0	0.3	3.6	17.4	37.8	56.7	67.3	84.4
OSCORE (%)	28.8	0.0	0.0	0.0	0.9	10.2	50.1	96.5	99.9	99.9
Valuation, Prior Returns, Idiosyncratic Volatility, and Factor Loadings										
ME ($ mil.)	3,581	40	76	113	238	646	2,088	6,940	13,926	58,863
BM (%)	59.0	5.4	12.1	17.5	30.1	50.4	76.7	107.2	131.7	207.5
Price	40.3	5.4	6.9	8.6	13.7	22.7	35.3	51.0	63.2	98.0
MOM (%)	18.60	−57.22	−39.08	−28.12	−9.64	10.37	34.33	68.71	101.49	208.66
RET_{-1} (%)	1.51	−25.86	−15.45	−11.01	−4.91	0.85	7.03	14.50	20.41	36.64
IVOL (%)	2.10	0.54	0.77	0.93	1.26	1.82	2.62	3.59	4.33	6.28
β_{MKT}	1.08	−1.07	−0.12	0.17	0.57	1.00	1.51	2.10	2.56	3.78
β_{SMB}	0.67	−2.06	−0.94	−0.59	−0.08	0.53	1.29	2.15	2.79	4.45
β_{HML}	0.05	−4.20	−2.31	−1.58	−0.62	0.15	0.82	1.53	2.08	3.65
β_{UMD}	−0.08	−2.83	−1.53	−1.09	−0.52	−0.05	0.39	0.86	1.25	2.42
β_{LIQ}	−0.03	−2.34	−1.31	−0.92	−0.42	−0.01	0.39	0.84	1.21	2.20
β_{CoSkew}	0.28	−41.13	−20.49	−14.11	−6.47	−0.01	6.51	14.64	21.80	46.39

Summary statistics (1/2)

	Baseline Dataset: Number of Analysts ≥ 2									
	S1	S2	S3	S4	S5	D1	D2	D3	D4	D5
SKEW	0.26	0.52	0.69	0.95	1.25	0.65	0.64	0.68	0.72	0.82
DISP (%)	5.6	5.9	6.1	7.0	9.8	1.5	3.4	6.5	13.5	40.1
Number of Analysts	10.0	9.2	8.4	5.7	3.8	5.9	7.5	7.3	7.1	6.6
Market Frictions and Financial Distress Proxies										
TURN (%)	13.0	11.0	10.0	8.8	7.8	7.8	9.0	10.1	11.4	12.2
ILLIQ ($\times 1,000$)	0.5	0.8	0.9	2.7	14.7	1.4	0.9	1.2	1.7	2.4
IO (%)	49.2	55.8	53.4	44.7	34.6	47.4	49.0	49.1	48.3	44.5
LEV (%)	19.5	16.1	17.9	18.5	16.7	16.3	17.5	17.7	17.4	19.6
OSCORE (%)	9.2	7.0	9.5	14.1	19.1	7.7	8.1	9.6	13.1	26.1
Valuation, Prior Returns, Idiosyncratic Volatility, and Factor Loadings										
ME ($ mil.)	1,463	1,302	1,085	401	162	952	922	704	534	379
BM (%)	47.8	44.3	48.5	53.7	60.2	43.0	47.2	50.7	53.6	61.0
Price	30.0	28.8	26.3	19.6	12.1	29.5	27.4	23.5	19.1	14.1
MOM (%)	23.01	13.95	10.98	7.41	−3.31	15.52	13.77	10.80	7.56	−1.04
RET_{-1} (%)	1.21	1.05	0.88	0.85	0.17	1.12	1.03	0.92	0.65	0.29
IVOL (%)	1.64	1.67	1.65	1.89	2.42	1.48	1.56	1.77	2.04	2.36
β_{MKT}	0.95	1.03	1.04	0.98	0.98	0.87	0.93	1.01	1.09	1.16
β_{SMB}	0.28	0.44	0.46	0.67	0.92	0.37	0.41	0.52	0.66	0.79
β_{HML}	0.18	0.07	0.14	0.18	0.15	0.14	0.15	0.16	0.15	0.14
β_{UMD}	0.05	−0.05	−0.07	−0.07	−0.11	0.03	−0.01	−0.06	−0.09	−0.16
β_{LIQ}	0.00	−0.03	−0.01	0.00	−0.03	−0.01	−0.01	−0.02	0.00	−0.03
β_{CoSkew}	0.08	0.10	0.24	−0.20	−0.34	−0.08	0.00	0.04	0.08	−0.07

Summary statistics (2/2)

	Number of Analysts ≥ 2					Number of Analysts ≤ 1				
	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
SKEW	0.26	0.52	0.69	0.95	1.25	0.58	0.93	1.26	1.49	1.75
DISP (%)	5.6	5.9	6.1	7.0	9.8	—	—	—	—	—
Number of Analysts	10.0	9.2	8.4	5.7	3.8	—	—	—	—	—
Market Frictions and Financial Distress Proxies										
TURN (%)	13.0	11.0	10.0	8.8	7.8	6.8	4.4	3.5	2.6	2.5
ILLIQ ($\times 1,000$)	0.5	0.8	0.9	2.7	14.7	6.3	28.6	54.0	103.8	190.5
IO (%)	49.2	55.8	53.4	44.7	34.6	31.2	22.6	17.2	12.3	8.5
LEV (%)	19.5	16.1	17.9	18.5	16.7	17.3	18.6	18.8	24.8	24.0
OSCORE (%)	9.2	7.0	9.5	14.1	19.1	15.4	23.7	25.1	35.1	34.4
Valuation, Prior Returns, Idiosyncratic Volatility, and Factor Loadings										
ME ($ mil.)	1,463	1,302	1,085	401	162	405	165	78	50	35
BM (%)	47.8	44.3	48.5	53.7	60.2	51.9	59.9	65.5	73.0	78.3
Price	30.0	28.8	26.3	19.6	12.1	21.8	15.9	11.8	11.0	8.6
MOM (%)	23.01	13.95	10.98	7.41	−3.31	22.63	15.16	10.96	8.32	3.76
RET_{-1} (%)	1.21	1.05	0.88	0.85	0.17	1.03	0.81	0.50	0.31	0.60
IVOL (%)	1.64	1.67	1.65	1.89	2.42	1.86	2.01	2.29	2.29	2.62
β_{MKT}	0.95	1.03	1.04	0.98	0.98	0.82	0.76	0.69	0.61	0.57
β_{SMB}	0.28	0.44	0.46	0.67	0.92	0.59	0.71	0.71	0.65	0.68
β_{HML}	0.18	0.07	0.14	0.18	0.15	0.22	0.25	0.23	0.25	0.25
β_{UMD}	0.05	−0.05	−0.07	−0.07	−0.11	−0.02	−0.02	−0.06	−0.07	−0.09
β_{LIQ}	0.00	−0.03	−0.01	0.00	−0.03	0.01	0.01	0.02	0.04	0.02
β_{CoSkew}	0.08	0.10	0.24	−0.20	−0.34	−0.41	−0.93	−1.14	−0.61	−0.89

Summary statistics: cross correlations

	SKEW	DISP	ILLIQ	TURN	IO	LEV	OSCORE	log(ME)	log(BM)	MOM	IVOL	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	β_{CoSkew}
SKEW	1.00	0.14	0.23	−0.18	−0.15	0.02	0.12	−0.51	0.14	−0.25	0.23	0.01	0.16	0.01	−0.05	−0.01
DISP	0.14	1.00	0.04	0.10	−0.05	0.06	0.21	−0.16	0.11	−0.12	0.23	0.08	0.10	0.00	−0.07	0.00
ILLIQ	0.23	0.04	1.00	−0.10	−0.11	0.03	0.06	−0.26	0.09	−0.07	0.17	−0.05	0.03	0.02	−0.01	−0.01
TURN	−0.18	0.10	−0.10	1.00	0.14	−0.16	−0.04	0.01	−0.20	0.16	0.44	0.15	0.10	−0.15	−0.01	0.04
IO	−0.15	−0.05	−0.11	0.14	1.00	−0.09	−0.09	0.20	−0.05	0.02	−0.08	0.05	−0.03	−0.02	0.00	0.01
LEV	0.02	0.06	0.03	−0.16	−0.09	1.00	0.27	0.02	0.47	−0.12	−0.16	−0.02	−0.07	0.24	−0.06	−0.01
OSCORE	0.12	0.21	0.06	−0.04	−0.09	0.27	1.00	−0.16	0.14	−0.07	0.09	0.01	0.05	0.05	−0.06	0.00
log(ME)	−0.51	−0.16	−0.26	0.01	0.20	0.02	−0.16	1.00	−0.19	0.09	−0.40	−0.01	−0.30	0.00	0.05	0.02
log(BM)	0.14	0.11	−0.20	0.09	−0.05	0.47	0.14	−0.19	1.00	−0.36	−0.09	−0.05	−0.03	0.19	−0.08	−0.01
MOM	−0.25	−0.12	−0.07	0.16	0.02	−0.12	−0.07	0.09	−0.36	1.00	−0.02	0.02	0.01	−0.01	0.04	0.02
IVOL	0.23	0.23	0.17	0.44	−0.08	−0.16	0.09	−0.40	−0.09	−0.02	1.00	0.13	0.23	−0.11	−0.05	0.02
β_{MKT}	0.01	0.08	−0.05	0.15	0.05	−0.02	0.01	−0.01	−0.05	0.02	0.13	1.00	0.08	0.25	0.02	0.09
β_{SMB}	0.16	0.10	0.03	0.10	−0.03	−0.07	0.05	−0.30	−0.03	0.01	0.23	0.08	1.00	0.18	0.00	0.14
β_{HML}	0.01	0.00	0.02	−0.15	−0.02	0.24	0.05	0.00	0.19	−0.01	−0.11	0.25	0.18	1.00	0.06	0.10
β_{UMD}	−0.05	−0.07	−0.01	−0.01	0.00	−0.06	−0.06	0.05	−0.08	0.04	−0.05	0.02	0.00	0.06	1.00	0.16
β_{CoSkew}	−0.01	0.00	−0.01	0.04	0.01	−0.01	0.00	0.02	−0.01	0.02	0.02	0.09	0.14	0.10	0.16	1.00

Skewness effect — long history

$\Uparrow$ Return Skewness $\Downarrow$ Expected Returns

Preference for skewness in portfolio choice context:

- Arditti (1967); Scott & Horvath (1980)
- Take prices & return distributions as given—don't speak to general equilibrium pricing effects

Higher moments' (e.g., **co-skewness**) effect on stochastic discount factors (SDFs)

- Rubenstein (1973); Kraus & Litzenberger (1976, 1983); Harvey & Siddique (2000)
- Full diversification with respect to three or more moments in general equilibirum
- Agents have preferences specifically over third moments
- Imply idiosyncratic characteristics irrelevant

Skewness effect — properties of individual securities matter

$\Uparrow$ Return Skewness $\Downarrow$ Expected Returns

Why look to properties of individual securities?

- Full diversification counterfactual (Mitton & Vorkink (2007))
- Full diversification is extremely fragile (Malkiel & Xu (2006))
- Diversification erodes skewness exposure—some investors underdiversify to capture return skewness, so **idiosyncratic skewness relevant** (Simkowitz & Beedles (1978); Conine & Tamarkin (1981))
- **Idiosyncratic return volatility important** for future returns (Ang et al. (2006, 2009))

Fischer Black (1986):

> *"...if there is little or no trading in* **individual** *shares, there can be no trading in mutual funds or portfolios or index futures or index options, because there will be no practical way to price them. The whole structure of financial markets depends on relatively liquid markets in the shares of* **individual** *firms."*

Skewness effect — explanations & evidence in individual securities

$\Uparrow$ Return Skewness $\Downarrow$ Expected Returns

Explanations — recent theories show not just coskewness with the market that can be priced, but also a security's own skewness:

- Behavioral — Optimal expectations, structural models of subjective beliefs: Brunnermeier & Parker (2005); Brunnermeier, Gollier & Parker (2007)
- Behavioral — Cumulative prospect theory: Barberis & Huang (2008)
- Non-standard preferences: Mitton & Vorkink (2007)
- **Financial distress** strongly associated with positive skewness: Campbell, Hilscher, & Szilagyi (2008)

Evidence — very recent because expected skewness has been difficult to measure:

- Boyer, Mitton, & Vorkink (2010): Firm-level variables to predict idiosyncratic skewness (size, industry, idiosyncratic volatility, momentum, turnover, etc.).
- Conrad, Dittmar, & Ghysels (2013): Options-implied ex-ante skewness.
- Amaya, Christoffersen, Jacobs, & Vasquez (2013): Realized skewness.
- and many others ...

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Dispersion effect — background

$\Uparrow$ Dispersion in Analysts' Forecasts $\Downarrow$ Expected Returns

Financial analysts are important market participants:

- Reach millions of investors through media
- Analysts provide forecasts of a firm's earnings
- Investors regard forecasts (rather than stock recommendations) as important input to their valuations[2]
- Large fraction of stocks are followed by several analysts[3]

Forecast dispersion is an important measure:

- Relationship to prices is long-standing & fundamental issue in finance
- Since 1990, dispersion measure used increasingly in papers in top finance and accounting journals[4]

[2] Block (1999, Financial Analysts Journal)
[3] 40.5% of all CRSP stocks in 2000; 84% of $>$median size firms.
[4] Barron, Stanford, and Yu (2009, Contemporary Accounting Research)

Dispersion effect — evidence & explanations

$\Uparrow$ Dispersion in Analysts' Forecasts $\Downarrow$ Expected Returns

Diether, Malloy, and Scherbina (2002, highly cited re: financial anomalies):

- Investor belief heterogeneity (proxied by forecast dispersion) and **market frictions** (short-selling constraints) prevent the revelation of negative opinions (Miller (1977))

Johnson (2004):

- Dispersion proxies for idiosyncratic parameter risk, negatively related to returns for **levered** firms

Sadka & Scherbina (2007):

- **Trading costs** and **illiquidity** (incl. short-selling risks) — positive relation between forecast dispersion & trading costs

Avramov, Chordia, Jostova and Philipov (2009):

- Linked to **financial distress** and not to market frictions.

and many others...

Back

Related Literature

Disagreement in Financial Markets

- Cross section: Miller (1977); Jarrow (1980); Diamond and Verrecchia (1987); Morris (1996); Chen, Hong, and Stein (2002)
- Financial bubbles: Harrison and Kreps (1978); Scheinkman and Xiong (2003); Hong, Scheinkman, and Xiong (2006)
- Momentum, reversal, information, volume, volatility, and comovement: Harris and Raviv (1993); Kandel and Pearson (1995); Cao and Ou-Yang (2008); Dumas, Kurshev, and Uppal (2009); Banerjee and Kremer (2010); Ottaviani and Sørensen (2015); Atmaz and Basak (2018); Banerjee, Davis, and Gondhi (2018); Chabakauri and Han (2020) among others
- Martin and Papadimitriou (2019); Yan (2010); Gao, Lu, Song, Yan (2019)

Robustness tests: Overview

- Robustness to mis-specification via nonlinearities. Interaction term is not capturing left-out squared terms.
- Orthogonalizing the regressors. Interaction term is not capturing all sorts of interactions or nonlinearities of other regressors.
- Alternative measures of forecast dispersion and skewness.
- Winsorization of all explanatory variables (at 1st and 99th percentiles) to ascertain whether outliers distort the estimates.
- Other explanations of dispersion effect via interactions.

Robustness to nonlinear mis-specification

Potential problem:

- If SKEW and DISP are correlated, e.g., DISP $= 0.5 \times$SKEW $+$ noise,
- Then SKEW$\times$DISP $= 0.5 \times (\text{SKEW})^2 +$ noise.
- So, interaction coefficient could be picking up mis-specified non-linearities.

Robustness test:

- Include squared terms for each main effect as well.
- Should still see strong significance for SKEW$\times$DISP, but squared terms should be insignificant.

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Robustness test:

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- Should still see strong significance for SKEW$\times$DISP, but squared terms should be insignificant.

Results:

	SKEW	DISP	SKEW$\times$DISP	(SKEW)^2	(DISP)^2
XXVII	-0.093	-0.702	-0.971^{***}	-0.218	0.607
	(-0.24)	(-1.09)	(-3.54)	(-1.40)	(1.52)

- Significance for SKEW$\times$DISP, virtually unaltered (a bit stronger) ✓
- Squared terms insignificant ✓
- Main effects interpretation not comparable to model without squared terms

Robustness to confounding interactions/nonlinearities of other regressors

Potential problem:

- If SKEW or DISP are correlated with other regressors, then SKEW$\times$DISP could be picking up mis-specified non-linearities of, or interactions between, other regressors.
- Is interaction of parts of SKEW and DISP **unexplained by other regressors** still significant?

Orthogonalization robustness test:

- Balli & Sorensen (2013, Empirical Economics, "Interaction Effects in Economics")
- First, orthogonalize SKEW and DISP with respect to all other regressors.
- Run regressions with SKEW, DISP, SKEW.$\perp\times$DISP.$\perp$, and other regressors.
- Caveat: May truly be interaction of non-orthogonalized SKEW and DISP that affect returns; But, orthogonalized interaction significant $\Rightarrow$ less likely spurious interaction

Robustness to confounding interactions/nonlinearities of other regressors

	VIII.⊥	IX.⊥	X.⊥	XII.⊥	XV.⊥	XVI.⊥
SKEW.⊥ ×DISP.⊥	-0.871^{***}	-0.824^{***}	-0.814^{***}	-0.637^{***}	-0.900^{***}	-0.928^{***}
	(-3.62)	(-3.06)	(-3.00)	(-2.72)	(-3.45)	(-3.36)
Controls	TURN	ILLIQ	IO	TURN ILLIQ IO	LEV	OSCORE

	XVIII.⊥	XXII.⊥	XXIII.⊥	XXIV.⊥	XXV.⊥	XXVI.⊥
SKEW.⊥ ×DISP.⊥	-0.907^{***}	-0.643^{**}	-0.640^{**}	-0.553^{**}	-0.614^{**}	-0.509^{*}
	(-3.42)	(-2.07)	(-2.41)	(-2.46)	(-2.18)	(-1.82)
Controls	LEV OSCORE	log(ME) log(BM) MOM RET_{-1}	IVOL	β_{MKT} β_{SMB} β_{HML} β_{UMD} β_{LIQ} β_{CoSkew}	log(ME) log(BM) MOM RET_{-1} IVOL β_{MKT} β_{SMB} β_{HML} β_{UMD} β_{LIQ} β_{CoSkew}	TURN ILLIQ IO LEV OSCORE log(ME) log(BM) MOM RET_{-1} IVOL β_{MKT} β_{SMB} β_{HML} β_{UMD} β_{LIQ} β_{CoSkew}

- SKEW.⊥×DISP.⊥ significant in all prior tests that included them ✓

Alternative measures

Table: FM Regressions: Alternative Skewness Measure

	I.S_2	II.S_2	III.S_2	IV.S_2	XII.S_2	XVIII.S_2	XXII.S_2	XXIII.S_2	XXIV.S_2	XXVI.S_2
SKEW.2	-0.623^{***}		-0.539^{***}	-0.413^{**}	-0.313^{*}	-0.283	-0.218	-0.288^{*}	-0.282^{*}	-0.072
	(-3.00)		(-2.68)	(-2.05)	(-1.70)	(-1.60)	(-1.22)	(-1.65)	(-1.80)	(-0.59)
DISP		-0.830^{***}	-0.750^{***}	0.069	0.072	0.156	0.007	0.322	-0.064	0.014
		(-3.76)	(-3.60)	(0.17)	(0.21)	(0.41)	(0.02)	(0.89)	(-0.20)	(0.05)
SKEW.2 $\times$ DISP				-0.937^{***}	-0.883^{***}	-0.996^{***}	-0.739^{**}	-0.999^{***}	-0.752^{***}	-0.590^{**}
				(-3.01)	(-3.15)	(-3.26)	(-2.50)	(-3.29)	(-2.75)	(-2.44)
Market Frictions	no	no	no	no	yes	no	no	no	no	yes
Fin. Distress	no	no	no	no	no	yes	no	no	no	yes
Valuation and Prior Returns	no	no	no	no	no	no	yes	no	no	yes
Idiosync. Vol.	no	no	no	no	no	no	no	yes	no	yes
Factor Loadings	no	no	no	no	no	no	no	no	yes	yes

Boyer, Mitton and Vorkink (2010) also produce a measure of expected **idiosyncratic** skewness (skewness unexplained by FF factors):

$$\text{SKEW.2} := \widehat{\mathbf{E}}_t[\text{Idiosyncratic Skewness}_{t+1\rightarrow t+60}] \quad \text{(BMV)}$$

Alternative measures

Table: FM Regressions: Alternative Forecast Dispersion Measure

	I.D_2	II.D_2	III.D_2	IV.D_2	XII.D_2	XVIII.D_2	XXII.D_2	XXIII.D_2	XXIV.D_2	XXVI.D_2
SKEW	-0.634^{***}		-0.592^{***}	-0.467^{**}	-0.326^{*}	-0.337^{*}	-0.230	-0.286	-0.286^{*}	-0.043
	(-2.96)		(-2.84)	(-2.21)	(-1.72)	(-1.82)	(-1.20)	(-1.57)	(-1.78)	(-0.33)
DISP.2		-7.975^{***}	-6.385^{**}	2.879	2.906	3.022	0.230	6.282	1.417	1.088
		(-2.63)	(-2.28)	(0.51)	(0.57)	(0.56)	(0.04)	(1.22)	(0.29)	(0.27)
SKEW$\times$DISP.2				-12.244^{**}	-11.698^{***}	-12.753^{***}	-9.781^{**}	-13.339^{***}	-10.593^{**}	-8.260^{**}
				(-2.48)	(-2.62)	(-2.80)	(-2.20)	(-2.97)	(-2.53)	(-2.19)
Market Frictions	no	no	no	no	yes	no	no	no	no	yes
Fin. Distress	no	no	no	no	no	yes	no	no	no	yes
Valuation and Prior Returns	no	no	no	no	no	no	yes	no	no	yes
Idiosync. Vol.	no	no	no	no	no	no	no	yes	no	yes
Factor Loadings	no	no	no	no	no	no	no	no	yes	yes

Normalizing by meanF introduces numerical issues when $|\text{meanF}| \approx 0$. We follow many studies which normalize by book-value-per-share or by price:

$$\text{DISP.2} := \frac{\text{stdF}}{\frac{1}{2}\left(\frac{\text{BE}}{\text{SHROUT}} + \text{Price}\right)}$$

Alternative measures

Table: FM Regressions: Alternative Skewness and Forecast Dispersion Measures

	I.2	II.2	III.2	IV.2	XII.2	XVIII.2	XXII.2	XXIII.2	XXIV.2	XXVI.2
SKEW.2	-0.623^{***}		-0.584^{***}	-0.453^{**}	-0.345^{*}	-0.316^{*}	-0.226	-0.306^{*}	-0.295^{*}	-0.077
	(-3.00)		(-2.90)	(-2.20)	(-1.88)	(-1.79)	(-1.29)	(-1.72)	(-1.90)	(-0.64)
DISP.2		-7.975^{***}	-6.506^{**}	4.564	4.602	4.784	1.757	8.742	2.506	2.104
		(-2.63)	(-2.30)	(0.72)	(0.80)	(0.78)	(0.31)	(1.49)	(0.47)	(0.48)
SKEW.2				-13.017^{**}	-12.401^{**}	-13.599^{***}	-10.430^{**}	-14.590^{***}	-11.156^{**}	-8.876^{**}
×DISP.2				(-2.45)	(-2.58)	(-2.75)	(-2.22)	(-3.01)	(-2.52)	(-2.25)
Market Frictions	no	no	no	no	yes	no	no	no	no	yes
Fin. Distress	no	no	no	no	no	yes	no	no	no	yes
Valuation and Prior Returns	no	no	no	no	no	no	yes	no	no	yes
Idiosync. Vol.	no	no	no	no	no	no	no	yes	no	yes
Factor Loadings	no	no	no	no	no	no	no	no	yes	yes

Previous findings upheld:

- Interaction term is negative, significant (✓H1)
- No significant dispersion effect independent of skewness (✓H2)
- Skewness negative, economically significant, partially picked up by other controls, stronger than main proxies case

Robustness to influence of outliers

Table: FM Regressions: Winsorization

	I.W	II.W	III.W	IV.W	XII.W	XVIII.W	XXII.W	XXIII.W	XXIV.W	XXVI.W
SKEW.$_{P01}^{P99}$	−0.671***		−0.585***	−0.444**	−0.258	−0.311	−0.228	−0.270	−0.271*	−0.027
	(−3.00)		(−2.69)	(−2.03)	(−1.34)	(−1.60)	(−1.14)	(−1.45)	(−1.67)	(−0.20)
DISP.$^{P97.5}$		−0.829***	−0.739***	0.034	−0.007	0.130	−0.063	0.257	−0.021	0.009
		(−3.75)	(−3.57)	(0.10)	(−0.03)	(0.41)	(−0.20)	(0.85)	(−0.08)	(0.04)
SKEW.$_{P01}^{P99}$				−0.979***	−0.875***	−1.050***	−0.742***	−1.054***	−0.844***	−0.636***
×DISP.$^{P97.5}$				(−3.49)	(−3.50)	(−3.79)	(−2.65)	(−3.85)	(−3.38)	(−2.76)
Market Frictions$_{P01}^{P99}$	no	no	no	no	yes	no	no	no	no	yes
Financial Distress$_{P01}^{P99}$	no	no	no	no	no	yes	no	no	no	yes
Valuation$_{P01}^{P99}$ and Prior Returns$_{P01}^{P99}$	no	no	no	no	no	no	yes	no	no	yes
Idiosync. Vol.$_{P01}^{P99}$	no	no	no	no	no	no	no	yes	no	yes
Factor Loadings$_{P01}^{P99}$	no	no	no	no	no	no	no	no	yes	yes

Winsorization of all explanatory variables (at 1st and 99th percentiles) to ascertain whether outliers distort the estimates . Findings upheld. ✓

Other explanations of dispersion effect via interactions

	I	II	III	IV	XXVIII	XXIX	XXX	XXXI	XXXIII
SKEW	-0.634^{***}		-0.550^{***}	-0.415^{**}		-0.368^{*}		-0.327^{*}	-0.050
	(-2.96)		(-2.65)	(-1.98)		(-1.69)		(-1.68)	(-0.39)
DISP		-0.829^{***}	-0.743^{***}	0.001	-0.742^{***}	-0.122	-0.573^{**}	0.272	0.157
		(-3.75)	(-3.58)	(0.00)	(-3.23)	(-0.36)	(-2.27)	(0.72)	(0.61)
SKEW ×DISP				-0.956^{***}		-0.747^{***}		-0.956^{***}	-0.396^{*}
				(-3.38)		(-2.71)		(-3.43)	(-1.78)
ILLIQ					-0.646	-0.071			-0.770
					(-1.06)	(-0.08)			(-1.12)
DISP ×ILLIQ					-7.622	-4.693			-6.567
					(-1.44)	(-1.24)			(-1.44)
LEV							0.240	0.287	-0.317
							(0.55)	(0.68)	(-1.34)
DISP ×LEV							-0.793^{*}	-0.887^{*}	-0.999^{**}
							(-1.74)	(-1.96)	(-2.44)
Other Market Frictions and Financial Distress Controls			no	no	no	no	no	no	yes
Valuation, Prior Returns, Idiosyncratic Volatility, and Factor Loadings Controls			no	no	no	no	no	no	yes

Sadka and Scherbina (2007):

- Less liquid stocks tend to be more severely overpriced
- Argue that analyst disagreement coincides with high trading costs
- Interaction between liquidity and dispersion should absorb dispersion effect

Other explanations of dispersion effect via interactions

	I	II	III	IV	XXVIII	XXIX	XXX	XXXI	XXXIII
SKEW	-0.634^{***}		-0.550^{***}	-0.415^{**}		-0.368^{*}		-0.327^{*}	-0.050
	(-2.96)		(-2.65)	(-1.98)		(-1.69)		(-1.68)	(-0.39)
DISP		-0.829^{***}	-0.743^{***}	0.001	-0.742^{***}	-0.122	-0.573^{**}	0.272	0.157
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DISP $\times$LEV							-0.793^{*}	-0.887^{*}	-0.999^{**}
							(-1.74)	(-1.96)	(-2.44)
Other Market Frictions and Financial Distress Controls			no	no	no	no	no	no	yes
Valuation, Prior Returns, Idiosyncratic Volatility, and Factor Loadings Controls			no	no	no	no	no	no	yes

Johnson (2004):

- Interaction between leverage and dispersion absorbs dispersion effect

Other explanations of dispersion effect via interactions

	I	II	III	IV	XXVIII	XXIX	XXX	XXXI	XXXIII
SKEW	-0.634^{***}		-0.550^{***}	-0.415^{**}		-0.368^{*}		-0.327^{*}	-0.050
	(-2.96)		(-2.65)	(-1.98)		(-1.69)		(-1.68)	(-0.39)
DISP		-0.829^{***}	-0.743^{***}	0.001	-0.742^{***}	-0.122	-0.573^{**}	0.272	0.157
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							(0.55)	(0.68)	(-1.34)
DISP							-0.793^{*}	-0.887^{*}	-0.999^{**}
$\times$LEV							(-1.74)	(-1.96)	(-2.44)
Other Market Frictions and Financial Distress Controls			no	no	no	no	no	no	yes
Valuation, Prior Returns, Idiosyncratic Volatility, and Factor Loadings Controls			no	no	no	no	no	no	yes

- Some support for these other explanations, but not strong
- SKEW and DISP interaction effect robust to these other explanations

Model predicts layer underlying, not competing, with other explanations.