Expected Returns and Risk in the Stock Market

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Abstract

We present new evidence on the predictability of aggregate market returns by developing two new prediction models, one risk-based, and the other purely statistical. The *pricing kernel model* expresses the expected return as the covariance of the market return with a pricing kernel that is a linear function of portfolio returns. The *discount rate model* predicts the expected return directly as a function of weighted past portfolio returns. These models provide independent evidence of predictability, with R^2 of 16-19% for 1-year returns. We show that innovations in the pricing kernel are associated with the cash flow component of the market return.

Keywords: Predictability, Expected returns, Risk, Sentiment

JEL Classification Codes: G12, G14, G17

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1 Introduction

The now extensive literature on the predictability of stock market returns can be classified according to the nature of the predictor variables that are used. Yield-based models use some measure of yields or relative prices as instruments to predict future returns; examples include the Treasury Bill rate, the dividend yield, and the book-to-market ratio. Information-based models rely on extracting information about future returns from either a representative investor or from a subset of investors who may be presumed to possess superior information. Examples include the *cay* model of Lettau and Ludvigson (2001) which assumes that the consumption-wealth ratio is a function of unobserved expectations about future asset returns, and the short interest predictor of Rapach *et al.* (2016) which assumes that short sellers have superior information about future stock returns. Sentiment-based models use indicators of sentiment among market participants as predictors of future stock returns; examples include Baker and Wurgler (2006) and Huang *al* (2015). Finally, risk-based models assume that expected returns are a function of risk and use measures of market risk to predict returns. Examples include Merton (1980), Ghysels *et al.* (2005) Scruggs (1998), and Guo *et al.* (2009). While Merton and Ghysels *al.* use the variance of market returns to predict future returns, both Scruggs and Guo *et al.* use the covariance of market returns with another portfolio return as a predictor.

Within the risk-based framework of our first model, which we refer to as the *pricing kernel model*, the time-variation in expected returns is driven solely by time variation in risk, as measured by the covariance of the market return with a set of portfolio returns that span innovations in the pricing kernel. For the whole sample period, 1954-2016, the in-sample R^2 of this *pricing kernel model* ranges from 7 to 9% for 1-quarter returns, and from 16 to 19% for 1-year returns, depending on the precise specification of the kernel. The R^2 for the *pricing kernel model* spanned by the 3 Fama-French factors is 19% for 1-year returns. This predictive power exceeds virtually all previous predictors of expected returns. The out-of sample R^2 of the *pricing kernel model* is of the order of 9-16% for the period 1965-2016. This compares with the both in- and out-of-sample R^2 of around 13% for the Rapach *et* *al.* (2016) short interest predictor which they show is greater than that of earlier popular predictor variables.

The second model, which we refer to as the *discount rate model*, exploits the accounting identity of the log-linear present value model of Campbell and Shiller (1988), and combines this with the assumption of a factor structure of returns to identify shocks to the discount rate. The model relies on the fact that in a world of time varying discount rates, returns on common stock portfolios reflect shocks to discount rates as well as to cash flow expectations. By assuming that the expected return follows an AR(1) process, we are able to express the expected return as a weighted average of past returns on a portfolio, whose weights are chosen so that its return has exposure only to aggregate discount rate innovations. This *discount rate model* yields in sample R^2 as high as 6% for quarterly returns and 18% for 1-year returns.

The two models offer largely independent evidence on the existence of predictability since, although they are similar in that they both extract information from past portfolio returns and assume the same AR(1) process for expected returns, they imply different sets of predictor variables. Empirically, we find only a marginal improvement in the predictive R^2 when both predictors are included in the regression, indicating that both models appear to be identifying the same predictable component of market returns. Indeed bootstrap simulations show that the probability of obtaining correlated predictors with a R^2 as high as we obtain for the two models is less than 1 in 1000 under the null hypothesis. Overall the results from the two models point to the existence of a predictable component of market returns, driven by time-variation in risk exposure, and characterised by an R^2 of 6-9% for 1-quarter returns, and with medium-term persistence in the expected return, as indicated by an quarterly autoregressive coefficient in the range 0.7-0.8.[1]

The discipline of the risk-based framework suggests further tests of the risk-return trade-off that underlies the observed pattern of return predictability. In particular, estimates of

[1] To be compared with values of 0.97 for highly persistent predictor variables such as D/P, and 0.4 for predictors with relatively low persistence such as the variance of market returns.

the expected return from the *pricing kernel model* should predict not only market returns but also future risk, as measured by the covariance of market returns and the pricing kernel. We verify this aspect of the model by regressing estimates of the realized covariance on the model's estimate of expected returns. Depending on the precise specification of the kernel the R^2 for the 1-year version of this regression is as high as 9%.

Further analysis of the *pricing kernel model* shows that the set of portfolio returns used to span the pricing kernel identifies only a component of the kernel. Components that are uncorrelated with market returns are undetermined, as are risk factors that have constant covariance with the market. Empirically this means the kernel can be parsimoniously represented by a simple combination of two assets: the market portfolio and a growth portfolio as defined by either a low book-to-market or low dividend yield portfolio. These reduced form models explain 16-19% of the time variation in the 1-year market excess return and capture the same levels of predictability as models with larger numbers of spanning portfolios. We find no evidence that either the *pricing kernel model* spanned by the Fama-French 3 factors or its reduced form variants are correlated with news about future discount rates. On the other hand, we find strong evidence that these pricing kernels are highly correlated with the component of the market return that is orthogonal to news about future discount rates. Following Campbell and Vuolteenaho (2004) we interpret this as cash flow news, and then our basic finding is that the predictability we observe is driven by time variation of the covariation of the market return with news about cash flows.

Using data from Ken French's website we also find support for the *pricing kernel model* from Global (ex-US) and European returns, although the Japanese data are less supportive and there is no support from the Asia-Pacific (ex-Japan) returns, perhaps because the 3 Fama-French factors are less meaningful in these heterogeneous markets.

The paper is organized as follows. In Section 2 we discuss how the paper is related to the existing literature on return predictability. Section 3 presents the basic model of expected returns and the *pricing kernel model*. Section 4 describes the data. Section 5 presents the main empirical results for the *pricing kernel model*. Section 6 is concerned with the estimation of the *discount rate model*. Section 7 compares the time series of risk premium estimates from the two models. Section 8 relates the pricing kernel to aggregate discount rate and cash flow news. Section 9 reports further empirical findings and robustness tests, and Section 10 concludes.

2 Related Literature

The *pricing kernel model* originates with Merton (1980) who uses the simple *CAPM* pricing kernel to forecast the expected return on the market portfolio: under the *CAPM* the covariance of the pricing kernel with the market return is proportional to the variance of the market return. Subsequent efforts to model the equity risk premium in terms of the volatility of the market return have met with mixed success. Several authors have reported a positive but insignificant relation between the variance of the market return and its expected value; others find a significant but *negative* relation; and some find both a positive and a negative relation depending on the method used.[2] Ghysels *et al.* (2005) establish a significant positive relation between the monthly market risk premium and the variance of returns estimated using daily data. We confirm the existence of a significant positive risk return relation at the quarterly frequency, with the market variance being captured by a distributed lag on past squared quarterly returns. Bandi *et al.* (2014) report a positive relation between a low frequency component of market return variance and a similarly slow moving component of the market excess return although the economic model that gives rise to this relation is not specified.

Campbell and Vuolteenaho (2004), Brennan, Wang, and Xia (2004), and Petkova (2005) all show that the value premium is correlated with innovations in their measures of investment opportunities. Scruggs (1998) employs a two-factor '*ICAPM*-type' pricing kernel in which the second factor is the return on a bond portfolio to capture time-variation in the equity premium; he finds that the equity premium is related to the covariance of the market return

[2] For references, see Ghysels *et al.* (2005).

with the bond return, although the results are sensitive to the assumption of a constant correlation between bond and stock returns as pointed out by Scruggs and Glabadanidis (2003). Guo *et al.* (2009) follow a similar approach, using the return on the Fama-French HML portfolio as a second factor, and find that the lagged market volatility and covariance of its return with the return on the Fama-French (1993) HML portfolio predict market excess returns over the period 1963-2005, but not over earlier periods.

Unlike these papers which allow the predictor variable to be pre-determined by an $ICAPM$ interpretation of their role in cross-sectional asset pricing tests, our general *pricing kernel model* identifies directly the component of the pricing kernel that is correlated with market returns, and we show that only the sensitivity of the pricing kernel to the market return together with the variance of the market return is required for forecasting the expected market return.

Ross (2005) develops an upper bound on the predictability of stock returns which depends on the volatility of the pricing kernel, and tighter bounds that require further specification of the pricing kernel have been provided by Zhou (2010) and Huang (2013). These tighter bounds 'provide a new way to diagnose asset pricing models' (Huang, 2013, p1). Our levels of predictability fall well within the Ross bounds, and the level of predictability that we find from the *pricing kernel model* is precisely that delivered by the (partial) specifications of the pricing kernel that we propose. Our approach does not require the specification of the complete stochastic discount factor so that our goal is not to test any particular asset pricing model or stochastic discount factor specification.

Our *discount rate model* builds on the distinction between between discount rate news and cash flow news that was developed by Campbell (1991), and was used in a similar context by Campbell and Ammer (1993) and Campbell and Vuolteenaho (2004). Their approach is to extract the discount rate news from the coefficients of a VAR in which the state variables are variables that are known to predict stock returns. In contrast, our state variables are constructed as distributed lags of returns on portfolios that are chosen to capture shocks to the discount rate and eliminate exposure to cashflow factors. This model is also related to research by van Binsbergen and Koijen (2009) that uses a Kalman filter to estimate the expected market return and dividend growth rate from market returns and the price-dividend ratio. However, we use a linear combination of portfolio returns instead of the dividend growth rate to filter out the cash flow news. Like van Binsbergen and Koijen (2009), we assume that the expected excess return follows an AR(1) process.

Our focus on the information about discount rate innovations that is contained in portfolio returns is related to Pastor and Stambaugh (2009) who use prior beliefs on the correlation between discount rate shocks and portfolio returns to develop a Bayesian approach to predictive regression systems. However, while we focus on the information contained in past portfolio returns, Pastor and Stambaugh are concerned primarily with the predictive power of the dividend yield and the *cay* variable.

In an interesting paper, Kelly and Pruitt (2013) combine cross-sectional information on book-to-market ratios to forecast 1-year stock returns and obtain out of sample R^2 as high as 13%. In this paper we use both 1-quarter and 1-year stock returns: for 1-year returns the *pricing kernel model* yields in sample R^2 of 16-19%, and out of sample R^2 of 9-16%. Like ours, their estimates of the time series of expected excess returns have low persistence relative to previous findings. However, the predictability that we identify is not strongly related to that of the Kelly and Pruitt (2013) model.[3] The *cay* predictor of Lettau and Ludvigson (2001) also identifies a component of the expected return that is essentially orthogonal to our model predictions.

Two important issues that arise in the extensive literature on predictability are the inference problems caused by highly persistent predictor variables, and the effects of data-mining arising from the collective search for predictor variables by the research community. The same concerns over data-mining potentially arise in our empirical analysis. However, a major advantage of our approach is that it allows us to assess whether the levels of predictability that we find can be explained by overfitting of the data. First, since we use only portfolio returns as predictor variables, it is straightforward to compute significance levels by simulation

[3]The correlation between the predicted excess return series is less than 0.25.

under the null hypothesis of no predictability or serial independence of returns. In contrast, when macro-economic series are used as predictor variables more extended assumptions are required to simulate the data under the null hypothesis. Secondly, whereas the previous literature has involved search over an undefined domain of potential predictors which does not lend itself to an assessment of the effects of data mining on levels of significance, in our approach, for a candidate set of spanning portfolios, the search is over a well-defined set of predictor variables characterized by a single weighting parameter, β. This allows us to assess the effects of data-mining on significance levels. Our analysis indicates that the level of predictability found cannot be explained by data-mining or the persistence of the predictor variables.

3 The Model of Expected Returns

Let $R_{M,t+1}$ denote the realized *excess* return on the market portfolio from time t to $t+1$ and write:

$$R_{M,t+1} = \mu_t + \xi_{t+1} \tag{1}$$

where μ_t is the mean excess return, and ξ_{t+1} is a mean zero error term.

The basic assumption underlying our estimations is that the mean excess return on the market portfolio, μ_t, follows an $AR(1)$ process:

$$\mu_t = \bar{\mu} + \rho[\mu_{t-1} - \bar{\mu}] + z_t = \bar{\mu} + \Sigma_{s=0}^{\infty} \rho^s z_{t-s} \tag{2}$$

where z_t is the innovation in the mean.

Our primary purpose is to estimate a rational risk-based model to explain the process (2). We shall approach this in the following section. Later, we shall also make use of a purely statistical approach to modeling (2); this is described in Section 6.

3.1 The pricing kernel model

It follows from the definition of the pricing kernel that μ_t is given by:

$$\mu_t = -cov_t(\tilde{m}_{t+1}, R_{M,t+1}) \tag{3}$$

where $\tilde{m}_{t+1}$ is the pricing kernel and we have imposed $E_t(\tilde{m}_{t+1}) = 1$.

We can write the kernel, $\tilde{m}_{t+1}$, as the sum of a time-varying linear function of the market return, $f_{m,t+1}$, and a component, υ_{t+1}, that is orthogonal to the market return:

$$\begin{aligned} \tilde{m}_{t+1} &\equiv a_m(t) - f_{m,t+1} + \upsilon_{t+1} \\ &= a_m(t) - b_m(t)R_{M,t+1} + \upsilon_{t+1} \end{aligned} \tag{4}$$

where $cov_t(\upsilon_{t+1}, R_{M,t+1}) = 0$, and $b_m(t)$ captures time variation in the sensitivity of the pricing kernel to the market return.Then

$$\mu_t = cov_t(b_m(t)R_{M,t+1}, R_{M,t+1}) \equiv b_m(t)\sigma_t^2(R_{M,t+1}) \tag{5}$$

so that the expected market excess return is equal to the product of the sensitivity of the pricing kernel to the market return, $b_m(t)$, and the variance of the market return, $\sigma_t^2(R_{M,t+1})$.

Assume that there exists a set of P 'loading-spanning portfolios' and a set of constant portfolio weights $\delta_p^c, p = 1, \cdots, P$ such that $b_m(t) = \Sigma_{p=1}^P \delta_p^c b_p(t)$, where $b_p(t)$ is the beta coefficient of portfolio p. Then, denoting the excess return on spanning portfolio p by $R_{p,t}$,

$$\mu_t = cov_t(\Sigma_{p=1}^P \delta_p^c b_p(t) R_{M,t+1}, R_{M,t+1}) = \Sigma_{p=1}^P \delta_p^c cov_t(R_{p,t+1}, R_{M,t+1}) \tag{6}$$

It follows that a sufficient condition for the risk premium, μ_t, to follow the $AR(1)$ process (2) is that the conditional covariances of the loading-spanning portfolio returns with the market return follow AR(1) processes with the same persistence parameter, ρ. We assume further that the innovations to the covariance processes are given by the product of the innovations in the spanning portfolio and market returns:

$$cov_t(R_{p,t+1}, R_{M,t+1}) = a_p + \rho[cov_{t-1}(R_{p,t}, R_{M,t}) - a_p] + (R_{p,t} - \mu_{p,t-1})(R_{M,t} - \mu_{t-1}) \tag{7}$$

where $\mu_{p,t-1}$ is the conditional expected excess return on portfolio p.

Then it is shown in Lemma 1 of the Appendix that μ_t can be written as:

$$\mu_t = \bar{a} + \sum_{p=1}^{P} \delta_p^c x_{p,t}^c(\beta) \tag{8}$$

where

$$x_{p,t}^c(\beta) \equiv \sum_{j=0}^{\infty} \beta^j [(R_{p,t-j} - \mu_{p,t-j-1})(R_{M,t-j} - \mu_{t-j-1})] \tag{9}$$

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Our primary estimations treat μ_{t-j-1} and $\mu_{p,t-j-1}$ as zero which is tantamount to replacing the covariances in (7) by the inner product, $\langle R_{p,t} R_{M,t} \rangle$. We show in the Appendix that this is a second order approximation whose importance we explore empirically in Section 9. Employing the inner product approximation, the predictive system for the market excess return becomes:

$$R_{M,t+1} = a_0 + \sum_{p=1}^{P} \delta_p^c x_{pt}^c(\beta) + \epsilon_{t+1} \tag{10}$$

$$x_{pt}^c(\beta) = \sum_{s=0}^{\infty} \beta^s R_{p,t-s} R_{M,t-s} \tag{11}$$

An attractive feature of the model is that the parameters a_0, δ_p^c, β can be estimated relatively easily since, given a value of β, the estimation reduces to a standard predictive regression with predictor variables $x_{pt}^c(\beta)$. Values of β over a grid ranging from 0.001 to 0.999 are considered. Further estimation details are discussed in Section 5.1.

Having estimated the model coefficients the fitted expected return is given by:

$$\hat{\mu}_t = \hat{a}_0 + \sum_{p=1}^{P} \hat{\delta}_p^c x_{pt}^c(\hat{\beta}) \tag{12}$$

Finally, the (excess) return on the pricing kernel portfolio, which mimics the estimated time-varying component of the pricing kernel, is defined by:

$$\hat{f}_{m,t} = \sum_{p=1}^{P} \hat{\delta}_p^c R_{p,t} \tag{13}$$

3.2 The return interval

Before taking the model to the data it is necessary to define the time interval or horizon over which returns are measured. In our model μ is equal to the negative of the covariance of the market return with the pricing kernel return. Brennan and Xia (2008) have shown that when the mean return follows an $AR(1)$ process as we have assumed, the R^2 of a predictive regression of long horizon (log) returns on μ is a hump-shaped function of the horizon and approaches zero with the horizon. This suggests that long horizon returns will be easier to predict. On the other hand, in principle more precise estimates of the comoments are possible with short horizon returns. However, the theory implies that the return interval for the comoments should correspond to the forecast return horizon. If log returns within a year were approximately *iid* then it would be possible to estimate the annual covariance of the log of the market return with the log of the pricing kernel return by appropriately scaling estimates of the covariance derived from high frequency data. However, the covariance that we require is the covariance of the arithmetic not the log returns, and there is also extensive evidence of lagged cross auto-correlations between security and portfolio returns,[4] these factors, as well as the predictability of returns that we are investigating, make the relation between the short run covariance that can be estimated using short horizon returns and the covariance of quarterly or annual returns uncertain. Therefore we cannot estimate covariances of long interval returns by simply scaling up estimates of the covariance obtained from short interval or high frequency returns.

As a compromise, in most of our analysis we adopt a 1-quarter horizon and estimate covariances for 1-quarter returns in order to forecast 1-quarter returns. Because of the persistence in μ the estimated 1-quarter μ will also forecast returns over 1-year, which permits us to use estimates of the 1-quarter covariances to predict 1-year returns. While

[4]Levhari and Levi (1977) showed that CAPM betas vary systematically with the return interval even if returns are *iid*. Lo and MacKinlay (1990) show that the returns on large firms systematically lead the returns on small firms. Gilbert *et al.* (2014) show that the difference between daily and quarterly betas depends on the opacity of the firm accounts. Brennan and Zhang (2014) show that the ratio of long to short horizon betas depends on firm size, book-to-market ratio, number of analysts etc.

it is theoretically preferable to use estimates of the 1-year covariances to predict 1-year returns the difficulty of estimating 1-year covariances leads us to stick with the 1-quarter covariances. In general we find that the R^2 and significance levels are higher for the 1-year return predictions. In Section 9 we also report the result of forecasting 1-month, 1-quarter and 1-year returns, using estimates of 1-month covariances.

4 Data

Our basic data are 1-month stock returns from the website of Ken French and the 1-month Treasury bill rate from Ibbotson Associates for the period from March 1946.01 to December 2016.12. 1-quarter and 1-year returns are computed by compounding the monthly returns in each quarter or year, and excess returns calculated by subtracting the compounded risk free rate. The aggregate market excess return, $R_{M,t}$, is defined as the difference between the return on the Fama-French aggregate market factor and the risk free rate, $R_{F,t}$. The estimation (prediction) period is from 1954.1 to 2016.4. The earlier returns are used to calculate the value of the predictor variables at the end of 1953.

The predictor variables for the *pricing kernel model*, $x^c_{pt}(\beta)$, are calculated from equation (11) using different sets of spanning portfolios. $FF3F$ denotes the 3 Fama-French factors. $4BM$-S denotes the market portfolio plus the four corner portfolios formed by a 2 by 3 sort on size and book-to-market ratio (*sl*, *sh*, *bl* and *bh*). $Zero$ denotes the market portfolio and a zero dividend yield portfolio. $Growth$ denotes the market portfolio and a Growth portfolio which is the average return on the two growth portfolios (*sl,bl*).

The forecasts from our models are compared with those from predictor variables that have been used in previous studies. These include the Dividend (Earnings) yield, the book-to-market value ratio for the Dow Jones Industrial Average, Stock Variance, the 3 month Treasury Bill rate, the Long Term Yield, the Term Spread, Inflation, the Default Yield Spread, and *cay* which is the consumption, wealth, income ratio of Lettau and Ludvigson (2001). These variables are described in Goyal and Welch (2008) and the data series were taken from the website of Amit Goyal.

The small-stock value spread is defined as the log(BE/ME) of the small high book-to-market portfolio minus the log(BE/ME) of the small low book-to-market portfolio. The book-to-market values for these portfolios are defined on a yearly basis and the method described in Campbell and Vuolteenaho (2004) is used to construct monthly values of the value spread. The glamor portfolio is defined as the quintile of stocks with the lowest book-to-market ratio and its cumulative log return over the past 36 months is used as the predictor variable. We use the portfolio data from Ken French's website rather than construct quintiles in the manner described in Eleswarapu and Reinganum (2004) and obtain results qualitatively equivalent to theirs with an $R^2 \approx 5\%$ for predicting 1-year excess returns.

The model of Guo and Savickas (2009) requires estimation the variance of the market over the quarter and the covariance of the market with the HML factor over the quarter. These are calculated from daily return data on Ken French's website. To simplify reporting of the results the bivariate predictive regression is estimated and then a time series of the fitted expected return used as the predictor variable. We also compare our model predictions with the Kelly and Pruitt (2013) 12 month in-sample forecasts constructed using the 6 Fama-French size and book-to-market portfolios.

5 The Pricing Kernel and Expected Returns

5.1 Estimation

We start by estimating equations (10) and (11) using different sets of spanning portfolios to form the predictor variables $x_p^c(\beta)$, truncating the summation in equation (11) at 1946.1. The predictive regression (10) is estimated over the period from 1954.1 and the last returns to be predicted are the 1-year or 1-quarter returns realized at the end of 2016.

A non-linear maximum likelihood estimator can be implemented by choosing values of β, forming the predictor variables from equation (11), and then running an OLS regression of market excess returns on the predictor variables.This procedure is followed for $0 \leq \beta \leq 0.999$, and the MLE estimator of β is the value that minimizes the sum of squared residuals (or equivalently maximizes the R^2) in (20). However, a problem with this MLE estimator is that, as Stambaugh (1986) shows, the small sample bias in the R^2 is increasing in the persistence of the predictor variable and therefore in the weighting parameter β which is being estimated. The higher bias in R^2 associated with higher values of β will tend to result in estimates of β that are too high. Therefore we employ a bias-adjustment procedure.

First we estimate the bias in the estimate of R^2 as a function of the estimated β under the null hypothesis of no predictability, $B_{R^2}(\beta)$. Then our bias-corrected estimator of β is given by $\hat{\beta} = argmax_{\beta}[R_c^2]$ where $R_c^2 = R^2(\beta) - B_{R^2}(\beta)$. To calculate the bias in the estimated R^2 under the null, $B_{R^2}(\beta)$, we adopt a bootstrap approach which reflects the null hypothesis of no predictability. Specifically, we fit a GARCH(1,1) to the returns on the market portfolio and each of the spanning portfolios and save both the (TxP) matrix of the fitted volatilities and the (TxP) matrix of standardized return innovations. To construct each bootstrap data sample we randomly select T $(Px1)$-vectors of standardized innovations and construct the period t vector of returns by multiplying the fitted volatilities for period t by the randomly selected standardized innovations and adding the intercepts from the GARCH estimation. In this way we preserve the exact time series of volatilities for the portfolios and the vector of mean returns, as well as the cross-sectional correlation structure, while ensuring that the returns are serially independent. Then for each sample of the simulated spanning portfolio returns we generate the predictor variables, $x_{pt}^c(\beta)$, for values of β in the range $0 \leq \beta \leq 0.999$, and calculate the R^2 from regressing the market excess return on the generated predictor variables. Repeating this 10,000 times we calculate $B_{R^2}(\beta)$ as the average value of R^2 in the bootstrapped samples for that value of β.

Having estimated the parameters (a_0, δ_p^c, β), we form time series estimates of the expected market excess return, $\hat{\mu}_t$, from Equation (12), and use these estimates to compute the autocorrelation of the expected market excess return, ρ. We assess the statistical significance of our results by determining the proportion of the 10,000 bootstrap samples in which the calculated value of the corrected $R_c^2 \equiv R^2 - B_{R^2}(\beta)$ exceeds that calculated using the actual data. Standard errors for the estimated coefficients are calculated from using a bootstrap under the alternative hypothesis.[5]

We also estimate 1-year expected excess returns using standard overlapping predictive regressions by repeating the above procedure, replacing the 1-quarter excess return as dependent variable with the 1-year excess return starting in the same quarter. Although this induces overlap in the dependent variable this is accounted for in the bootstrap simulations used to calculate standard errors and significance.

5.2 Empirical results

Table 1 reports the results of estimating equations (10) and (11). Motivated by the *CAPM* and the canonical status of the Fama-French (1993) pricing kernel, we initially consider two pricing kernels or sets of spanning portfolios: the market portfolio, M, as proxied by the Fama-French market factor, and the 3 Fama-French factors, $FF3F$. Panel A reports the results for prediction of the 1-quarter excess return on the market portfolio, and Panel B for prediction of the 1-year excess return for the period 1954.1 to 2016.4.[6]

For the 1-quarter horizon the null hypothesis of no predictability is rejected at the 1% level for both sets of spanning portfolios. After bias correction, the fraction of the variance of returns explained by the model, R_c^2, is 4.1% for M and 6.4% for $FF3F$. The significance of the results for the single spanning portfolio, M, implies that time variation in the volatility of the simple *CAPM* pricing kernel has predictive power for the equity premium and that the market excess return is predicted by a weighted average of past squared market returns. This result contrasts with the findings of Goyal and Welch (2008), but is consistent with the results of Ghysels *et al.* (2005) who estimate the market variance from daily data.

The weighting parameter, β, is 0.61 for $FF3F$ but only 0.45 for M and the estimated autocorrelations, ρ, are 0.77 ($FF3F$) and 0.58 (M), implying half lives for shocks to the

[5] We estimate the standard errors under the alternative hypothesis because β is undefined under the null.

[6] We start the predictions in 1954 because this gives us a long enough period to form the sums $x_{pt}^c(\beta)$, starting the data in 1946.1.

discount rate of 1.29 (M) and 2.67 ($FF3F$) quarters. Thus the $CAPM$ or market volatility model identifies a high frequency component of the expected return series and, as a result, fails to capture significant time-variation in the 1-year expected return series: as seen in Panel B, the R^2 is less than 4% and the p-value for the regression is 21%. On the other hand, the model using the $FF3F$ spanning portfolios explains almost 17% of the time-variation in the 1-year expected return series after bias correction, and close to 19% before the bias correction. This level of explanatory power is higher than that reported in prior studies,[7] and is highly statistically significant (p-value$= 0.005$)

For the 1-year predictions in Panel B the estimated coefficients of the $FF3F$ pricing kernel are all highly significant. The strong predictive power of the model suggests that the loadings of the returns of the $FF3F$ portfolios on the market returns do a good job of spanning the loading of the pricing kernel on the market return: $b_m(t) \approx \Sigma_{p=1}^{3} \delta_p^c(t)$.

Note that the pricing kernel that we identify is not the full kernel and therefore cannot be used for general asset pricing. First, the estimation is concerned only with the part of the kernel that is correlated with R_M as shown in equations (4) and (5); secondly, the estimated kernel may contain components orthogonal to R_M that are not in the full kernel; thirdly, the estimation only pins down a component of the kernel that has a time-varying covariance with the market return since a constant covariance does not give rise to return predictability.

5.3 Dissecting the Fama-French pricing kernel

The Fama-French 3-factor model of the pricing kernel lacks a strong theoretical foundation, and its 3-factor structure arbitrarily constrains the relative weights on big and small firms and on growth and value firms. Therefore in Table 2 we repeat the analysis using different combinations of firm size and growth/value portfolios.

In regression (i) we include the four Fama-French ‘corner’ portfolios, big, high and low

[7] Rapach *et al.* (2016) report a simple R^2 of 13% for 1-year returns for the period 1973-2014, while Kelly and Pruitt (2013) report an R^2 of 6% (18%) for the period 1930 to 2010 using 6 (100) predictor book-to-market ratios.

book-to-market, firm portfolios (*bh* and *bl*), and small, high and low book-to-market, firms portfolios (*sh* and *sl*), as well as the market portfolio itself. For the 1-year predictions shown in Panel B the coefficients of both big firm portfolio returns as well as of the small low book-to-market firm portfolio are significant, in addition to the market portfolio. And the R^2 has risen modestly from 18.6% for the $FF3F$ kernel in Table 1 to 22%. In regressions (ii) and (iii) we combine the two low (high) book-to-market portfolios into a single *Growth* (*Value*) factor and in regression (iv) we combine the three small (big) firm portfolios into a single *Small* (*Large*) firm factor.[8] In regression (ii) we find that *Growth* is highly significant, while *Value* is not. Regression (iii) shows that the explanatory power of the model is virtually unchanged when only the market return and the *Growth* return are included in the regression.

Regression (v) shows that very similar results are obtained when *Growth* is replaced by a portfolio that captures the growth dimension by a sort on dividend yield: *Zero* is the Fama-French zero dividend yield portfolio. Once again we find that the coefficient of the growth firm portfolio return, represented by zero dividend yield firms, is highly significant. Regressions (iii) ($Growth$) and (v)($Zero$) are also highly significant in the 1-quarter predictions in Panel A. $Zero$ yields R^2 of 8.5% after bias correction.

When we include the returns on portfolios formed on firm size in regression (iv), the coefficients of both *Small* and *Large* are highly significant for the 1-year predictions in Panel B. Moreover, the predictive power of this 3 factor pricing kernel is only modestly greater (and for the quarterly predictions less than) than that of the 2 factor kernels, *Growth* and *Zero*. Since the pricing kernel obtained with Size portfolios does not yield a two factor representation, in the interest of parsimony we concentrate on the two two-factor 'growth prospect' pricing kernels *Growth* and *Zero* which do almost as well as, or better than, the $FF3F$ kernel. $Zero$ has a bias adjusted R^2 of 18% for 1-year returns.

Table 3 reports subsample results for 1-year predictions. For the $FF3F$ kernel the R^2 for the two halves of the sample period increase to 24% and 20%. However, in the second half

[8]The factors are defined in terms of the FF portfolios as follows: $Growth \equiv (sl + bl)/2$; $Value \equiv (sh + bh)/2)$; $Small \equiv (sl + sn + sh)/3$; $Large \equiv (bl + bl + bh)/3$.

the coefficients of R_M and SMB approximately halve, while that of HML approximately doubles. In the first half, the coefficient of SMB is significant, but not that of HML, while this reverses in the second half. The regressions are significant in both subsamples.

For the *Growth* and *Zero* kernels the simple R^2 ranges from 14% to 24% and are significant except for *Growth* in the second half of the sample period. The coefficients of both pricing kernels are quite stable across the two subperiods. This contrasts with the instability of the coefficients of the $FF3F$ pricing kernel described in the previous paragraph.

$FF3F$, *Growth* and *Zero* have very similar weighting parameters, β, and the autocorrelations of the predicted risk premia series are almost identical, suggesting that these may be almost identical series. This is confirmed by the evidence in Panel A of Table 4 which shows that the correlations between 1-year (1-quarter) expected return series generated from the $FF3F$, *Growth* and *Zero* sets of spanning portfolios range from 0.87 to 0.96 (0.86 to 0.94). Similarly, Panel B shows that for $FF3F$, *Growth* and *Zero*, $f_{m,t}$, the returns on the pricing kernel portfolios defined in equation (13), have correlations of 0.79-0.95 (0.76-0.93). In contrast, the correlation of the market portfolio return, M, with the other pricing kernel portfolios is less than 0.7 (0.4) for the 1-quarter (1-year) predictions.

Overall, we note that the *Zero* pricing kernel yields the best performance for predicting both 1-quarter and 1-year expected returns. The results imply that 18% (8.5%) of time-variation in the 1-year (1-quarter) market return is explained by time variation in risk as captured by time-variation in the covariance of the market return with the pricing kernel that we have identified.

Figure 1, Panel A, plots the quarterly time-series of the estimated 1-year expected returns from the *pricing kernel model* for the $FF3F$, $4BM$-S and *Growth* sets of spanning portfolios. Consistent with the high correlations reported in Panel A of Table 4, the series track each other closely. There are five pronounced peaks: for each we report equity premium estimates from the $FF3F$ and *Zero* models which can be compared with the mean annual equity premium of 7.4%. In the second quarter of 1955 27% (21%); this coincides with the Formosa crisis. In the fourth quarter of 1962 26% (20%) during the Cuban Missile Crisis. In the third quarter of 1974 32% (40%) following the first oil crisis. In the fourth quarter of 1987 22% (32%) following the 1987 Crash. During 2009 40% (33%) following the collapse of Lehman Brothers in September 2008. Interestingly, the estimated equity premium was negative around 1973-4 as well as around the millennium:[9] it seems that at least part of the runup in equity prices around the millennium can be attributed to a sharp decline in the equity premium.[10]

5.4 Time-variation in kernel portfolio betas

We have seen in Section 3.1 that the expected excess return on the market can be written as:

$$\mu_t = b_m(t)\sigma_t^2(R_{M,t+1}) \tag{14}$$

where $b_m(t)$ is the time-varying or state-dependent sensitivity of the kernel to the market return, or beta: $\tilde{m}_{t+1} = a_m(t) - b_m(t)R_{M,t+1} + \upsilon_{t+1}$, and $cov_t(\upsilon_{t+1}, R_{M,t+1}) = 0$. If $b_m(t)$ is constant then (14) implies that the expected excess return is proportional to the market variance as implied by model M, in which the market is the single spanning portfolio. The other *pricing kernel* models imply that the kernel beta varies over time. In fact, equation (14) implies that $b_m(t) = \mu_t/\sigma_t^2$, so that we can write $b_m(t)$ as $b_m(s_t)$ where $s_t = \mu_t/\sigma_t^2(R_{M,t+1})$.

Then the loading-spanning portfolios are such that their weighted average beta is equal to the kernel beta, $b_m(s_t)$:

$$\Sigma_{p=1}^{P}\delta_p^c b_p(t) = b_m(s_t) = \frac{\mu_t}{\sigma_t^2} \tag{15}$$

[9] Boudoukh *et al.* (1993) report reliable evidence that the ex ante equity market risk premium is negative in some states of the world.

[10] It was in September 1999 that James K. Glassman and Kevin A. Hassett published their article Dow 36,000, which argued that future dividends on the market should be discounted at a rate below the Treasury bond rate.

It follows that the spanning portfolio betas can be written as linear functions of s_t plus an error term:

$$b_p(s_t) = b_{p0} + b_{p1}s_t + \zeta_{pt} \tag{16}$$

where ζ_{pt} is an orthogonal error term, such that (i) $\Sigma_{p=1}^{P}\delta_p^c b_{p0} = 0$, (ii) $\Sigma_{p=1}^{P}\delta_p^c b_{p1} = 1$, and (iii)$\Sigma_{p=1}^{P}\delta_p^c \zeta_{pt} = 0$.

Consider the regression:

$$R_{p,t+1} = \alpha_p + b_{p0}R_{M,t+1} + b_{p1}s_t R_{M,t+1} + \epsilon_{pt+1} \tag{17}$$

Condition (ii) implies that $b_{p1} \neq 0$ for a least one of each set of spanning portfolios. In Panel A of Table 5 we report estimates of equation (17) for three different definitions of s_t. To proxy s_t we take μ_t as the 1-year expected return from the *pricing kernel* model,[11] and for σ_t^2 we use Goyal's estimate of the quarterly variance ($svar$) scaled up to an annual variance under the assumptions that quarterly *log* returns are *iid*. In regressions (i) $s_t = \mu_t/\sigma_t^2$ which corresponds to the theoretical variable.

We note that for all of the spanning portfolios b_1 is significantly different from zero, being positive for one of the portfolios and negative for the other three. However, in regressions (ii) and (iii) where s_t is set equal to $\hat{\mu}_t$ and $1/\sigma_t^2$ respectively, b_1, except in the case of HML, is insignificantly different from zero. Thus, our spanning portfolio betas are driven by the state variable $s_t = \mu_t/\sigma^2$ as condition (ii) requires, and this condition is not satisfied for the other definitions of s_t except in the case of HML.

We can derive a stronger implication for the coefficients of (17) for the reduced form models ($Growth$, $Zero$), which have only two spanning portfolios. Since for the market, portfolio 1, $b_1(t) = 1$, equation (15) yields the following expression for the beta of the second portfolio (either $Growth$ or $Zero$): $b_2(t) = \frac{1}{\delta_2^c}\frac{\mu_t}{\sigma_t^2} - \frac{\delta_1^c}{\delta_2^c}$, which implies that b_{21} in equations (15) and (17) has the same sign as δ_2^c, the portfolio weight in the pricing kernel portfolio. Table

[11] For the HML and SMB regressions $\hat{\mu}_t$ is taken from the $FF3F$ *pricing kernel model*; for $Growth$ ($Zero$) regressions $\hat{\mu}_t$ is calculated from the *pricing kernel models* denoted by $Growth$ ($Zero$).

2 shows that for both *Growth* and *Zero* this weight is negative. Regressions (i) in Panel A of Table 5 show that $b_{p1} < 0$ for both *Growth* and *Zero*, confirming this prediction of the model.

For the pricing kernel portfolio regressions, reported in Panel B, $R_{p,t}$ in equation (17) is replaced by the pricing kernel portfolio return, $f^k_{m,t} = \sum_p \delta^c_p R_{p,t}$. We note first that the estimate of b_1 is insignificantly different from zero in regressions (ii) and (iii) for all three sets of spanning portfolios. In contrast, in regressions (i) which use a proxy for the theoretical state variable the estimates of b_1 are all positive and significantly different from zero as condition (ii) implies. On the other hand, the point estimates of b_1 are 0.13, 0.27, 0.24 which are significantly below the theoretical value of unity, and the estimates of b_0 are significantly above their theoretical value of zero. The failure of the point estimates of the coefficients to conform to their theoretical values points to a failure to exactly satisfy the joint requirement that the loading-spanning portfolios span the kernel beta and that the covariances of their returns with the market return satisfy (7). An additional factor is that the proxy we use for the market variance is imperfect. Nevertheless, it is encouraging to find that the pricing kernel portfolio betas are positively related to our proxy for μ_t/σ^2_t as the theory implies, and are not significantly related to either μ_t or $1/\sigma^2_t$ alone.

5.5 Predicting covariance with the pricing kernel

The *pricing kernel model* rests on the familiar result that the expected excess return on the market portfolio is equal to the negative of the conditional covariance of the pricing kernel with the market return as shown in equation (3). It follows that if the model forecasts the expected excess return it should also forecast the corresponding conditional covariance. Such an implication can be tested in principle by regressing an estimate of the conditional covariance on the expected return forecast:

$$\hat{C}_{t+1} = a + \gamma\hat{\mu}_t + \nu_{t+1}, \qquad for \;\; t = 1, \cdots, T-1 \tag{18}$$

where $\hat{C}_{t+1} = cov_t(R_{M,t+1}, -\tilde{m}_{t+1})$ is the (negative of the) estimated covariance of the market return in period $(t+1)$ with the pricing kernel, and $\hat{\mu}_t$ is the estimate formed at the end period t of the arithmetic excess market return over the next period.

As pointed out in Section 3.2 there is no simple relation between the covariance of returns estimated using short horizon returns and the same covariance estimated using long horizon returns. This implies that we have to be very cautious in estimating covariances of long interval returns by simply scaling up estimates of the covariance obtained from short interval.

Therefore we face a quandary: we cannot estimate the conditional covariance of returns over the next year from the observed one year return, and yet if we use more high frequency data we are uncertain of the relation between the covariance of the one year return and the covariance of higher frequency returns; the higher is the frequency of returns used the more efficient will be the estimator but also the more biased. In Table 6 we report the results of time-series estimates of equation (18) using the covariance, $\hat{C}_{t+1} = cov_t(R_{M,t+1}, \hat{f}_{m,t+1})$, calculated from the 12 monthly returns of the market and the pricing kernel portfolio over the following year.[12] For each set of spanning portfolios, we report the regression of the estimated covariance of the implied pricing kernel on the corresponding estimate of the expected 1-year return, $\hat{\mu}_t$.

First we note that the expected return from the *M pricing kernel model* is significantly related to $cov(R_M, -\tilde{m})$ despite the fact that the 1-year expected return predictions from this model are not themselves significant (see Table 1). However, the R^2 is only 3%. The expected returns formed using the other 3 sets of spanning portfolios, $FF3F$, $Growth$ and $Zero$, all predict the covariance of the market return with corresponding estimate of the pricing kernel with R^2 of $8-9\%$. Meanwhile we note that while the theoretical value of γ is unity since the risk premium is equal to the covariance of the (1-year) market return with the pricing kernel, the empirical estimates of γ are in the range $0.3-0.4$. This is consistent with errors in the expected return estimates as well as mis-specification of the measured

[12] Estimates calculated from monthly returns are scaled up to an annual covariance under the simplifying assumption that the vector of monthly *log* returns is distributed *iid*.

covariance. Nevertheless, we have established that as the model predicts, the *pricing kernel models* are able to predict future market returns because they are able to predict the future covariance of the market return with the pricing kernel.

5.6 Relation to classical predictors

To compare our risk-based predictor with some classical predictors Panel A of Table 7 reports estimates of the predictive equation:

$$R_{M,t+1} = \alpha + \beta X_t + \epsilon_{t+1}$$

$R_{M,t+1}$ is the 1-year market excess return starting in quarter t and X_t is the value of the predictor variable at the beginning of the quarter. The predictor variables are standardized to have unit variance, so that the β estimates are sufficient statistics for predictive power. The highest R^2 of the predictive regressions, are 7% (*cay*), 5% (Kelly-Pruitt (2013)), 5% (Glamor) and 4% (*dividend yield* and *term spread*) which compare with R^2 of 16-19% for the *pricing kernel models* shown in Panel B of Tables 1 and 2.

The table also reports time-series correlations between predicted returns from the *pricing kernel models* and the classical predictors. The classical predictors with the highest correlations with our kernel based predictors are those of Guo *et al.*(2009)(0.5) and *svar* (0.4). Both of these predictors are conceptually as well as empirically related to our pricing kernel predictor. Guo *et al.*'s (2009) predictor is the variance of the market return and its covariance with the Fama-French HML factor, which is similar to our $FF3F$ kernel model, except that they estimate variances and covariances using daily data over the previous quarter. And *svar* is simply our M model which is one component of the $FF3F$ kernel.

Panel B of Table 7 reports the results of OLS regressions of realized 1-year market excess returns on the *pricing kernel model* predictions and the four classical predictor variables with the highest R^2 in Panel A. Column 1 shows that *dp*, *glam*, and *kp* jointly capture 8.4% of the variation in the market excess return. Columns 2-4 show that the *pricing kernel model* captures 16-19% of the variation: the slope coefficients in these regressions are unity because the independent variable is the predictor yielded by the optimally selected kernel weights (Equation (12)). In columns 5-7 the regressions include the classical predictors (excluding *cay*) and one of the *pricing kernel model* predictions. The coefficients of the *pricing kernel model* predictions remain highly significant and close to their theoretical value of unity. In contrast, with the exception of *dp* in regression 7, none of the individual classical predictors is significant, although they jointly raise the R^2 for the $FF3F$ pricing kernel from 19.3% to 23.8%. In columns 8-11 the Lettau-Ludvigson *cay* is added to the regressions. This variable is significant, raises the R^2 of the $FF3F$ model from 23.8% to 28.5% and reduces the coefficient of the pricing kernel predictions to around 0.82; however, this value is not significantly different from its theoretical value of unity. The addition of *cay* to the regression also makes *glam* significant, but the *pricing kernel model* predictions remain by far the most significant predictors in the presence of theses other predictors.

5.7 Out of sample return prediction

Table 8 reports for the *pricing kernel* models pseudo-R^2 for rolling out of sample 1-quarter and 1-year forecasts: the models are estimated initially over the period 1954.1 to 1964.4 and the parameter estimates are then used to forecast the 1-year expected return starting in 1965.1. Then the estimation period is extended for one period for the next forecast, and so on.[13] The pseudo-R^2 is one minus the ratio of the mean square forecasting error of the model to the mean square error of a naive forecast equal to the historical mean.[14] We also report results for predictions starting in 1980.1.

For 1-year forecasts starting in 1965.1 the pseudo-R^2 for the three models range from 9 to 16%. When the forecasts start in 1980.1 the R^2 is significantly lower; for $Zero$ the R^2 falls from 16.1% to 7.7%. This compares with an out of sample R^2 of 3.5-13.1% reported by

[13]To mitigate the effects of over-fitting in small samples at high values of β, the range of β is limited so that the R^2 in the estimation period does not exceed 20% for 1-year, and 5% for 1-quarter, forecasts.

[14]The historical mean estimate uses data from 1946.1 although the results do not qualitatively change when the historical mean is calculated from 1927.1.

Kelly and Pruitt (2013) for the period 1980-2010[15], and out-of-sample R^2 of 13.2% reported by Rapach *et al.* (2016) for their short interest predictor for the period 1990-2014. The R^2 for the 1-quarter forecasts are in some cases higher and in other cases lower than for the corresponding 1-year forecasts.

6 The Discount Rate Model

So far we have assumed that the expected return on the market portfolio is determined by the time-varying covariance of the return with a pricing kernel. We consider next an estimator of the expected return which relies only on the assumed $AR(1)$ structure of the expected return process (2) and, following Campbell and Vuolteenaho (2004), a decomposition of the market return into cash flow news and discount rate news. This *discount rate model* allows us to validate results for the *pricing kernel model* and to explore the nature of the risks underlying the time variation in expected returns.

The discount rate model is described in the following lemma, which is proved in the Appendix and is a consequence of the fact that an $AR(1)$ process can be written as a geometrically weighted sum of past innovations to the process, as shown in equation (2).

Lemma 2: Discount Rate Model

If (i) the market excess return, $R_{M,t+1}$, is given by $R_{M,t+1} = \mu_t + \xi_{t+1}$, where ξ_{t+1} is a mean zero error term and μ_t follows the $AR(1)$ process (2).

(ii) there exists a set of P well-diversified portfolios whose excess returns, $R_{p,t}, p = 1, \cdots, P$ follow an exact factor model and span the space of (possibly correlated) innovations in aggregate cash flow expectations ($y_{j,t}$) and the discount rate (z_t) so that:

$$R_{p,t} = \beta_{p0} + k_p \mu_{t-1} + \Sigma_{j=1}^{M} \beta_{pj} y_{j,t} + \gamma_p z_t \tag{19}$$

and $P \geq M$.

[15]The R^2 varies with the number of predictor portfolios, but not in a monotonic fashion.

Then z_t, the innovation to the process (2), can be written as a linear function of the spanning portfolio returns: $z_t = \Sigma_{p=1}^{P} \delta_p^d R_{pt}$, and

$$R_{M,t+1} = a_0 + \sum_{p=1}^{P} \delta_p^d x_{pt}^d(\beta) + \epsilon_{t+1} \tag{20}$$

where

$$x_{pt}^d(\beta) = \sum_{s=0}^{\infty} \beta^s R_{p,t-s} \tag{21}$$

and a_0 and β are constants.

Equation (19) is motivated by the log-linear model of Campbell and Vuolteenaho (2004)[16] which decomposes the unexpected return on a portfolio into cash flow news and discount rate news. Given the $AR(1)$ process (2) for expected returns, discount rate news is a function of the innovation to this process, z_t. Then, if there is a single common aggregate cash flow factor that affects all securities, we should expect z_t to be be spanned by any two well-diversified portfolios that have different loadings on the cash flow factor and the discount rate factor, z_t. This motivates us to choose as spanning portfolios the market portfolio and either $Growth$ or $Zero$, which were described in Section 5 and whose returns we expect to load relatively heavily on the discount rate innovation z_t because of the long duration of their cash flows. For comparison, and to allow for multiple cash flow factors, we also consider the following sets of spanning portfolios: M (the market portfolio), $FF3F$ (the Fama-French 3 factors), and 4-BMS (the market portfolio and the 4 'corner' *size* and *book-to-market portfolios).* We do not expect the market portfolio alone to span z_t since a single portfolio return does not permit separation of cash flow news and discount rate news.

The predictor variables for each of the spanning portfolios, $x_p^d(\beta)$, are formed by truncating the summation in equation (21) at 1946.1. Standard errors, p-values and bias-adjusted R^2 are calculated using the same bootstrap procedure as for the *pricing kernel model.* The predictive regressions and the return vectors which are sampled for the bootstrap start in 1954.1 and the last returns to be predicted are the annual and quarterly returns realized at

[16] See also Campbell and Shiller (1988), Campbell (1991), and Campbell and Ammer (1993).

the end of 2016. Having estimated the parameters (a_0, δ^c, β), we form quarterly time-series estimates of the expected market excess return, $\hat{\mu}_t = \hat{a}_0 + \sum_{p=1}^{P} \hat{\delta}_p^c x_{pt}^d(\hat{\beta})$, and use these estimates to compute the autocorrelation of the expected market excess return, ρ.

Table 9 reports the results of tests of predictability for the *discount rate model* for 1-quarter and 1-year market excess returns, along with estimates of the weighting parameter, β, and the persistence parameter of the market risk premium, ρ. R_c^2 is the bias-corrected R^2. As expected, the predictive model is not significant for either the 1-quarter or the 1-year prediction when the market is the single spanning portfolio (M); nor does the significance improve for the 1-year predictions when the market portfolio is replaced by the three Fama-French factors ($FF3F$) although the unadjusted R^2 for 1-year predictions is as high as 10%.

When we replace the Fama-French factors by 4 size and book-to-market portfolios ($4BM$-S) the bias-adjusted R^2 rises to 18% for the 1-year prediction and both the 1-year and 1-quarter predictions reject the null of no-predictability at the 2% level or better. However, while the *Zero* set of spanning portfolios performs no better than the Fama-French factors, the *Growth* spanning portfolios yield predictions which are significant at the 7% level for 1-quarter predictions and at the 3% level for 1-year predictions. We emphasize that these significance levels are conservative in that they take full account of the bias due to the persistence of the predictor variables. The R^2 for the 1-year prediction using the *Growth* spanning portfolios is 17% before bias adjustment and 13% after adjustment. In Section 8 below we shall use the estimates of the innovation in the expected return, z_t, implied by the *discount rate model* to discuss the nature of the risks captured by the *pricing kernel model*.[17]

7 A Comparison of Predictors

Panel A of Table 10 shows that the two *discount rate model* series have correlation of 0.77, while their correlations with the *pricing kernel model* estimates are 0.50 and 0.44. Bootstrap

[17] The out-of-sample performance of the *discount rate model* does not improve on that of a naive forecast. We hypothesize this is due to the additional bias in estimating the model in small training samples, particularly when sampling at the higher values of β required to fit the *discount rate model*

simulations show that the probability of obtaining R^2 as high as we obtain for the two models and a correlation of 0.5 or above is less than 1 in 1000 under the null hypothesis, so that the two models are identifying a common component of the expected return process. However correlation is not a good measure of the common information in two predictions when the predictions contain correlated errors.

Therefore Panel B of Table 10 reports the results of regressing the 1-year realized market return on the fitted values of the $FF3F$ version of the *pricing kernel model* and the $4BMS$ and *Growth* versions of the *discount rate model*. It can be seen that the predictions of both models can be improved upon by combining them, so that neither drives out the other. However, the marginal increase in R^2 from adding the *pricing kernel model* $FF3F$ prediction to the *discount rate model* 4-BMS prediction is only about 13%, showing that these two predictors carry mainly the same information about future returns.[18] Thus the *discount rate model* lends additional weight to our findings of predictability from the *pricing kernel model* and indeed, when both predictions are combined in a single regression, the *discount rate model* predictions slightly dominate.

Figure 1, Panel B, plots the 1-year expected return series from the *discount rate model* for the $4BMS$ spanning portfolios along with the *pricing kernel model* $FF3F$ series. There are periods when the series from the two model track each other very closely, for example, both models capture the spikes in expected returns in 1962, 1974, 2002, and 2008-9. And both models yield small or negative estimates of the expected return around the millennium and the dot-com crash.

[18]Similarly the PKM(FF3F) and DRM(Growth) have univariate R^2 of approximately 19% and 17% respectively, which only increases to a R^2 value of 25% in the predictive regression with both predictors.

8 Time Varying Risk of Discount Rate or Cash Flow News?

The intertemporal CAPM of Merton (1973) implies that the pricing kernel will be a linear combination of the return on the market portfolio and innovations in variables that affect the future investment opportunity set such as interest rates. Campbell (1993) derives an approximate discrete-time version of Merton (1973). Campbell and Vuolteenaho (2004) use this model to show that the pricing kernel can be written as a linear function of the return on the market portfolio and 'discount rate news', which can be represented as the innovation in the discounted expected future return on the market portfolio; they also show that the market return can be written as the sum of cash flow news and discount rate news. In our model of expected returns, (1) and (2), discount rate news is proportional to z_t, the innovation in the $AR(1)$ process that drives mean returns so that, if discount rate news is priced, z_t must be correlated with the pricing kernel. We define (orthogonal) cash flow news as the component of the market return that is orthogonal to the estimate of z_t, and write it as $R_{M,t}^{\perp\hat{z}_t}$.

It is natural therefore to ask how the time-varying components of the pricing kernels that we identified in Section 5 are related to the discount rate news implied by the *discount rate model* whose estimates we have reported in Section 6, and to cash flow news as defined above. To calculate discount rate news from the *discount rate model* we employ four different sets of spanning portfolios for robustness: $FF3F$, $4BM\text{-}S$, $Growth$, and $Zero$. For each set of spanning portfolios, discount rate news is defined as the estimated innovation in the 1-year expected excess return, $\hat{z}_t$, which is calculated by fitting an $AR(1)$ process to the estimated series $\hat{\mu}_t$. Similarly, to estimate the pricing kernel[19] from the *pricing kernel model* we consider three sets of spanning portfolios, $k = FF3F$, $Growth$ and $Zero$, and for each set k calculate the pricing kernel portfolio return, $\hat{f}_{m,t}^k = \Sigma_{p=1}^P \delta_p^c R_{p,t}$.

[19]We emphasize that our procedure identifies only that component of the kernel which has a time-varying covariance with the market portfolio return.

When we regress the 1-quarter pricing kernel portfolio return, $\hat{f}^k_{m,t}$, on the market return, $R_{M,t}$, the market return explains 11-13% of pricing kernel return depending on the set of spanning portfolios. Table 11 reports the result of regressing the kernel portfolio return on the discount rate news variable, $\hat{z}_t$, and the cash flow news variable, $R^{\perp\hat{z}_t}_{M,t}$, where these variables are defined for each of the four sets of spanning portfolios. For the three pricing kernel portfolio returns and the four estimates of discount rate news two features stand out. First, the pricing kernel portfolio return is virtually independent of discount rate news: the highest R^2 in the regressions is 2% despite the fact that the market portfolio is a component of both the pricing kernel portfolio and the discount rate news proxy, $\hat{z}_t$. Secondly, and in contrast, the pricing kernel portfolio return loads strongly on the cash flow news variable, $R^{\perp\hat{z}_t}_{M,t}$. Using the $FF3F$ proxy for cash flow news, the R^2 of the regressions ranges from 43 to 66%, with the exception of the $4BM$-S proxy for cash flow news where the R^2 is only 19%. Indeed for all three kernel portfolio returns the $4BM$-S proxy for cash flow news has the lowest explanatory power. The reduced explanatory power of the $4BM$-S version of $R^{\perp\hat{z}_t}_{M,t}$ may be due to the fact that this version of the *discount rate model* overfits the expected return series. This conjecture is supported by the very high raw R^2 of 27.4% for this model and the very large bias adjustment of 9.7% seen in Panel B of Table 8.[20]

Since the risk premium associated with a variable is proportional to its covariance with the pricing kernel, these results support the hypothesis that the risk whose time-variation is responsible for time variation in the expected market excess return is primarily the co-variation of the market return with the component of the market return that is orthogonal to discount rate news which we have labeled cash flow news. It is of course possible that our proxy for cash flow news contains news about future discount rates that we have not identified with the *discount rate model*.

[20]Ferson *et al.* (2004) discuss the problem of model selection and spurious regressors in a setting in which the dependent variable has a persistent component as here.

9 Further Results and Robustness Tests

9.1 Refining the pricing kernel estimates

Our primary estimates of the *pricing kernel model*, reported in Tables 1 and 2, employ a second order approximation that is equivalent to setting μ_t and $\mu_{p,t}$ to zero when forming the predictor variables $x^c_{p,t}$. To explore whether the predictions change substantially when re-estimated with time-varying μ's we adopt a two-step estimation in which estimates of the μ's from the first stage calculation are used in a second stage estimation of the full model defined by Equations (8) and (9).

The first step is to estimate the 1-quarter expected returns, $\mu_{p,t}$, by estimating equations (10) and (11) using the $FF3F$ spanning portfolios as in Section 5. The results of these first stage regressions are reported in Panel A of Table 12, which repeats the estimates for the market portfolio, M, from Table 1.

In the second stage regressions the predictor variables take account of this predictability in the $FF3F$ spanning portfolio returns by including the lagged conditional expected returns calculated from the first stage regressions so that: $x^c_{pt}(\beta) = \sum_{s=0}^{\infty} \beta^s (R_{p,t-s} - \hat{\mu}_{p,t-s-1})(R_{M,t-s} - \hat{\mu}_{t-s-1})$. Comparing the two stage results reported in Panel B of Table 12 with the one stage results reported in Table 1, we see that for the 1-quarter forecasts the two-stage estimator results in a modest improvement: the bias-adjusted R^2 increases from 6.4% to 8.3%. On the other hand there is little change in the estimates of the δ^c coefficients. For the 1-year forecasts, the adjusted R^2 is virtually unchanged from the single stage estimator.[21]

Thus, aligning the statistical model more closely to the theoretical *pricing kernel model* leads to a modest but significant improvement in the performance of the model although the signs of the coefficients and the overall significance of the results are unchanged as a result of the two-stage estimation.

[21] Similar results apply for the *pricing kernel models Growth* and *Zero* where the quarterly bias-adjusted R^2 increase to 7.6% and 10% respectively in the two-stage estimation.

9.2 Monthly return data

As discussed in Section 3.2 above, there is a trade-off in using high frequency data to estimate the comoment of R_{Mt} and R_{pt}. On the one hand, more precise estimates of the comoments are theoretically possible with high frequency data, although thin trading and adjustment lags may limit the advantages of high frequency data in practice. On the other hand, the theory implies that the return interval for the comoments should correspond to the forecast return horizon, and the R^2 tends to increase with the return horizon for short horizons. To this point we have used estimates of comoments of 1-quarter returns to forecast both 1-quarter and 1-year excess returns. In this section we report the results of using 1-month returns to calculate the comoments of returns that form the predictor variables: $x^c_{pt}(\beta) = \sum_{s=0}^{\infty} \beta^s R^m_{p,t-s} R^m_{M,t-s}$, where the superscript m denotes a 1-month return. The results for monthly predictions, of 1-month, 1-quarter and 1-year returns, using monthly data to form the predictor variables are reported in Table 13. We compare them with the predictions that use 1-quarter returns to form the predictor variables and are reported in Tables 1 and 2.

First, we note that the coefficients in Table 13 have the same signs for all forecast horizons. Moreover, the signs of the coefficients are the same as for the corresponding coefficients for the predictions that use 1-quarter returns to form the predictor variables reported in Tables 1 and 2. As expected from the discussion in Section 3.2, the R^2 for the 1-month return predictions shown in Panel A are very small, and only the $Zero$ spanning portfolio prediction is significant at the 5% level. For the 1-quarter return predictions reported in Panel B the bias-corrected R^2 are all smaller than those reported using 1-quarter return data to form the predictors: for $FF3F$ the R^2_c falls from 6.4% to 4.1%, and for $Growth$ ($Zero$) from 5.7% (8.5%) to 2.7% (4.7%). Despite this, the predictions for three out of the four sets of spanning portfolios are significant at the 5% level on account of the increased number of observations.[22] However, the increased number of observations is insufficient to rescue the significance of the results for 1-year predictions shown in Panel C and the bias-corrected

[22]The increase in the overlap of the predictions is taken into account by the bootstrap used to compute significance.

R^2 is in all cases well-below that obtained using quarterly returns to form the predictor variables: for $Growth$ ($Zero$) from 14.7% (17.9%) to 5.8% (8.6%).

In summary, the results obtained using 1-month returns to form the predictors are consistent with, but weaker than, those we have reported that use 1-quarter returns.

9.3 International returns

Table 14 shows the results of estimating the *pricing kernel model* on international returns taken from Ken French's website. Because data are available only since July 1990 we use 1-month returns to construct the predictor variables and predictions of 1-quarter and 1-year excess returns are made monthly. Given the relatively short sample period and the relatively poor performance of the Fama-French 3-factor model in pricing international stocks[23] the $FF3F$ spanning portfolios are surprisingly successful. For 1-quarter predictions the null hypothesis of no-predictability is rejected at the 5% level for Global, Global (ex-US) and Europe and at the 10% level for Japan. For N. America the p-value is 12%, although the R^2 of 8% is well above the 5.1% for the comparable US estimation in Panel B of Table 12 which yielded a p-value of 2.3%. The higher p-value and R^2 for N. America may both reflect the short sample period as well as the inclusion of Canadian stocks. For Asia-Pacific the R^2 is only 3% and the p-value is 55% so that there is no evidence of predictability. This may reflect the poor performance of the $FF3F$ local kernel in pricing these stocks noted by Fama and French (2012). A notable feature of the 1-quarter prediction results is the relative importance of HML in the pricing kernel; this is consistent with a similar finding for the US $FF3F$ kernel seen in Panels A of Tab1e 1 and B of Table 12.

The relative importance of HML continues in the 1-year prediction results shown in Panel B: the t-statistic on this portfolio return is above 2 for Global (ex-US), Europe and Japan whereas SMB is significant only for Japan, and R_M for none of the regions. However, for none of the regions are the 1-year predictions significant: the lowest p-value is for Global (13.5%). We note that in Panel C of Table 12 the $FF3F$ kernel was not significant for the

[23] See Fama and French (2012).

US either when 1-month returns were used to construct the predictor variables for 1-year forecasts, despite the much longer sample period.

Overall, the 1-quarter prediction results provide some additional support for the $FF3F$ version of the *pricing kernel model* of return prediction. Taking account of the short sample period, only the results for Asia-Pacific stand out as offering no support.

10 Conclusion

We have shown that it is possible to extract the expected returns on the market portfolio from the lagged returns on a set of spanning portfolios, and have provided new evidence on the predictability of (excess) stock returns. Our first model, the *pricing kernel model*, assumes that time variation in expected returns is driven solely by time variation in risk, where the risk of the market return is measured by its covariance with a set of portfolio returns that span the innovations in the pricing kernel. This model expresses the expected excess market return as a weighted average of past *cross-products* of the returns on the spanning portfolios and the market portfolio. The second model, the *discount rate model* relies on the fact that in a world of time varying discount rates, returns on common stock portfolios reflect shocks to discount rates as well as to cash flow expectations. By assuming that the expected return follows an AR(1) process we are able to express the expected return as a weighted average of past returns on a portfolio whose weights are chosen so that its return has exposure only to discount rate innovations.

The *pricing kernel model* predicts quarterly returns with a corrected R^2 of 6-8% and annual returns with a corrected R^2 of 17-18%. Out of sample, it reduces the mean square forecast error by 10-12% for quarterly forecasts and 9-16% for 1-year forecasts. These predictions and forecast improvements are highly statistically significant. The predictive power of the model is also robust to the inclusion of other predictor variables that have been examined previously, and the component of predictability that we identify has a correlation of only 0.13-0.17 with the Lettau-Ludvigson (2001) *cay* variable.

The *discount rate model* yields the highest R^2 with the $4BM$-S spanning portfolios. This predicts 1-quarter returns with a bias-adjusted R^2 of 6% (p-val =2%) and 1-year returns with a biased-adjusted R^2 of 18% (p-val =1.5%). However, the large bias adjustment for this set of spanning portfolios points to model overfitting, and the next highest bias-adjusted R^2 is for the *Growth* version for which the 1-year R_c^2 is 13% with a p-value of 3%. The model allows us distinguish between discount rate news which is perfectly correlated with the discount rate shock and the cash flow news that is orthogonal to the discount rate shock. Making this distinction, we find that the risk whose time-variation is responsible for time variation in the expected market excess return, is the covariation of the market return with the component of the market return that is orthogonal to discount rate news which we have labeled cash flow news.

While we estimate the models using 1-quarter portfolio returns to forecast either 1-quarter or 1-year market returns, we find that 1-month returns may also be used in the *pricing kernel model* to calculate the predictor variables although the results are less significant than when 1-quarter returns are used. We also find in results not reported here that the expected market excess return calculated from the *pricing kernel model* also forecasts the returns on portfolios formed on the basis of size and book-to-market or dividend yield, with the returns on 'growth' portfolios being the most sensitive to the expected market return. We also estimate the model on six different regional stock market indices for a shorter sample period and find evidence of predictability at the 5% level for three of them, at the 10% level for one, and at the 12% level for another.

Our evidence that a substantial component of the time variation in market excess returns can be explained by a rational risk-based model suggests that caution should be exercised in attributing time-variation in expected returns to sentiment or other non-rational factors.

11 Appendix

Lemma 1

If (i) the expected excess return on the market index, μ_t, is given by:

$$\mu_t = \Sigma_{p=1}^{P} \delta_p^c cov_t(R_{p,t+1}, R_{M,t+1}) \tag{22}$$

and (ii) the conditional covariances of the loading-spanning portfolio returns follow AR(1) processes with the same persistence parameter, ρ, and with innovations which are equal up to a constant to the product of the innovations in the spanning portfolio and market returns:

$$cov_t(R_{p,t+1}, R_{M,t+1}) = a_p + \rho[cov_{t-1}(R_{p,t}, R_{M,t}) - a_p] + (R_{p,t} - \mu_{p,t-1})(R_{M,t} - \mu_{t-1}) \tag{23}$$

where $\mu_{p,t}$ is the mean excess return on portfolio p at time t,

then the expected market excess return, μ_t, can be written as:

$$\mu_t = \bar{a} + \quad \sum_{p=1}^{P} \delta_p^c x_{p,t}^c(\rho) \tag{24}$$

where $x_{p,t}^c(\rho) \equiv \sum_{j=0}^{\infty} \rho^j [(R_{p,t-j} - \mu_{p,t-j-1})(R_{M,t-j} - \mu_{t-j-1})]$.

Proof

Equations (22) and (23) imply that:

$$\mu_t = \bar{a} + \rho[\sum_{p=1}^{P} \delta_p^c cov_{t-1}(R_{p,t} R_{M,t}) - \bar{a}] + \xi_{\alpha_t} \tag{25}$$

where $\bar{a} = \sum_{p=1}^{P} \delta_p^c a_p$, and $\xi_{\alpha,t} = \sum_{p=1}^{P} \delta_p^c (R_{p,t} - \mu_{p,t-1})(R_{M,t} - \mu_{t-1})$.

Then (25) implies that μ_t can be written as an affine function of the geometrically weighted average of past values of products of spanning portfolio and market return innovations:

$$\begin{aligned} \mu_t &= \bar{a} + \quad \sum_{j=0}^{\infty} \rho^j \xi_{\alpha,t-j} = \bar{a} + \sum_{j=0}^{\infty} \rho^j \sum_{p=1}^{P} \delta_p^c (R_{p,t-j} - \mu_{p,t-j})(R_{M,t-j} - \mu_{t-j}) \\ &= \bar{a} + \quad \sum_{p=1}^{P} \delta_p^c x_{p,t}^c(\rho) \end{aligned} \tag{26}$$

where $x^c_{p,t}(\rho) \equiv \sum_{j=0}^{\infty} \rho^j [(R_{p,t-j} - \mu_{p,t-j-1})(R_{M,t-j} - \mu_{t-j-1})]$.

Lemma 1a *If we set $\mu_{p,t} = \mu_t = 0$ in (23) so that (23) becomes the process for the inner product, $\langle R_{p,t+1} R_{M,t+1} \rangle_t \equiv E_t[R_{p,t+1} R_{M,t+1}]$:*

$$\langle R_{p,t+1} R_{M,t+1} \rangle_t = a_p + \rho[\langle R_{p,t} R_{M,t} \rangle_{t-1} - a_p] + R_{p,t} R_{M,t} \tag{27}$$

and spanning portfolio expected returns are linearly related to the market expected return: $\mu_{p,t} = \alpha_p + \kappa_p \mu_t$, then (25) implies that μ_t can be written as an approximate affine function of the geometrically weighted average of past values of products of spanning portfolio and market returns:

$$\begin{aligned} \mu_t &\approx a^* + \sum_{j=0}^{\infty} \rho^j \xi_{\alpha,t-j} = a^* + \sum_{j=0}^{\infty} \rho^j \sum_{p=1}^{P} \delta_p^c R_{p,t-j} R_{M,t-j} \\ &= a^* + \sum_{p=1}^{P} \delta_p^c x^{c*}_{p,t}(\rho) \end{aligned} \tag{28}$$

where $x^{c*}_{p,t}(\rho) \equiv \sum_{j=0}^{\infty} \rho^j [R_{p,t-j} R_{M,t-j}]$.

Proof

Using the definition of covariance in (22)

$$\begin{aligned} \mu_t &= \Sigma_{p=1}^{P} \delta_p^c cov_t(R_{p,t+1}, R_{M,t+1}) = \Sigma_{p=1}^{P} \delta_p^c (\langle R_{p,t+1} R_{M,t+1} \rangle_t - \mu_{p,t} \mu_t) \\ &= \Sigma_{p=1}^{P} \delta_p^c \langle R_{p,t+1} R_{M,t+1} \rangle_t - \mu_t \Sigma_{p=1}^{P} \delta_p^c (\alpha_p + \kappa_p \mu_t) \\ &\approx \Sigma_{p=1}^{P} \delta_p^{c*} \langle R_{p,t+1} R_{M,t+1} \rangle_t \end{aligned} \tag{29}$$

where $\delta_p^{c*} = \delta_p^c / (1 - \Sigma_{p=1}^{P} \delta_p^{c*} \alpha_p)$, and we have neglected squared terms in μ_t. Then successive substitution from (27) yields (28).

Proof of Lemma 2

Multiply equation (19) by δ_p^d and sum over p:

$$\sum_{p=1}^{P} \delta_p^d R_{p,t} = \sum_{p=1}^{P} \delta_p^d \beta_{p0} + \sum_{p=1}^{P} \delta_p^d k_p \mu_{t-1} + \sum_{p=1}^{P} \delta_p^d \sum_{j=1}^{M} \beta_{pj} y_{j,t} + \sum_{p=1}^{P} \delta_p^d \gamma_p z_t$$

Choose the P coefficients, δ_p^d, so that : $\sum_{p=1}^{P} \delta_p^d \beta_{pj} = 0, j = 1, \cdots, M; \sum_{p=1}^{P} \delta_p^d \gamma_p = 1$. Then:

$$\sum_{p=1}^{P} \delta_p^d R_{p,t} = -\delta_0^d + w\mu_{t-1} + z_t \tag{30}$$

where $\delta_0^d = -\sum_{p=1}^{P} \delta_p^d \beta_{p0}$, $w = \sum_{p=1}^{P} \delta_p^d k_p$.

Finally,

$$z_t = \delta_0^d + \sum_{p=1}^{P} \delta_p^d R_{p,t} - w\mu_{t-1}, \tag{31}$$

Then combining equations (2) and (31),

$$\mu_t = (1-\rho)\bar{\mu} + (\rho - w)\mu_{t-1} + \delta_0^d + \sum_{p=1}^{P} \delta_p^d R_{p,t} \tag{32}$$

Defining $\beta \equiv \rho - w$ we have:

$$\begin{aligned} \mu_t &= (1-\beta-w)\bar{\mu} + \delta_0^d + \sum_{p=1}^{P} \delta_p^d R_{pt} \\ &+ \beta[(1-\beta-w)\bar{\mu} + \delta_0^d + \sum_{p=1}^{P} \delta_p^d R_{p,t-1} + \beta\mu_{t-2}] \end{aligned} \tag{33}$$

Substituting recursively for μ_{t-j}, yields (20) and (21).

12 References

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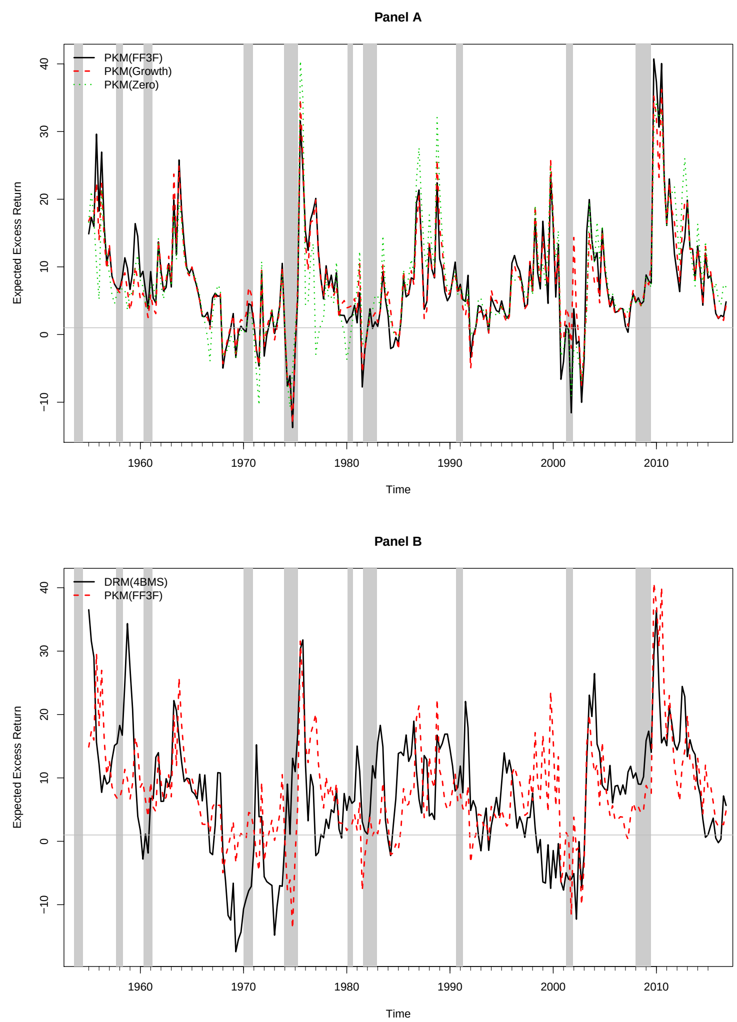

Figure 1: Expected 1-year Market Excess Returns

Panel A shows estimates of the 1-year expected market excess returns from the *pricing kernel model* using different sets of spanning portfolios denoted by $FF3F$ (the Fama-French 3 factors), *Growth* (the market and a growth portfolio), and *Zero* (the market and a zero-dividend portfolio). *Panel B* shows estimates of the 1-year expected market excess returns from the *discount rate model* denoted by $DRM(4BMS)$. The spanning portfolios used in the *discount rate model* are the market portfolio plus 4 portfolios (*sl*, *sh*, *bl* and *bh*) which are the four 'corner' portfolios formed by a 2 by 3 sort on size and book-to-market ratio. Corresponding estimates of the expected market return from the *pricing kernel model*, PKM(FF3F), are included for comparison. The date, which represents the quarter in which the expectations are formed, runs from 1954.1 to 2015.4. Shaded areas indicate NBER recession dates.

Table 1: **Pricing Kernel Model: Tests of Predictability and Parameter Estimates**
This table reports tests of predictability of market excess returns using the *pricing kernel model*: $R_{M,t+1} = a_0 + \sum_{p=1}^{P} \delta_p^c x_{pt}^c(\beta) + \epsilon_{t+1}$ with predictor variables defined as $x_{pt}^c(\beta) = \sum_{s=0}^{\infty} \beta^s R_{p,t-s} R_{M,t-s}$. The 1-quarter (or 1-year) excess return on the market index is denoted by $R_{M,t+1}$; The quarterly excess returns on a set of spanning portfolios is denoted by $R_{p,t}, p = 1, \cdots, P$. Panel A presents coefficient estimates (a_0, δ_p^c, β) for predicting 1-quarter returns and panel B for 1-year returns. We estimate the model with four different sets of spanning portfolios denoted by: M (the market portfolio); $FF3F$ (the 3 Fama-French factors); *Growth* (the market portfolio and a Growth portfolio that consists of equal proportions of the small and big low book-to-market portfolios); *Zero* (the market portfolio and a zero dividend-yield portfolio). The prediction period is 1954.1 to 2016.4. Estimations are performed by a grid search over $0 \leq \beta \leq 0.999$. The parameters are chosen to maximize the R_c^2 of the predictive regression, where R_c^2 denotes the R^2 of the predictive regression adjusted to correct for small sample bias. The significance level of the R_c^2, denoted by the p-value, is determined by a bootstrap procedure in which returns over the period 1946.1 to 2016.4 are sampled under the null hypothesis. The bootstrap s.e. of the estimates are in brackets. ρ is the first order autocorrelation of the estimated market risk premium.

Panel A: Predicting 1-quarter market excess returns

Spanning Portfolios	a_0	R_M	SMB	HML	β	ρ	R^2	R_c^2	p-value
(i) M	0.000	1.337			0.454	0.584	0.041	0.041	0.008
	(0.01)	(0.44)			(0.26)	(0.22)	(0.03)		
(ii) $FF3F$	-0.003	1.645	-1.184	1.496	0.608	0.771	0.066	0.064	0.004
	(0.01)	(0.51)	(0.97)	(0.82)	(0.23)	(0.19)	(0.04)		

Panel B: Predicting 1-year market excess returns

Spanning Portfolios	a_0	R_M	SMB	HML	β	ρ	R^2	R_c^2	p-value
(i) M	0.041	2.995			0.345	0.467	0.038	0.033	0.208
	(0.09)	(1.36)			(0.30)	(0.25)	(0.03)		
(ii) $FF3F$	0.034	6.113	-10.018	5.522	0.516	0.688	0.186	0.169	0.005
	(0.12)	(2.15)	(3.82)	(3.02)	(0.22)	(0.16)	(0.07)		

Table 2: **Dissecting the Pricing Kernel Models**

This table reports tests of predictability of market excess returns using the *pricing kernel model*: $R_{M,t+1} = a_0 + \sum_{p=1}^{P} \delta_p^c x_{pt}^c(\beta) + \epsilon_{t+1}$ with predictor variables defined as $x_{pt}^c(\beta) = \sum_{s=0}^{\infty} \beta^s R_{p,t-s} R_{M,t-s}$. The 1-quarter (or 1-year) excess return on the market index is denoted by $R_{M,t+1}$; The quarterly excess returns on a set of spanning portfolios are denoted by $R_{p,t}, p = 1, \cdots, P$. Panel A presents results for predicting 1-quarter returns and panel B for 1-year returns. The spanning portfolios are combinations of the market portfolio and four 'corner' portfolios from the Fama-French size and book-to-market sort: *sl* is the small low book-to-market firm portfolio, *sh* the small high book-to-market firm portfolio, *bl* the big low book-to-market firm portfolio, *bh* the big high book-to-market firm portfolio. *Growth* is a portfolio that consists of equal proportions of the small and big low book-to-market portfolios. *Zero* is the zero dividend yield portfolio. For estimation details see legend to Table 1.

Panel A: Predicting 1-quarter market excess returns

Spanning Portfolios	a_0	R_M	*sl*	*sh*	*bl*	*bh*	β	ρ	R^2	R_c^2	p-value
(i) *4BM-S*	-0.004	-3.280	-1.560	0.676	3.880	1.775	0.650	0.771	0.072	0.069	0.007
	(0.04)	(1.49)	(0.88)	(1.21)	(1.34)	(1.25)	(0.19)	(0.14)	(0.05)		
	a_0	R_M	*Growth*	*Value*							
(ii) *G+V*	-0.003	3.607	-3.035	1.174			0.569	0.737	0.064	0.062	0.004
	(0.02)	(1.45)	(1.21)	(0.92)			(0.25)	(0.19)	(0.04)		
(iii) *Growth*	-0.003	5.151	-3.381				0.624	0.764	0.059	0.058	0.003
	(0.01)	(1.44)	(1.23)				(0.26)	(0.18)	(0.04)		
	a_0	R_M	*Small*	*Large*							
(iv) *S+L*	-0.005	-2.697	-1.128	5.247			0.685	0.827	0.067	0.066	0.003
	(0.04)	(1.16)	(0.76)	(1.67)			(0.17)	(0.14)	(0.04)		
	a_0	R_M	*Zero*								
(v) *Zero*	-0.004	4.578	-2.431				0.643	0.763	0.086	0.085	0.000
	(0.01)	(1.10)	(0.76)				(0.24)	(0.16)	(0.04)		

Panel B: Predicting 1-year market excess returns

Spanning Portfolios	a_0	R_M	*sl*	*sh*	*bl*	*bh*	β	ρ	R^2	R_c^2	p-value
(i) *4BM-S*	0.035	-12.895	-7.429	-1.345	16.154	11.026	0.515	0.641	0.219	0.190	0.005
	(0.17)	(5.26)	(3.83)	(4.36)	(5.18)	(4.81)	(0.19)	(0.15)	(0.07)		
	a_0	R_M	*Growth*	*Value*							
(ii) *G+V*	0.034	23.024	-18.755	2.339			0.463	0.663	0.160	0.145	0.013
	(0.13)	(5.89)	(5.10)	(3.28)			(0.26)	(0.16)	(0.06)		
(iii) *Growth*	0.034	25.195	-18.646				0.510	0.673	0.159	0.147	0.006
	(0.10)	(5.99)	(4.79)				(0.29)	(0.17)	(0.06)		
	a_0	R_M	*Small*	*Large*							
(iv)*S+L*	0.032	-11.478	-9.236	26.288			0.569	0.729	0.201	0.182	0.004
	(0.12)	(4.60)	(3.65)	(7.39)			(0.15)	(0.14)	(0.07)		
	a_0	R_M	*Zero*								
(v) *Zero*	0.032	16.794	-9.868				0.577	0.697	0.193	0.179	0.002
	(0.07)	(4.25)	(2.73)				(0.26)	(0.16)	(0.07)		

Table 3: **Pricing Kernel Model: Subsample analysis**

This table reports subsample analysis for tests of predictability of market excess returns using the *pricing kernel model*: $R_{M,t+1} = a_0 + \sum_{p=1}^{P} \delta_p^c x_{pt}^c(\beta) + \epsilon_{t+1}$ with predictor variables defined as $x_{pt}^c(\beta) = \sum_{s=0}^{\infty} \beta^s R_{p,t-s} R_{M,t-s}$. $R_{M,t+1}$ is the 1-year excess return on the market index; $R_{p,t}, p = 1, \cdots, P$, are the quarterly excess returns on a set of spanning portfolios. We estimate the model with three sets of spanning portfolios denoted by: $FF3F$ (the 3 Fama-French factors); $Growth$ (the market portfolio and a Growth portfolio that consists of equal proportions of the small and big low book-to-market portfolios); $Zero$ (the market portfolio and a zero dividend-yield portfolio). The full sample period, 1954.1 to 2016.4, is divided into two subsample periods for the tests. For estimation details see legend to Table 1.

Predicting 1-year market excess returns

Spanning Portfolios	*Sample Period*	a_0	R_M	SMB	HML	β	ρ	R^2	R_c^2	p-value
$FF3F$	1955-1985	0.008	8.721	-12.820	2.992	0.474	0.639	0.238	0.211	0.034
		(0.32)	(3.18)	(4.97)	(4.18)	(0.25)	(0.16)	(0.10)		
	1986-2016	0.071	3.302	-6.780	7.303	0.573	0.768	0.203	0.167	0.081
		(0.75)	(2.43)	(5.22)	(4.55)	(0.18)	(0.13)	(0.10)		
		a_0	R_M	$Growth$						
$Growth$	1955-1985	0.008	30.505	-20.997		0.422	0.615	0.210	0.195	0.027
		(0.26)	(8.12)	(6.20)		(0.30)	(0.18)	(0.08)		
	1986-2016	0.068	24.495	-19.952		0.541	0.688	0.136	0.113	0.150
		(0.64)	(8.22)	(7.11)		(0.25)	(0.16)	(0.08)		
		a_0	R_M	$Zero$						
$Zero$	1955-1985	0.006	20.783	-11.039		0.572	0.632	0.242	0.220	0.013
		(0.16)	(6.22)	(3.79)		(0.33)	(0.19)	(0.09)		
	1986-2016	0.065	18.182	-12.601		0.531	0.736	0.204	0.181	0.043
		(0.61)	(5.68)	(4.16)		(0.23)	(0.16)	(0.09)		

Table 4: **Pricing Kernel Model: correlations between market return predictions**

This table reports correlations between estimates of the expected returns on the aggregate market for various specifications of the *pricing kernel model*. Expected excess returns are estimated for both 1-quarter and 1-year returns over the period 1954.1 to 2016.4. The fitted values for the expected returns are used to calculate correlations reported in Panel A. The correlations between the pricing kernel mimicking portfolios are reported in Panel B. The *pricing kernel model* is estimated on four different sets of spanning portfolios denoted by M (the market portfolio), $FF3F$ (the 3 Fama-French factors), $Growth$ (the market portfolio and a portfolio of low book-to-market stocks), and $Zero$ (the market portfolio and a portfolio of zero dividend yield stocks).

	1-quarter expected returns				1-year expected returns			
	M	$FF3F$	$Growth$	$Zero$	M	$FF3F$	$Growth$	$Zero$
Volatility(%)	1.77	2.32	2.15	2.55	3.65	7.95	7.31	8.01
Mean(%)	1.73	1.73	1.73	1.73	7.44	7.44	7.44	7.44
Panel A: Correlations of expected returns								
1-quarter								
M	1.00							
$FF3F$	0.71	1.00						
$Growth$	0.76	0.94	1.00					
$Zero$	0.64	0.86	0.92	1.00				
1-year								
M	0.33	0.42	0.39	0.31	1.00			
$FF3F$	0.09	0.49	0.44	0.41	0.44	1.00		
$Growth$	0.13	0.47	0.45	0.39	0.48	0.96	1.00	
$Zero$	0.15	0.46	0.45	0.46	0.43	0.87	0.89	1.00
Panel B: Correlations of pricing kernel portfolio returns								
1-quarter								
M	1.00							
$FF3F$	0.68	1.00						
$Growth$	0.67	0.93	1.00					
$Zero$	0.53	0.76	0.84	1.00				
1-year								
M	1.00	0.68	0.67	0.53	1.00			
$FF3F$	0.34	0.87	0.89	0.79	0.34	1.00		
$Growth$	0.36	0.84	0.94	0.81	0.36	0.95	1.00	
$Zero$	0.33	0.67	0.77	0.98	0.33	0.79	0.80	1.00

Table 5: **Time-varying betas for spanning and kernel portfolios**

This table reports estimates of market model regressions with state-dependent betas for 4 spanning portfolios and 3 pricing kernel portfolios. The regressions are of the form

$$R_{p,t+1} = \alpha_p + b_{p0} R_{M,t+1} + b_{p1} s_{k,t} R_{M,t+1} + \epsilon_{t+1}, \qquad k = 1, \cdots, 3$$

where $R_{p,t}$ is the 1-year excess return on the portfolio and $R_{M,t}$ is the 1-year return on the market index. The beta of the portfolio is a linear function of the state variable $s_{k,t}$; $\beta_{p,t} = b_{p0} + b_{p1} s_{k,t}$. Results are reported for three different state variables. In regressions (i) $s_{1,t} = \hat{\mu}_t / \hat{\sigma}_t^2$. In regressions (ii) $s_{2,t} = \hat{\mu}_t$, and in regressions (iii) $s_{3,t} = 1/\hat{\sigma}_t^2$. $\hat{\mu}_t$ is the 1-year expected market excess return from the *pricing kernel* model. $\hat{\sigma}_t^2$ is an estimate of the 1-year variance of the market return computed by scaling up Goyal's estimate of the quarterly variance (*svar*) under the assumption that quarterly *log* returns are *iid.* For the pricing kernel portfolio regressions, reported in Panel B, $R_{p,t}$ is replaced by the pricing kernel portfolio return, $\hat{f}_{m,t}$, for the 3 sets of spanning portfolios $FF3F$, $Growth$ and $Zero$. Regressions are estimated using quarterly overlapping data and the t-statistics, in parentheses, are adjusted for serial correlation in the residuals using the Newey-West correction (1987) with 4 lags. The sample period is 1954.1 to 2016.4.

Panel A. Spanning Portfolio Returns

Portfolio	Regression	α	b_0	b_1	R^2
SMB	(i)	1.886	0.239	-1.662	0.076
		(1.20)	(3.20)	(-2.44)	
	(ii)	1.361	0.204	-5.808	0.054
		(0.93)	(2.40)	(-0.59)	
	(iii)	1.276	0.171	0.000	0.052
		(0.82)	(2.20)	(0.01)	
HML	(i)	4.643	-0.237	2.951	0.092
		(2.91)	(-2.45)	(3.78)	
	(ii)	5.450	-0.226	19.142	0.042
		(3.41)	(-1.86)	(2.06)	
	(iii)	4.794	-0.226	0.015	0.065
		(3.18)	(-2.30)	(2.67)	
$Growth$	(i)	3.939	1.168	-1.589	0.881
		(4.45)	(26.76)	(-3.65)	
	(ii)	3.516	1.141	-6.174	0.874
		(4.23)	(22.78)	(-1.07)	
	(iii)	3.701	1.142	-0.005	0.875
		4.24)	(22.55)	(-1.15)	
$Zero$	(i)	3.571	1.403	-1.798	0.745
		(1.96)	(18.16)	(-2.10)	
	(ii)	3.139	1.431	-17.071	0.744
		(1.82)	(16.25)	(-1.64)	
	(iii)	2.905	1.334	0.000	0.740
		(1.60)	(16.61)	(-0.03)	

Table 5 continued

Panel B. Pricing Kernel Portfolio Returns, $\hat{f}_{m,t}$

Portfolio	Regression	α	b_0	b_1	R^2
$\hat{f}_m(FF3F)$	(i)	0.044	0.026	0.273	0.233
		(0.25)	(2.92)	(3.66)	
	(ii)	0.127	0.030	1.217	0.194
		(0.75)	(2.84)	(1.17)	
	(iii)	0.102	0.032	0.001	0.195
		(0.59)	(3.48)	(1.19)	
$\hat{f}_m(Growth)$	(i)	0.141	0.030	0.201	0.259
		(0.92)	(3.97)	(3.45)	
	(ii)	0.198	0.035	0.544	0.234
		(1.35)	(3.96)	(0.58)	
	(iii)	0.178	0.034	0.001	0.236
		(1.16)	(4.34)	(0.91)	
$\hat{f}_m(Zero)$	(i)	0.126	0.025	0.134	0.163
		(0.73)	(3.24)	(1.78)	
	(ii)	0.158	0.023	1.307	0.161
		(0.93)	(2.59)	(1.33)	
	(iii)	0.181	0.031	0.000	0.152
		(1.04)	(3.91)	(-0.17)	

Table 6: **Predicting kernel risk using the expected return from the pricing kernel model**

This table reports the results of regressions:

$$\hat{C}_{t+1} = a + \gamma \hat{\mu}_t + \nu_{t+1}, \qquad for \quad t = 1, \cdots, T-1$$

$\hat{\mu}_t$ is the 1-year expected return from the *pricing kernel model* with $FF3F$ spanning portfolios. $\hat{C}_{t+1}$ is the estimated covariance over the following year of the market return with $-\tilde{m}$, the negative of the pricing kernel estimated for that set of spanning portfolios. Results are reported for 4 different sets of spanning portfolios: M (the market portfolio); $FF3F$ (the 3 Fama-French factors); $Growth$ (the market portfolio and a Growth portfolio that consists of equal proportions of the small and big low book-to-market portfolios); $Zero$ (the market and a zero dividend-yield portfolio). The covariances are calculated from monthly returns, in each case being scaled up to an annual covariance. t-statistics are in parentheses and are adjusted for serial correlation in the residuals using the Newey-West correction (1987) with 4 lags. The sample period is 1954.1 to 2016.4.

	a	γ	R^2
Spanning Portfolios			
M	0.050	0.283	0.030
	(4.88)	(2.59)	
FF3F	0.053	0.368	0.083
	(4.37)	(3.73)	
Growth	0.054	0.377	0.081
	(4.59)	(3.44)	
Zero	0.040	0.403	0.093
	(3.24)	(4.12)	

Table 7: **Classical Predictor variables**

Panel A reports estimates of the equation: $R_{M,t+1} = \alpha + \beta X_t + \epsilon_{t+1}$ for the period 1954.1 to 2016.4. $R_{M,t+1}$ is the 1-year aggregate market excess return starting in quarter t and X_t is the value of the predictor variable at the beginning of the quarter. ρ is the auto-correlation of the predictor variable. t-statistics are in parentheses and are adjusted for serial correlation in the residuals using the Newey-West correction (1987). Panel B reports the results of regressing the 1-year aggregate market excess return on selected classical predictor variables as well as the expected return prediction from the *pricing kernel model* using $FF3F$, $Growth$ and $Zero$ as spanning portfolios. The predictor variables are: the Dividend (Earnings) yield, *dp* (*ep*),the Value Spread VS,Glamour, *glam*, the Stock Variance, *svar*, the 3 month Treasury Bill rate, *tbl*, the Long Term Yield, *lty*, the Term Spread, *tms*,Inflation, $infl$, the Default Yield Spread, dfy, and *cay* is the consumption, wealth, income ratio of Lettau and Ludvigson (2001). Descriptions of these variables are in Goyal and Welch (2008) and the data series are from the website of Amit Goyal. *kp* is a Kelly-Pruitt (2013) predictor, and *guo* is the Guo and Savickas (2009) prediction.

A. Predictive Regressions

Predictor	α	β	R^2	ρ	Correlation with pricing kernel model prediction: $FF3F$	$Growth$	$Zero$
dp	0.430	0.039	0.04	0.974	0.03	-0.01	-0.03
	(2.45)	(2.01)					
ep	0.216	0.021	0.01	0.939	-0.20	-0.16	-0.17
	(1.42)	(0.92)					
b/m	0.025	0.024	0.01	0.979	-0.04	-0.05	-0.08
	(0.54)	(1.16)					
VS	0.443	-0.035	0.03	0.818	-0.10	-0.08	0.01
	(2.44)	(-2.01)					
glam	0.114	-0.040	0.05	0.899	-0.22	-0.17	-0.15
	(5.33)	(-2.62)					
svar	0.060	0.024	0.01	0.415	0.38	0.41	0.41
	(3.16)	(2.47)					
tbl	0.110	-0.024	0.01	0.951	-0.38	-0.34	-0.33
	(3.63)	(-1.25)					
lty	0.090	-0.007	-0.00	0.980	-0.29	-0.26	-0.21
	(1.90)	(-0.35)					
tms	0.028	0.039	0.04	0.839	0.26	0.25	0.31
	(0.92)	(2.28)					
$infl$	0.101	-0.028	0.02	0.450	-0.27	-0.24	-0.22
	(4.88)	(-1.70)					
dfy	0.016	0.026	0.02	0.874	0.19	0.19	0.23
	(0.38)	(1.50)					
kp	-0.054	0.043	0.05	0.876	0.24	0.20	0.14
	(-0.96)	(2.46)					
guo	0.000	0.032	0.03	0.779	0.51	0.46	0.45
	(0.00)	(2.51)					
cay	0.072	0.050	0.07	0.981	0.16	0.13	0.17
	(3.99)	(2.86)					

Table 7 continued

B. Predicting the 1-year market excess return: a comparison of pricing kernel predictions with classical predictors

	1	2	3	4	5	6	7	8	9	10	11
constant	0.260	0.000	0.000	0.000	0.361	0.370	0.347	0.174	0.284	0.289	0.274
	(0.87)	(0.00)	(0.00)	(0.00)	(1.72)	(1.59)	(1.72)	(0.59)	(1.31)	(1.21)	(1.32)
dp	0.024				0.036	0.038	0.037	0.016	0.029	0.030	0.029
	(0.88)				(1.97)	(1.82)	(2.07)	(0.58)	(1.47)	(1.37)	(1.56)
glam	-0.032				-0.020	-0.024	-0.024	-0.038	-0.026	-0.030	-0.029
	(-1.91)				(-1.36)	(-1.54)	(-1.66)	(-2.46)	(-1.90)	(-2.12)	(-2.19)
kp	0.020				-0.002	0.000	0.003	0.026	0.005	0.007	0.010
	(0.81)				(-0.11)	(-0.02)	(0.16)	(1.15)	(0.24)	(0.30)	(0.49)
cay								0.055	0.041	0.044	0.041
								(2.76)	(2.37)	(2.47)	(2.35)
Pricing kernel models:											
FF3F		1.000			0.938				0.823		
		(5.94)			(4.98)				(4.52)		
Growth			1.000			0.950				0.837	
			(5.52)			(4.54)				(4.21)	
Zero				1.000			0.962				0.855
				(5.72)			(5.17)				(4.63)
R^2	0.084	0.193	0.163	0.196	0.238	0.220	0.259	0.172	0.285	0.274	0.304

Table 8: **Out of sample return forecasts**

This table reports the pseudo-R^2 for rolling out of sample forecasts starting in 1965.1 and in 1980.1. The pseudo-R^2 is one minus the ratio of the mean square forecasting error of the model to the mean square error of a naive forecast equal to the historical mean. The training period for the forecasting model is from 1954.1 to either 1964.4 or 1979.4 The out-of-sample performance of the *pricing kernel model* is evaluated for three sets of spanning portfolios denoted by: $FF3F$, *Growth* and *Zero*.

	$FF3F$	*Growth*	*Zero*
		1-quarter forecasts	
Forecast period			
1965.1-2016.4	9.9%	9.5%	11.9%
1980.1-2016.4	10.1%	3.3%	10.2%
		1-year forecasts	
1965.1-2016.4	9.4%	13.8%	16.1%
1980.1-2016.4	6.2%	6.6%	7.7%

Table 9: **Discount Rate Model: Tests of Predictability and Parameter Estimates**

This table reports tests of predictability of market excess returns using the *discount rate model*: $R_{M,t+1} = a_0 + \sum_{p=1}^{P} \delta_p^d x_{pt}^d(\beta) + \epsilon_{t+1}$ with predictor variables defined as $x_{pt}^d(\beta) = \sum_{s=0}^{\infty} \beta^s R_{p,t-s}$. The 1-quarter (or 1-year) excess return on the market index is denoted by $R_{M,t+1}$; The quarterly excess returns on a set of spanning portfolios is denoted by $R_{p,t}, p = 1, \cdots, P$. Panel A presents coefficient estimates (a_0, δ_p^d, β) for predicting 1-quarter returns and panel B for 1-year returns. We estimate the model using various combinations of spanning portfolios: these are the market portfolio (M), the 3 Fama-French factors ($FF3F$), the market portfolio and 4 portfolios constructed on size and book-to-market ($4BM$-S), and the market portfolio and in turn a growth portfolio ($Growth$) and a zero dividend yield portfolio ($Zero$). The model is estimated over the sample period 1954.1 to 2016.4 (the prediction period). The bootstrap s.e. of the estimates are in brackets. ρ is the first order autocorrelation of the estimated market risk premium. See legend to Table 1 for remaining definitions.

Panel A: Predicting 1-quarter market excess returns

Spanning Portfolios	a_0	R_M	SMB	HML			β	ρ	R^2	R_c^2	p-value
M	0.071	-0.045					0.987	0.941	0.013	0.010	0.356
	(0.05)	(0.05)					(0.23)	(0.24)	(0.01)		
$FF3F$	0.019	0.125	-0.315	-0.123			0.066	0.155	0.033	0.032	0.102
	(0.03)	(0.09)	(0.15)	(0.11)			(0.35)	(0.31)	(0.03)		
	a_0	R_M	sl	sh	bl	bh					
$4BM$-S	0.153	0.545	-0.083	-0.053	-0.359	-0.160	0.975	0.812	0.070	0.058	0.020
	(0.13)	(0.38)	(0.12)	(0.15)	(0.26)	(0.17)	(0.37)	(0.14)	(0.03)		
	a_0	R_M	$Growth$	$Zero$							
$Growth$	0.087	0.076	-0.148				0.985	0.853	0.038	0.033	0.066
	(0.08)	(0.18)	(0.15)				(0.24)	(0.17)	(0.02)		
$Zero$	0.063	-0.011		-0.038			0.981	0.899	0.017	0.013	0.385
	(0.07)	(0.09)		(0.06)			(0.17)	(0.17)	(0.02)		

Panel B: Predicting 1-year market excess returns

Spanning Portfolios	a_0	R_M	SMB	HML			β	ρ	R^2	R_c^2	p-value
M	0.342	-0.216					0.988	0.937	0.084	0.063	0.146
	(0.20)	(0.14)					(0.07)	(0.09)	(0.05)		
$FF3$	0.385	-0.293	-0.174	-0.019			0.981	0.874	0.106	0.048	0.339
	(0.31)	(0.18)	(0.21)	(0.21)			(0.05)	(0.08)	(0.06)		
	a_0	R_M	sl	sh	bl	bh					
$4BM$-S	0.624	1.683	-0.337	-0.048	-1.129	-0.593	0.981	0.818	0.274	0.177	0.015
	(0.68)	(1.10)	(0.36)	(0.46)	(0.76)	(0.52)	(0.24)	(0.09)	(0.07)		
	a_0	R_M	$Growth$	$Zero$							
$Growth$	0.397	0.231	-0.545				0.986	0.855	0.169	0.128	0.034
	(0.38)	(0.44)	(0.36)				(0.13)	(0.09)	(0.06)		
Zero	0.313	-0.108		-0.125			0.984	0.903	0.100	0.060	0.228
	(0.30)	(0.23)		(0.12)			(0.06)	(0.08)	(0.06)		

Table 10: **Comparison of expected return estimates from the *pricing kernel* and *discount rate* models**

Panel A reports the volatilities and correlations of three estimates of the expected 1-year excess return, $\hat{\mu}_t$. The first, $\hat{\mu}_{pkm}(FF3F)$, is constructed from the *pricing kernel model* using the $FF3F$ portfolios as spanning portfolios; the model parameters are shown in Panel B of Table 1. The second, $\hat{\mu}_{drm}(4BMS)$, is constructed from the *discount rate model* using the market return and 4 book-to-market and size sorted portfolios to span z_t, the innovation in μ. The third, $\hat{\mu}_{drm}(Growth)$, is constructed from the *discount rate model* using the market return and the *Growth* portfolio to span z_t, the innovation in μ. The parameters used to construct μ_{drm} are shown in Panel B of Table 9. Panel B reports the results of regressions of the 1-year market excess return, $R_{M,t+1}$, on the three estimates of the expected 1-year excess return, $\hat{\mu}_t$. The sample period is 1954.1 to 2016.4.

Panel A. Volatilities and correlations of expected return estimates

	$\hat{\mu}_{pkm}(FF3F)$	$\hat{\mu}_{drm}(4BMS)$	$\hat{\mu}_{drm}(Growth)$
Volatility	7.95	9.64	7.53
		Correlations	
$\hat{\mu}_{pkm}(FF3F)$	1.00		
$\hat{\mu}_{drm}(4BMS)$	0.50	1.00	
$\hat{\mu}_{drm}(Growth)$	0.44	0.77	1.00

Panel B. Regression of $R_{M,t+1}$ on expected return estimates

	1	2	3	4	5	6
α				-2.288	-2.848	-2.189
				(-0.98)	(-1.25)	(-0.96)
$\hat{\mu}_{pk}(FF3F)$	1.000			0.524	0.721	0.527
	(5.94)			(2.41)	(2.95)	(2.32)
$\hat{\mu}_{drm}(4BMS)$		1.000		0.784		0.805
		(7.39)		(4.59)		(3.78)
$\hat{\mu}_{drm}(Growth)$			1.000		0.661	-0.037
			(4.52)		(2.48)	(-0.12)
R^2	0.193	0.286	0.172	0.323	0.252	0.320

Table 11: **The pricing kernel and cash flow news**

The estimated pricing kernel portfolio returns, $\hat{f}_{m,t}$, are regressed on the discount rate shocks, $\hat{z}_t$, and the component of the market return that is orthogonal to $\hat{z}_t$ and is the proxy for cash flow news, $R_{M,t}^{\perp\hat{z}_t}$. The pricing kernel portfolio estimates are for 1-year predictions and use as spanning portfolios either the 3 Fama-French factors ($FF3F$), the market return and a portfolio of growth stocks ($Growth$), or the market return and portfolio of non-dividend paying stocks ($Zero$). The parameters underlying these estimates are shown in Panel B of Tables 1 and 2. The innovation in the expected market excess return, $\hat{z}_t$, is estimated from the *discount rate model* using as spanning portfolios either $FF3F$, $4BMS$, $Growth$ or $Zero$; the parameters underlying these estimates are shown in Panel B of Table 9. The regressions use quarterly returns over the sample period 1954.1 to 2016.4.

Pricing kernel	a	$\hat{z}_t$	$R_{M,t}^{\perp\hat{z}_t}$	R^2	a	$\hat{z}_t$	$R_{M,t}^{\perp\hat{z}_t}$	R^2
	$\hat{z}_t(FF3F)$				$\hat{z}_t(4\text{-}BMS)$			
$\hat{f}_m(FF3F)$	0.11	-2.68		0.01	0.11	-1.48		0.01
	(2.8)	(-2.0)			(2.8)	(-2.0)		
	0.11		13.39	0.43	0.11		5.65	0.19
	(3.7)		(13.8)		(3.1)		(7.7)	
$\hat{f}_m(Growth)$	0.12	-3.31		0.02	0.12	-0.43		0.00
	(3.4)	(-2.6)			(3.4)	(-0.6)		
	0.12		11.73	0.37	0.12		7.34	0.36
	(4.3)		(12.1)		(4.2)		(11.9)	
$\hat{f}_m(Zero)$	0.10	-2.94		0.02	0.10	-0.65		0.00
	(2.8)	(-2.3)			(2.8)	(-0.9)		
	0.10		10.93	0.31	0.10		6.29	0.26
	(3.3)		(10.7)		(3.2)		(9.4)	
	$\hat{z}_t(Growth)$				$\hat{z}_t(Zero)$			
$\hat{f}_m(FF3F)$	0.11	0.44		0.00	0.11	-3.49		0.02
	(2.8)	(0.4)			(2.8)	(-2.3)		
	0.11		12.66	0.66	0.11		14.59	0.44
	(4.8)		(22.0)		(3.7)		(13.9)	
$\hat{f}_m(Growth)$	0.12	0.48		0.00	0.12	-3.63		0.02
	(3.4)	(0.5)			(3.4)	(-2.5)		
	0.12		12.70	0.74	0.12		14.15	0.46
	(6.7)		(26.8)		(4.6)		(14.6)	
$\hat{f}_m(Zero)$	0.10	-0.33		0.00	0.10	-1.59		0.00
	(2.8)	(-0.3)			(2.8)	(-1.1)		
	0.10		10.16	0.47	0.10		17.40	0.69
	(3.8)		(14.7)		(4.9)		(23.2)	

Table 12: **Pricing Kernel Model: Two Stage Estimates**

This table reports the results of two-stage tests of the null hypothesis of no predictability of the market excess return for the *pricing kernel model*: $R_{M,t+1} = a_0 + \sum_{p=1}^{P} \delta_p^c x_{pt}^c(\beta) + \epsilon_{t+1}$ where $x_{pt}^c(\beta) = \sum_{s=0}^{\infty} \beta^s (R_{p,t-s} - \mu_{p,t-s-1})(R_{M,t-s} - \hat{\mu}_{t-s-1})$. The 1-quarter expected excess return for the market, $\hat{\mu}_t$, and spanning portfolios, $\hat{\mu}_{p,t}$, are generated from a first stage estimation of the *pricing kernel model* and are reported in Panel A. The 2nd stage estimation of the full model, using $\hat{\mu}_t$ and $\hat{\mu}_{p,t}$ from the 1st stage, is reported in Panel B. Parameter estimates for prediction of 1-quarter and 1-year are shown and should be compared with those from the single stage estimator of expected returns reported in Table 1. $R_{M,t+1}$ is the 1-year excess return on the market index; $R_{p,t}, p = 1, \cdots, P$, are the quarterly excess returns on a set of spanning portfolios. The spanning portfolios in the PKM are the 3 Fama-French portfolios ($FF3F$). The sample period is 1954.1 to 2016.4. The bootstrap s.e. of the estimates are in brackets. See legend to Table 1 for remaining definitions.

Panel A: Stage 1 prediction of 1-quarter spanning portfolio excess returns

Portfolio Predicted	a_0	R_M	SMB	HML	β	ρ	R^2	R_c^2	p-value
M	-0.003	1.645	-1.184	1.496	0.608	0.771	0.066	0.064	0.004
	(0.01)	(0.51)	(0.97)	(0.82)	(0.23)	(0.19)	(0.04)		
SMB	-0.024	-0.871	0.746	0.308	0.988	0.938	0.034	0.034	0.059
	(0.02)	(0.50)	(0.59)	(0.70)	(0.26)	(0.25)	(0.03)		
HML	0.004	0.564	-2.021	0.823	0.667	0.758	0.035	0.034	0.067
	(0.02)	(0.63)	(1.11)	(0.55)	(0.24)	(0.22)	(0.03)		

Panel B: Stage 2 prediction of 1-quarter and 1-year market excess returns

Prediction Horizon	a_0	R_M	SMB	HML	β	ρ	R^2	R_c^2	p-value
1-*quarter*	-0.002	1.730	-0.698	1.872	0.603	0.722	0.085	0.083	0.000
	(0.02)	(0.59)	(1.17)	(0.98)	(0.16)	(0.16)	(0.04)		
1-*year*	0.041	6.106	-8.768	6.197	0.502	0.644	0.181	0.165	0.006
	(0.09)	(2.25)	(4.28)	(3.44)	(0.20)	(0.18)	(0.06)		

Table 13: **Pricing Kernel Model: Monthly Data**

The table reports the results of tests of the null hypothesis of no predictability of the market excess return for the *pricing kernel model*: $R_{M,t+1} = a_0 + \sum_{p=1}^{P} \delta_p^c x_{pt}^c(\beta) + \epsilon_{t+1}$ where $x_{pt}^c(\beta) = \sum_{s=0}^{\infty} \beta^s R_{p,t-s}^m R_{M,t-s}^m$. Predictions are made monthly and the predicted variable, $R_{M,t+1}$, is the 1-month (Panel A), 1-quarter (Panel B) or 1-year (Panel C) excess return on the market index; $R_{p,t}^m, p = 1, \cdots, P$, are the 1-month excess returns on a set of spanning portfolios, and $R_{M,t}^m$ is the 1-month excess return on the market index. See legend to Table 1 for remaining definitions.

A. Predicting 1-month market excess return

Spanning Portfolios	a_0	R_M	SMB	HML	β	ρ	R^2	R_c^2	p-value
M	0.000	0.362			0.878	0.938	0.006	0.006	0.146
	(0.01)	(0.32)			(0.19)	(0.17)	(0.01)		
FF3	-0.017	0.235	-0.264	0.208	0.987	0.992	0.014	0.013	0.067
	(0.01)	(0.28)	(0.52)	(0.50)	(0.21)	(0.19)	(0.02)		
	a_0	R_M	*Growth*						
Growth	-0.012	0.774	-0.555		0.986	0.992	0.010	0.010	0.086
	(0.01)	(0.90)	(0.75)		(0.21)	(0.19)	(0.01)		
	a_0	R_M	*Zero*						
Zero	0.000	1.321	-0.775		0.912	0.964	0.015	0.014	0.031
	(0.00)	(0.65)	(0.44)		(0.09)	(0.07)	(0.02)		

B. Predicting 1-quarter market excess return

	a_0	R_M	SMB	HML	β	ρ	R^2	R_c^2	p-value
M	0.000	1.670			0.812	0.893	0.027	0.023	0.041
	(0.01)	(0.83)			(0.12)	(0.09)	(0.04)		
FF3	-0.002	1.974	-0.852	2.651	0.850	0.934	0.051	0.041	0.023
	(0.02)	(0.82)	(1.44)	(1.50)	(0.07)	(0.06)	(0.05)		
	a_0	R_M	*Growth*						
Growth	-0.002	5.954	-4.090		0.873	0.939	0.034	0.027	0.058
	(0.01)	(2.66)	(2.16)		(0.09)	(0.06)	(0.04)		
	a_0	R_M	*Zero*						
Zero	0.001	5.143	-2.944		0.877	0.944	0.053	0.047	0.007
	(0.01)	(1.88)	(1.27)		(0.06)	(0.04)	(0.05)		

C. Predicting 1-year market excess return

	a_0	R_M	SMB	HML	β	ρ	R^2	R_c^2	p-value
M	0.040	4.931			0.713	0.815	0.029	0.021	0.301
	(0.08)	(2.72)			(0.14)	(0.11)	(0.04)		
FF3	0.036	8.666	-11.447	13.094	0.746	0.870	0.115	0.090	0.072
	(0.13)	(4.12)	(6.84)	(6.88)	(0.09)	(0.06)	(0.07)		
	a_0	R_M	*Growth*						
Growth	0.038	36.139	-27.601		0.765	0.856	0.076	0.058	0.140
	(0.10)	(14.02)	(11.18)		(0.12)	(0.07)	(0.06)		
	a_0	R_M	*Zero*						
Zero	0.044	20.930	-13.175		0.831	0.915	0.108	0.086	0.051
	(0.06)	(8.56)	(5.63)		(0.08)	(0.05)	(0.09)		

Table 14: **Pricing Kernel Model: International Data**

The table reports the results of tests of the null hypothesis of no predictability of the market excess return for the *pricing kernel model*: $R_{M,t+1} = a_0 + \sum_{p=1}^{P} \delta_p^c x_{pt}^c(\beta) + \epsilon_{t+1}$ where $x_{pt}^c(\beta) = \sum_{s=0}^{\infty} \beta^s R_{p,t-s}^m R_{M,t-s}^m$. Predictions are made monthly and Panel A predicts 1-quarter and panel B for 1-year returns. $R_{M,t+1}$ is the 1-quarter (Panel A) or 1-year (Panel B) excess return on the market index; $R_{p,t}^m, p = 1, \cdots, P$, are the 1-month excess returns on a set of spanning portfolios, and $R_{M,t}^m$ is the 1-month return on the market index. The model is estimated for 6 different regional indices using the 3 Fama-French portfolios ($FF3F$) for that region as spanning portfolios. The data are from Developed Markets and Factors on the website of Ken French where details on their construction may be found. The sample prediction period is 1992.Q1 to 2016.Q4 and returns starting in 1990.M7 are used to construct the predictor variables.
See legend to Table 1 for further estimation details.

Panel A: Predicting 1-quarter market excess returns

Index	a_0	R_M	SMB	HML	β	ρ	R^2	p-value
Global	0.012	0.899	-0.967	7.343	0.791	0.947	0.104	0.040
	(0.02)	(1.54)	(5.81)	(5.50)				
Global(ex-US)	0.002	0.983	-4.010	11.582	0.741	0.903	0.122	0.016
	(0.03)	(1.53)	(5.60)	(5.93)				
Europe	0.009	-0.781	-6.569	10.698	0.693	0.850	0.116	0.035
	(0.02)	(1.45)	(5.34)	(5.48)				
Japan	-0.004	1.266	-5.808	5.511	0.826	0.853	0.076	0.097
	(0.03)	(1.08)	(3.34)	(3.61)				
Asia-Pacific	0.024	-0.170	4.175	1.658	0.878	0.891	0.030	0.548
	(0.03)	(0.83)	(3.18)	(2.78)				
N. America	0.014	0.805	1.214	3.267	0.855	0.966	0.081	0.119
	(0.02)	(1.58)	(4.04)	(3.62)				

Panel B: Predicting 1-year market excess returns

Index	a_0	R_M	SMB	HML	β	ρ	R^2	p-value
Global	0.034	5.144	-13.856	22.541	0.763	0.926	0.219	0.135
	(0.11)	(3.86)	(15.20)	(13.97)				
Global(ex-US)	0.008	5.268	-13.400	33.083	0.688	0.872	0.190	0.193
	(0.16)	(3.75)	(13.55)	(13.63)				
Europe	0.047	-0.268	-8.084	29.479	0.633	0.838	0.138	0.409
	(0.11)	(3.31)	(12.63)	(12.48)				
Japan	0.005	4.147	-18.789	20.450	0.729	0.770	0.113	0.513
	(0.20)	(2.80)	(8.97)	(9.66)				
Asia-Pacific	0.075	0.796	12.737	2.654	0.847	0.898	0.076	0.692
	(0.13)	(2.22)	(8.94)	(7.35)				
N. America	0.059	4.686	-5.775	10.406	0.853	0.958	0.202	0.184
	(0.09)	(4.29)	(11.48)	(10.01)				