Identifying Price Informativeness

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How much information is revealed by asset prices?

- Price informativeness: measures the precision of information in price
- Disconnect between theoretical work and empirical measures of informativeness

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This paper

- Methodology to identify price informativeness from linear regressions of prices on asset payoffs
- Recover *exact* measures of stock specific price informativeness
  - Absolute and relative price informativeness

Main Results

1. Identifying price informativeness

- Identify exact measures of price informativeness (precision of info in price)
- Illustration of identification results
  - Regression of prices on current and future payoffs

$$p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \varepsilon_t$$

  - Price informativeness = signal to noise ratio

$$\tau_{\hat{p}} = \frac{\beta_2^2}{\mathbb{V}\text{ar}\left[\varepsilon_t\right]}$$

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2. Empirical implementation

- Prices are noisy: median updating weight is 0.02 and mean is 0.05
  - Dispersed and right-skewed distribution of informativeness measures
- Informativeness increases in market cap, in turnover, and over time.
- Informativeness is higher at lower frequencies

Why Price Informativeness?

"The economic problem of society is (...) rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know. Or, to put it briefly, it is a problem of the *utilization of knowledge which is not given to anyone in its totality*."

Hayek, 1945

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Hayek, 1945

- Which stocks/industries to learn about?
- Identify good investment opportunities more precisely

Outline

1. **Identifying price informativeness**
   - Framework
   - Identification
   - Extensions
2. **Empirical implementation**
   - Estimates of price informativeness
   - Distribution of price informativeness across stock characteristics

Framework

- $t = 0, 1, 2...$
- One asset traded at price p_t with an unobserved payoff

$$\theta_{t+1} = \mu_\theta + \rho\theta_t + \underbrace{\eta_t}_{\text{innovation}},$$

with $|\rho| < 1$ and η_t are i.i.d., mean zero with variance τ_η^{-1}

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  with $|\rho| < 1$ and η_t are i.i.d., mean zero with variance τ_η^{-1}
- Investors have two privately observed trading motives
  1. Private signal about the innovation to the payoff, s_t^i (information)
  2. Private idiosyncratic trading motive, n_t^i (noise)

Assumptions

1. **Additive noise**
   - Signals about innovation to payoff

$$s_t^i = \eta_t + \varepsilon_{st}^i$$

   - Idiosyncratic trading motives

$$n_t^i = \underbrace{n_t}_{\text{aggregate}} + \underbrace{\varepsilon_{nt}^i}_{\text{idiosyncratic}}$$

   where n_t is unobserved and $\mathbb{V}ar\left[n_t\right] = \tau_n^{-1}$

2. **Linear asset demands**

$$\Delta q_t^i = \alpha_\theta^i \theta_t + \alpha_s^i s_t^i + \alpha_n^i n_t^i - \alpha_p^i p_t + \psi^i$$

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- General structure
  - Agnostic about trading motives, distributional assumptions, learning technology, investor heterogeneity
  - First order approximation to more general demand functions

Price Informativeness

Linear quilibrium price

$$p_t = \pi_\theta \theta_t + \overbrace{\pi_s \eta_t}^{\text{signal}} + \overbrace{\pi_n n_t}^{\text{noise}} + \text{risk premium}$$

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- **Relative Price informativeness:**

$$\tau_{\hat{p}}^R = \frac{\text{precision of info in price}}{\text{precision of innovation}} = \frac{\tau_{\hat{p}}}{\tau_\eta}$$

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$$\tau_{\hat{p}}^R = \frac{\text{precision of info in price}}{\text{precision of innovation}} = \frac{\tau_{\hat{p}}}{\tau_\eta}$$

- **Updating weight** (Kalman Gain): $K = \frac{\tau_{\hat{p}}^R}{1+\tau_{\hat{p}}^R}$, where

$$\text{Posterior Belief} = (1-K) \times \text{Prior Belief} + K \times \text{Signal in Price}$$

Identifying Absolute Price Informativeness

Identification Result 1

Absolute price informativeness can be recovered by regressing prices on payoffs

$$p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \varepsilon_t, \tag{R1}$$

as

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- Equilibrium price

$$p_t = \underbrace{\text{risk premium}}_{=\beta_0} + \underbrace{(\pi_\theta - \pi_s \rho)}_{=\beta_1} \theta_t + \underbrace{\pi_s}_{=\beta_2} \theta_{t+1} + \underbrace{\pi_n n_t}_{=\varepsilon_t}$$

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Identifying Relative Price Informativeness

Identification Result 2

Relative price informativeness can be recovered from regressions of prices on payoffs

$$p_t = \beta_0 + \beta_1\theta_t + \beta_2\theta_{t+1} + \varepsilon_t \Rightarrow R^2_{|\theta_{t+1},\theta_t} \tag{R1}$$

$$p_t = \zeta_0 + \zeta_1\theta_t + \varepsilon_t^{\zeta} \qquad \Rightarrow R^2_{|\theta_t} \tag{R2}$$

using the R-squareds as follows

$$\tau^R_{\hat{p}} = \frac{R^2_{|\theta_{t+1},\theta_t} - R^2_{|\theta_t}}{1 - R^2_{|\theta_{t+1},\theta_t}}$$

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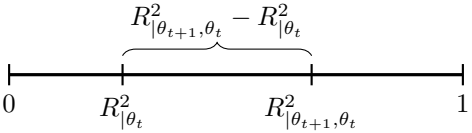

Why this definition of informativeness?

1. Blackwell's (1951) notion of informativeness to rank signals (experiments)
   - More informative signals are associated with lower expected losses for decision maker
   - Quadratic loss function + unbiased signals $\Rightarrow$ signal precision $\tau_{\hat{p}}$ induces an order

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2. Gaussian signal structure + Bayesian updating
   - $\tau_{\hat{p}}$ is the only relevant object to form a posterior, given the signal

Predictability is not Informativeness

- Predictability measures confound two objects
  - price informativeness: how well prices reveal asset payoffs
  - payoff volatility: how easy it is to forecast asset payoffs

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- Predictability measures confound two objects
  - price informativeness: how well prices reveal asset payoffs
  - payoff volatility: how easy it is to forecast asset payoffs
- Without a structural interpretation cannot interpret magnitudes
  - Need assumptions on distributions and updating rules

Extensions

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   - Extend results running regressions in differences

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2. Payoff correlated with noise
3. Multiple risky assets with correlated payoffs
4. Learnable and unlearnable components of asset payoff

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1. Public signals
   - All identification results are valid, implementation may differ

Non-stationary payoffs

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- Price informativeness

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Learnable and unlearnable payoff

- Two components of asset payoff: learnable and unlearnable

$$\eta_t = \eta_t^L + \eta_t^U$$

- Private signal of learnable component

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- Identification results remain valid, interpretation of noise differs

Public signals

- Investors receive a public signal of innovation to payoff

$$\pi_t = \eta_t + \varepsilon_{\pi t}$$

- Linear asset demands

$$\Delta q_t^i = \alpha_\theta^i \theta_t + \alpha_s^i s_t^i + \alpha_\pi^i \pi_t + \alpha_n^i n_t^i - \alpha_p^i p_t + \psi^i,$$

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- Observing the public signal
  - Price

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- Implementation changes, same identification

Empirical Implementation

- Implement regressions R1 and R2 in our identifications results at the stock level
  - CRSP/Compustat, 1963-2017, quarterly data
  - Total market value, M_t^j
  - Quarterly earnings as a measure of payoffs, E_t^j
  - $N = 839$ stocks with more than 80 observations

Statistic	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Mean Earnings	77.04	344.25	1.21	11.76	43.69
St. Dev. Earnings	81.06	371.29	3.41	12.23	39.49

in millions of 2008 USD, N=839 stocks with >80 observations

Distribution

Empirical Implementation

- Main specification (stationary earnings)

$$M_t^j = \beta_0^j + \beta_1^j E_{j,t} + \beta_2^j E_{j,t+1} + \varepsilon_t^j \Rightarrow R^{2j}_{|\theta_{t+1},\theta_t}$$
$$M_t^j = \zeta_0^j + \zeta_1^j E_{j,t} \qquad\qquad + \hat{\varepsilon}_t^j \Rightarrow R^{2j}_{|\theta_t}$$

- Measures of price informativeness

$$\tau_{\hat{p}}^j = \frac{\left(\beta_2^j\right)^2}{\mathbb{V}\text{ar}\left[\varepsilon_t^j\right]}, \quad \tau_{\hat{p}}^{Rj} = \frac{R^{2j}_{|\theta_{t+1},\theta_t} - R^{2j}_{|\theta_t}}{1 - R^{2j}_{|\theta_{t+1},\theta_t}} \quad \text{and} \quad K^j = \frac{\tau_{\hat{p}}^{Rj}}{1 + \tau_{\hat{p}}^{Rj}}$$

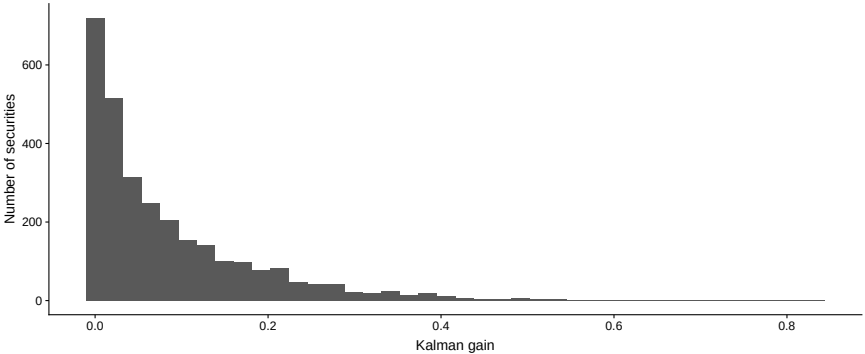
- We test for non-stationarity of earnings
  - Use adequate methodology given test
- Implicit assumption: structural stability of parameters

Diff

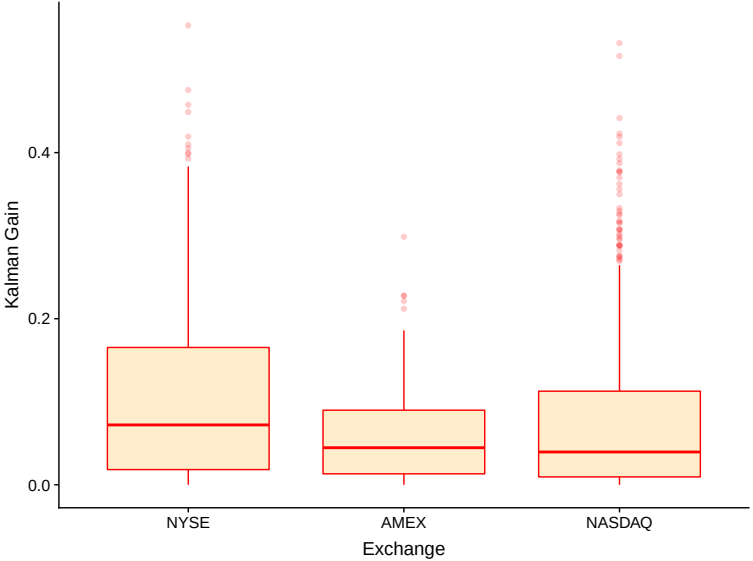
Empirical Results

Statistic	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Absolute Informativeness	0.03	0.17	0.0000	0.001	0.01
Relative Informativeness	0.08	0.20	0.01	0.02	0.08
Kalman Gain	0.06	0.09	0.01	0.02	0.08

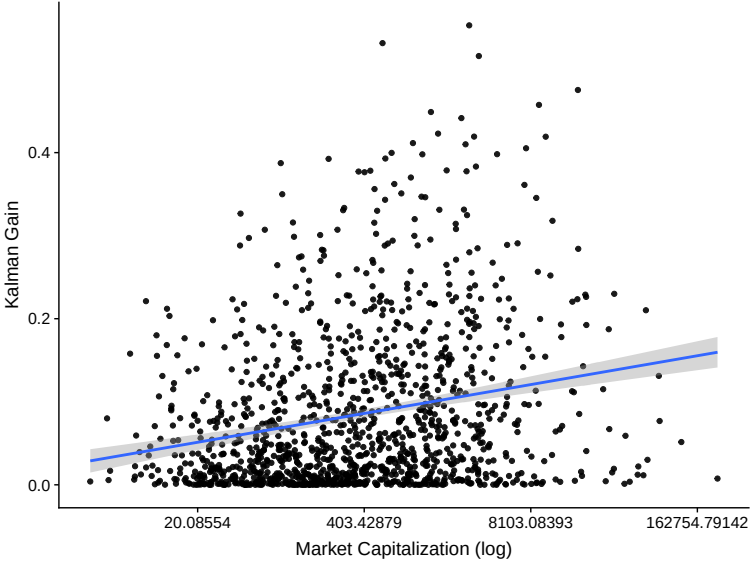
Price Informativeness



Price Informativeness by Exchange



Price Informativeness by Market Cap



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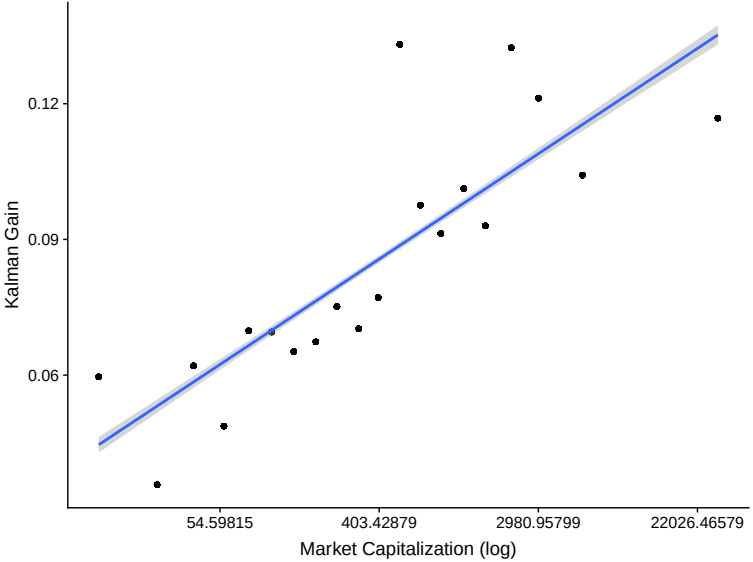
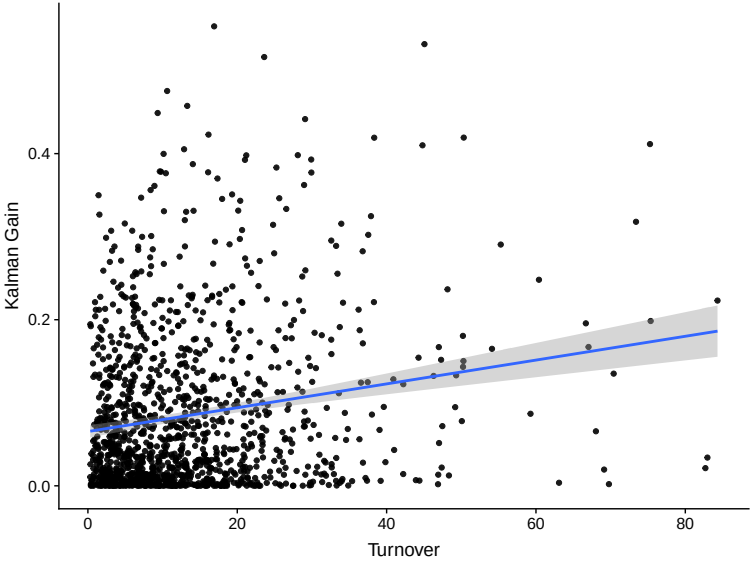


Price Informativeness by Turnover



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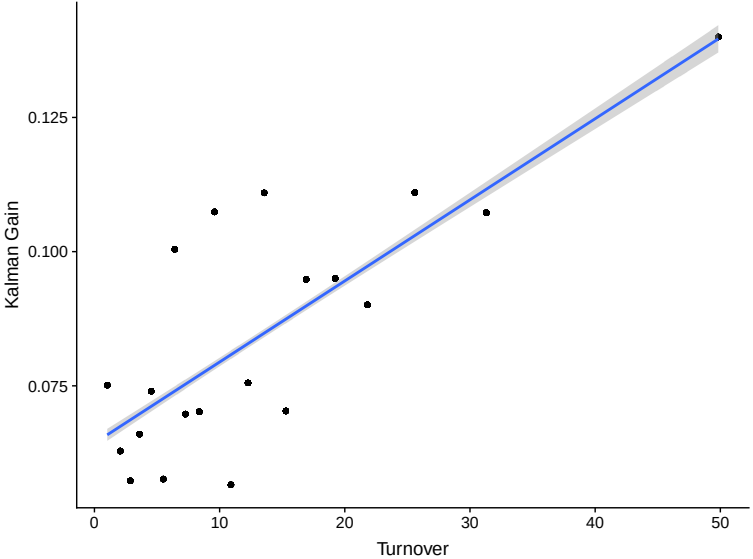
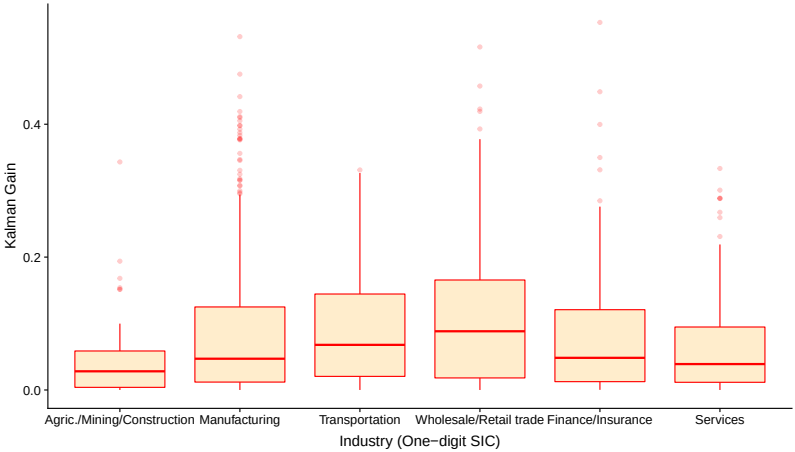
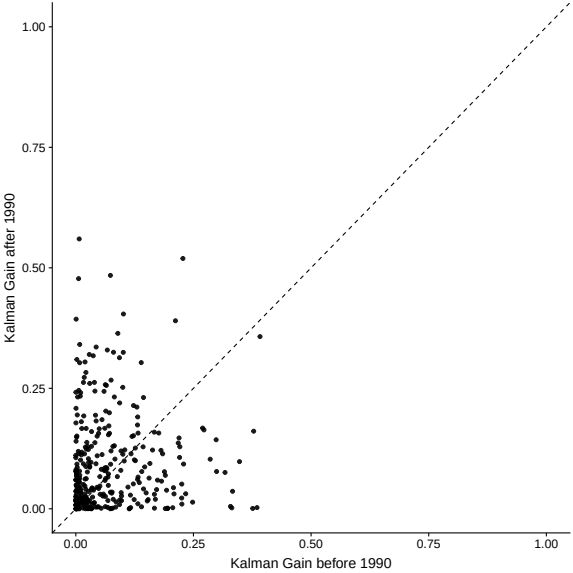


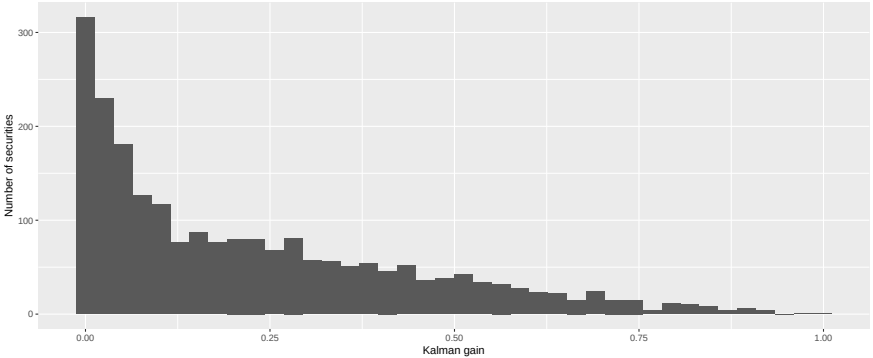
Price Informativeness by Industry



Price Informativeness over time



Price Informativeness Annual Frequency



Conclusion

- Methodology to recover exact measures of stock-specific price informativeness
  - General framework (rich heterogeneity, minimal distributional assumptions)
  - Stationary and non-stationary payoffs
  - Straight forward implementation

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  - General framework (rich heterogeneity, minimal distributional assumptions)
  - Stationary and non-stationary payoffs
  - Straight forward implementation
- Empirical application
  - Prices are not too informative
  - Substantial dispersion in price informativeness
  - Informativeness increases in market cap and turnover
  - Informativeness increased over time for most stocks
  - Informativeness is higher at lower frequencies

Measures of predictability

- Forecasting price efficiency (BPS)

$$V_{FPE} = \mathbb{V}\text{ar}\left[\mathbb{E}\left[\theta_{t+1} | \hat{p}_t, \theta_t\right]\right]$$

Measures of predictability

- Forecasting price efficiency (BPS)

$$V_{FPE} = \mathbb{V}\text{ar}\left[\mathbb{E}\left[\theta_{t+1} \middle| \hat{p}_t, \theta_t\right]\right] = \frac{\tau_{\hat{p}}}{\left(\tau_\eta + \tau_{\hat{p}}\right)^2} = \frac{\tau_{\hat{p}}^R}{\left(1 + \tau_{\hat{p}}^R\right)^2} \frac{1}{\tau_\eta}$$

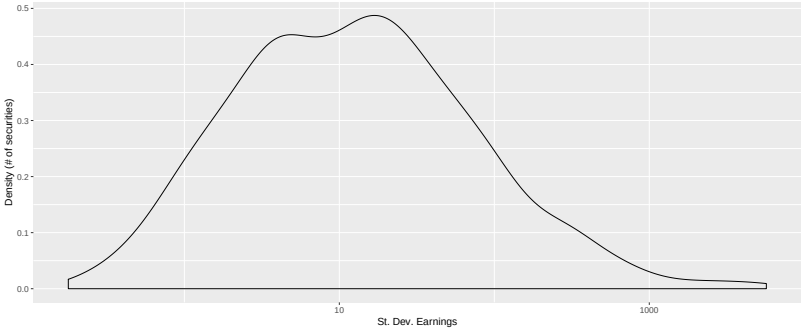
- Posterior variance

$$\mathbb{V}\text{ar}\left[\theta_{t+1} | \hat{p}_t, \theta_t\right] = \left(\tau_\eta + \tau_{\hat{p}}\right)^{-1} = \tau_\eta^{-1}\left(1 + \tau_{\hat{p}}^R\right)^{-1}$$

- These measures confound price informativeness and payoff uncertainty (τ_η)

Back

Payoff Volatility (Earnings)



Back

Correlated payoff and noise

- Aggregate noise component given by

$$n_t = \omega \eta_t + \varepsilon_{nt},$$

where $\varepsilon_{nt} \sim N\left(\mu_n, \tau_n^{-1}\right)$

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- Price informativeness

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- Identification results remain valid, interpretation of noise differs
  - Orthogonal component of aggregate noise n_t

Multiple risky assets

- N risky assets with correlated payoffs

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\mu}_\theta + C\boldsymbol{\theta}_t + \boldsymbol{\eta}_t,$$

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