Identifying Price Informativeness

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How much information is revealed by asset prices?

- Price informativeness: measures the precision of information in price
- Disconnect between theoretical work and empirical measures of informativeness
How much information is revealed by asset prices?

- Price informativeness: measures the precision of information in price
- Disconnect between theoretical work and empirical measures of informativeness

This paper

- Methodology to identify price informativeness from linear regressions of prices on asset payoffs
- Recover exact measures of stock specific price informativeness
  - Absolute and relative price informativeness
Main Results

1. Identifying price informativeness
   - Identify exact measures of price informativeness (precision of info in price)
   - Illustration of identification results
     - Regression of prices on current and future payoffs
       \[ p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \epsilon_t \]
     - Price informativeness = signal to noise ratio
       \[ \tau_p = \frac{\beta_2^2}{\text{Var}[\epsilon_t]} \]
Main Results

1. Identifying price informativeness
   ▶ Identify exact measures of price informativeness (precision of info in price)
   ▶ Illustration of identification results
     ▶ Regression of prices on current and future payoffs
       \[ p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \varepsilon_t \]
     ▶ Price informativeness = signal to noise ratio
       \[ \tau_p = \frac{\beta_2^2}{\text{Var}[\varepsilon_t]} \]

2. Empirical implementation
   ▶ Prices are noisy: median updating weight is 0.02 and mean is 0.05
     ▶ Dispersed and right-skewed distribution of informativeness measures
   ▶ Informativeness increases in market cap, in turnover, and over time.
   ▶ Informativeness is higher at lower frequencies
Why Price Informativeness?

“The economic problem of society is (...) rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know. Or, to put it briefly, it is a problem of the utilization of knowledge which is not given to anyone in its totality.”

Hayek, 1945
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Hayek, 1945

- Which stocks/industries to learn about?
- Identify good investment opportunities more precisely
Outline

1. **Identifying price informativeness**
   - Framework
   - Identification
   - Extensions

2. **Empirical implementation**
   - Estimates of price informativeness
   - Distribution of price informativeness across stock characteristics
Framework

- $t = 0, 1, 2...$
- One asset traded at price $p_t$ with an unobserved payoff

$$\theta_{t+1} = \mu + \rho \theta_t + \eta_t,$$

with $|\rho| < 1$ and $\eta_t$ are i.i.d., mean zero with variance $\tau_{\eta}^{-1}$
Framework

- $t = 0, 1, 2...$
- One asset traded at price $p_t$ with an unobserved payoff

$$
\theta_{t+1} = \mu_\theta + \rho \theta_t + \eta_t, \\
\text{innovation}
$$

with $|\rho| < 1$ and $\eta_t$ are i.i.d., mean zero with variance $\tau_\eta^{-1}$

- Investors have two privately observed trading motives
  1. Private signal about the innovation to the payoff, $s^i_t$ (information)
  2. Private idiosyncratic trading motive, $n^i_t$ (noise)
Assumptions

1. **Additive noise**
   - Signals about innovation to payoff
     \[ s_t^i = \eta_t + \varepsilon_{st}^i \]
   - Idiosyncratic trading motives
     \[ n_t^i = \underbrace{n_t}_{\text{aggregate}} + \underbrace{\varepsilon_{nt}^i}_{\text{idiosyncratic}} \]
     where \( n_t \) is unobserved and \( \text{Var}[n_t] = \tau_n^{-1} \)

2. **Linear asset demands**
   \[ \Delta q_t^i = \alpha_0 \theta_t + \alpha_s s_t^i + \alpha_n n_t^i - \alpha_p p_t + \psi^i \]
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1. **Additive noise**
   - Signals about innovation to payoff
     \[
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   \[
   \Delta q_t^i = \alpha_i \theta_t + \alpha_s^i s_t^i + \alpha_n^i n_t^i - \alpha_p^i p_t + \psi^i
   \]

   - **General structure**
     - Agnostic about trading motives, distributional assumptions, learning technology, investor heterogeneity
     - First order approximation to more general demand functions
Price Informativeness

Linear equilibrium price

\[ p_t = \pi_\theta \theta_t + \pi_s \eta_t + \pi_n n_t + \text{risk premium} \]
Price Informativeness

Linear equilibrium price

\[ p_t = \pi_{\theta} \theta_t + \pi_s \eta_t + \pi_n n_t + \text{risk premium} \]

- Absolute Price Informativeness:

\[ \tau_p \equiv \text{precision of information in price} \]
Price Informativeness

Linear equilibrium price

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\[ \tau_p \equiv \text{precision of information in price} = \left( \frac{\pi_s}{\pi_n} \right)^2 \tau_n \]
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▶ Absolute Price Informativeness:

\[ \tau_p \equiv \text{precision of information in price} = \left( \frac{\pi_s}{\pi_n} \right)^2 \tau_n \]

▶ Relative Price informativeness:

\[ \tau_p^R = \frac{\text{precision of info in price}}{\text{precision of innovation}} = \frac{\tau_p}{\tau_\eta} \]
Price Informativeness

Linear equilibrium price

\[ p_t = \pi_\theta \theta_t + \pi_s \eta_t + \pi_n n_t + \text{risk premium} \]

- **Absolute Price Informativeness:**

\[ \tau_p \equiv \text{precision of information in price} = \left( \frac{\pi_s}{\pi_n} \right)^2 \tau_n \]

- **Relative Price informativeness:**

\[ \tau_p^R = \frac{\text{precision of info in price}}{\text{precision of innovation}} = \frac{\tau_p}{\tau_\eta} \]

- **Updating weight (Kalman Gain):**

\[ K = \frac{\tau_p^R}{1 + \tau_p^R}, \text{ where} \]

\[ \text{Posterior Belief} = (1 - K) \times \text{Prior Belief} + K \times \text{Signal in Price} \]
Identifying Absolute Price Informativeness

Identification Result 1

Absolute price informativeness can be recovered by regressing prices on payoffs

\[ p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \varepsilon_t, \]  

\[ \hat{\tau}_p = \frac{\beta_2^2}{\text{Var} \left[ \varepsilon_t \right]} \]  

(R1)
Identifying Absolute Price Informativeness

Identification Result 1

Absolute price informativeness can be recovered by regressing prices on payoffs

\[ p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \varepsilon_t, \quad \text{(R1)} \]

as

\[ \tau_{\hat{p}} = \frac{\beta_2^2}{\text{Var} [\varepsilon_t]} \]

- Equilibrium price

\[ p_t = \beta_0 + (\pi_\theta - \pi_s \rho) \theta_t + \pi_s \theta_{t+1} + \pi_n n_t = \beta_0 + \beta_1 + \beta_2 \theta_{t+1} + \varepsilon_t \]
Identifying Absolute Price Informativeness

Identification Result 1

Absolute price informativeness can be recovered by regressing prices on payoffs

\[ p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \epsilon_t, \quad (R1) \]

as

\[ \hat{\tau}_p = \frac{\beta_2^2}{\text{Var}[\epsilon_t]} \]

▶ Equilibrium price

\[ p_t = \text{risk premium} + (\pi_\theta - \pi_s \rho) \theta_t + \pi_s \theta_{t+1} + \pi_n \eta_t \]

\[ = \beta_0 \]

\[ = \beta_1 \]

\[ = \beta_2 \]

\[ = \epsilon_t \]

▶ Price informativeness \( \hat{\tau}_p = \left( \frac{\pi_s}{\pi_n} \right)^2 \tau_n \)
Identifying Absolute Price Informativeness

Identification Result 1

Absolute price informativeness can be recovered by regressing prices on payoffs

\[ p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \epsilon_t, \]  
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as

\[ \tau_p = \frac{\beta_2^2}{\text{Var}[\epsilon_t]} \]

- Equilibrium price

\[ p_t = \underbrace{\text{risk premium}}_{=\beta_0} + \underbrace{(\pi_\theta - \pi_{s}\rho)\theta_t}_{=\beta_1} + \underbrace{\pi_s \theta_{t+1}}_{=\beta_2} + \underbrace{\pi_n n_t}_{=\epsilon_t} \]

- Price informativeness \( \tau_p = \left( \frac{\pi_s}{\pi_n} \right)^2 \tau_n = \frac{\beta_2^2}{\text{Var}[\epsilon_t]} \)
Identifying Relative Price Informativeness

Identification Result 2

Relative price informativeness can be recovered from regressions of prices on payoffs

\[ p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \epsilon_t \Rightarrow R^2_{|\theta_{t+1},\theta_t} \]  \hspace{1cm} (R1)

\[ p_t = \zeta_0 + \zeta_1 \theta_t + \epsilon_t^\zeta \Rightarrow R^2_{|\theta_t} \]  \hspace{1cm} (R2)

using the R-squareds as follows

\[ \tau^R_p = \frac{R^2_{|\theta_{t+1},\theta_t} - R^2_{|\theta_t}}{1 - R^2_{|\theta_{t+1},\theta_t}} \]
Identifying Relative Price Informativeness

Identification Result 2

Relative price informativeness can be recovered from regressions of prices on payoffs

\[ p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \varepsilon_t \Rightarrow R^2_{|\theta_{t+1},\theta_t} \quad (R1) \]

\[ p_t = \zeta_0 + \zeta_1 \theta_t + \tilde{\varepsilon}_t \Rightarrow R^2_{|\theta_t} \quad (R2) \]

using the R-squareds as follows

\[ \tau^R_p = \frac{R^2_{|\theta_{t+1},\theta_t} - R^2_{|\theta_t}}{1 - R^2_{|\theta_{t+1},\theta_t}} \]
Why this definition of informativeness?

1. Blackwell's (1951) notion of informativeness to rank signals (experiments)
   - More informative signals are associated with lower expected losses for decision maker
   - Quadratic loss function + unbiased signals $\Rightarrow$ signal precision $\tau_p$ induces an order
Why this definition of informativeness?

1. Blackwell’s (1951) notion of informativeness to rank signals (experiments)
   ▶ More informative signals are associated with lower expected losses for decision maker
   ▶ Quadratic loss function + unbiased signals $\Rightarrow$ signal precision $\tau_{\hat{p}}$ induces an order

2. Gaussian signal structure + Bayesian updating
   ▶ $\tau_{\hat{p}}$ is the only relevant object to form a posterior, given the signal
Predictability is not Informativeness

- Predictability measures confound two objects
  - price informativeness: how well prices reveal asset payoffs
  - payoff volatility: how easy it is to forecast asset payoffs

Without a structural interpretation cannot interpret magnitudes

Need assumptions on distributions and updating rules
Predictability is not Informativeness

- Predictability measures confound two objects
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- Without a structural interpretation cannot interpret magnitudes
  - Need assumptions on distributions and updating rules
Extensions

1. Non-stationary payoffs
   - Extend results running regressions in differences

2. Payoff correlated with noise

3. Multiple risky assets with correlated payoffs
   - All identification results are valid, interpretation of noise differs

4. Learnable and unleARNable components of asset payoffs

5. Public signals
   - All identification results are valid, implementation may differ
Extensions

1. Non-stationary payoffs
   ▶ Extend results running regressions in differences
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   ▶ All identification results are valid, implementation may differ
Non-stationary payoffs

- Non-stationary payoffs
  \[ \Delta \theta_t = \mu \theta + \eta_t \]

- Cannot estimate Regression R1 using OLS
Non-stationary payoffs

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- Regressions in differences
  \[ \Delta p_t = \beta_0 + \beta_1 \Delta \theta_t + \beta_2 \Delta \theta_{t+1} + \hat{\epsilon}_t \]
  \[ \Delta p_t = \zeta_0 + \zeta_1 \Delta \theta_t + \hat{\epsilon}_t \]

Price informativeness
\[ \tau_{\hat{p}} = 2 \beta_2 V \left[ \hat{\epsilon}_t \right] \] and
\[ \tau_{\hat{R}} = 2 R^2 | \Delta \theta_t + 1, \Delta \theta_{t+1} - R^2 | \Delta \theta_t - 1 \]
Non-stationary payoffs

- Non-stationary payoffs
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- Regressions in differences
  \[ \Delta p_t = \beta_0 + \beta_1 \Delta \theta_t + \beta_2 \Delta \theta_{t+1} + \hat{\epsilon}_t \Rightarrow R^2_{|\Delta \theta_{t+1}, \Delta \theta_t} \]
  \[ \Delta p_t = \zeta_0 + \zeta_1 \Delta \theta_t + \hat{\epsilon}_t^\zeta \Rightarrow R^2_{|\Delta \theta_t} \]

- Price informativeness
  \[ \tau_p = 2 \frac{\beta_2^2}{\text{Var}[\hat{\epsilon}_t]} \quad \text{and} \quad \tau^R_p = 2 \frac{R^2_{|\Delta \theta_{t+1}, \Delta \theta_t} - R^2_{|\Delta \theta_t}}{1 - R^2_{|\Delta \theta_{t+1}, \Delta \theta_t}} \]
Learnable and unlearnable payoff

- Two components of asset payoff: learnable and unlearnable
  \[ \eta_t = \eta_t^L + \eta_t^U \]

- Private signal of learnable component
  \[ s_t^i = \eta_t^L + \varepsilon_{st}^i \]
Learnable and unlearnable payoff

➢ Two components of asset payoff: learnable and unlearnable

\[ \eta_t = \eta_t^L + \eta_t^U \]

➢ Private signal of learnable component

\[ s_t^i = \eta_t^L + \epsilon_{st}^i = \eta_t - \eta_t^U + \epsilon_{st}^i = \eta_t + \epsilon_{st}^{i'} \]
Learnable and unlearnable payoff

Two components of asset payoff: learnable and unlearnable

\[ \eta_t = \eta_t^L + \eta_t^U \]

Private signal of learnable component

\[ s_t^i = \eta_t^L + \varepsilon^i_{st} = \eta_t - \eta_t^U + \varepsilon^i_{st} = \eta_t + \varepsilon^i_{st} \]

Equilibrium price

\[ p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \frac{\alpha_n}{\alpha_p} n_t - \frac{\alpha_s}{\alpha_p} \eta_t^U + \varepsilon_t \]
Learnable and unlearnable payoff

- Two components of asset payoff: learnable and unlearnable

$$\eta_t = \eta^L_t + \eta^U_t$$

- Private signal of learnable component

$$s^i_t = \eta^L_t + \varepsilon^i_{st} = \eta_t - \eta^U_t + \varepsilon^i_{st} = \eta_t + \varepsilon^i_{st}$$

- Equilibrium price

$$p_t = \beta_0 + \beta_1 \theta_t + \beta_2 \theta_{t+1} + \frac{\alpha_n}{\alpha_p} n_t - \frac{\alpha_s}{\alpha_p} \eta^U_{t+1}$$

- Identification results remain valid, interpretation of noise differs
Public signals

- Investors receive a public signal of innovation to payoff

\[ \pi_t = \eta_t + \varepsilon_{\pi t} \]

- Linear asset demands

\[ \Delta q^i_t = \alpha^i_0 \theta_t + \alpha^i_s s^i_t + \alpha^i_\pi \pi_t + \alpha^i_n n^i_t - \alpha^i_p p_t + \psi^i , \]
Public signals

- Investors receive a public signal of innovation to payoff
  \[ \pi_t = \eta_t + \varepsilon_{\pi t} \]

- Linear asset demands
  \[ \Delta q^i_t = \alpha_\theta^i \theta_t + \alpha_s^i s_t + \alpha_{\pi}^i \pi_t + \alpha_n^i n_t - \alpha_p^i p_t + \psi^i, \]

- Observing the public signal
  - Price
  \[ p_t = \chi_0 + \chi_1 \theta_t + \chi_2 \theta_{t+1} + \chi_3 \pi_t + \frac{\alpha_n}{\alpha_p} n_t \]
  \[ = \bar{\varepsilon}_t \]
Public signals

- Investors receive a public signal of innovation to payoff

\[ \pi_t = \eta_t + \epsilon_{\pi t} \]

- Linear asset demands

\[ \Delta q^i_t = \alpha^i_\theta \theta_t + \alpha^i_s s^i_t + \alpha^i_{\pi} \pi_t + \alpha^i_n n^i_t - \alpha^i_p p_t + \psi^i, \]

- Observing the public signal
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\[ p_t = \chi_0 + \chi_1 \theta_t + \chi_2 \theta_{t+1} + \chi_3 \pi_t + \frac{\bar{\alpha}_n}{\bar{\alpha}_p} n_t = \bar{\epsilon}_t \]

- Implementation changes, same identification
Empirical Implementation

- Implement regressions R1 and R2 in our identifications results at the stock level
  - CRSP/Compustat, 1963-2017, quarterly data
  - Total market value, $M_t^j$
  - Quarterly earnings as a measure of payoffs, $E_t^j$
  - $N = 839$ stocks with more than 80 observations

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Pctl(25)</th>
<th>Median</th>
<th>Pctl(75)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Earnings</td>
<td>77.04</td>
<td>344.25</td>
<td>1.21</td>
<td>11.76</td>
<td>43.69</td>
</tr>
<tr>
<td>St. Dev. Earnings</td>
<td>81.06</td>
<td>371.29</td>
<td>3.41</td>
<td>12.23</td>
<td>39.49</td>
</tr>
</tbody>
</table>

in millions of 2008

USD, $N=839$ stocks with $>80$ observations
Empirical Implementation

▶ Main specification (stationary earnings)

\[ M^j_t = \beta^j_0 + \beta^j_1 E_{j,t} + \beta^j_2 E_{j,t+1} + \epsilon^j_t \Rightarrow R^2_{j|\theta_t+1,\theta_t} \]

\[ M^j_t = \zeta^j_0 + \zeta^j_1 E_{j,t} + \hat{\epsilon}^j_t \Rightarrow R^2_{j|\theta_t} \]

▶ Measures of price informativeness

\[ \tau^j_p = \frac{\left( \beta^j_2 \right)^2}{\text{Var} \left[ \epsilon^j_t \right]}, \quad \tau^R_j = \frac{R^2_{j|\theta_t+1,\theta_t} - R^2_{j|\theta_t}}{1 - R^2_{j|\theta_t+1,\theta_t}} \quad \text{and} \quad K^j = \frac{\tau^R_j}{1 + \tau^R_j} \]

▶ We test for non-stationarity of earnings

▶ Use adequate methodology given test

▶ Implicit assumption: structural stability of parameters
Empirical Results

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>Absolute Informativeness</td>
<td>0.03</td>
<td>0.17</td>
<td>0.0000</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>Relative Informativeness</td>
<td>0.08</td>
<td>0.20</td>
<td>0.01</td>
<td>0.02</td>
<td>0.08</td>
</tr>
<tr>
<td>Kalman Gain</td>
<td>0.06</td>
<td>0.09</td>
<td>0.01</td>
<td>0.02</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Price Informativeness
Price Informativeness by Exchange

- NYSE
- AMEX
- NASDAQ
Price Informativeness by Market Cap

![Graph showing the relationship between Kalman Gain and Market Capitalization (log)].

The graph plots the Kalman Gain against Market Capitalization (log) on a scatter plot. The data points are distributed across the plot, indicating a correlation between the two variables. A line of best fit is also displayed, suggesting a trend in the data.
Price Informativeness by Market Cap

![Graph showing the relationship between Market Capitalization (log) and Kalman Gain.](image-url)
Price Informativeness by Turnover
Price Informativeness by Turnover
Price Informativeness by Industry

![Box plot showing price informativeness by industry](image-url)
Price Informativeness over time

Kalman Gain before 1990

Kalman Gain after 1990

0.00 0.25 0.50 0.75 1.00

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Price Informativeness Annual Frequency
Conclusion

- Methodology to recover exact measures of stock-specific price informativeness
  - General framework (rich heterogeneity, minimal distributional assumptions)
  - Stationary and non-stationary payoffs
  - Straightforward implementation
Conclusion

▶ Methodology to recover exact measures of stock-specific price informativeness
  ▶ General framework (rich heterogeneity, minimal distributional assumptions)
  ▶ Stationary and non-stationary payoffs
  ▶ Straightforward implementation

▶ Empirical application
  ▶ Prices are not too informative
  ▶ Substantial dispersion in price informativeness
  ▶ Informativeness increases in market cap and turnover
  ▶ Informativeness increased over time for most stocks
  ▶ Informativeness is higher at lower frequencies
Measures of predictability

▶ Forecasting price efficiency (BPS)

\[ V_{FPE} = \text{Var} \left[ \mathbb{E} \left[ \theta_{t+1} | \hat{p}_t, \theta_t \right] \right] \]
Measures of predictability

▶ Forecasting price efficiency (BPS)

\[ V_{FPE} = \text{Var} [\mathbb{E} [\theta_{t+1} | \hat{p}_t, \theta_t]] = \frac{\tau_{\hat{p}}}{(\tau_\eta + \tau_{\hat{p}})^2} = \frac{\tau_{\hat{p}}^R}{(1 + \tau_{\hat{p}}^R)^2} \frac{1}{\tau_\eta} \]

▶ Posterior variance

\[ \text{Var} [\theta_{t+1} | \hat{p}_t, \theta_t] = (\tau_\eta + \tau_{\hat{p}})^{-1} = \tau_\eta^{-1} (1 + \tau_{\hat{p}}^R)^{-1} \]

▶ These measures confound price informativeness and payoff uncertainty (\(\tau_\eta\))
Payoff Volatility (Earnings)
Correlated payoff and noise

- Aggregate noise component given by

\[ n_t = \omega \eta_t + \epsilon_{nt}, \]

where \( \epsilon_{nt} \sim N(\mu_n, \tau_n^{-1}) \)
Correlated payoff and noise

- Aggregate noise component given by

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where \( \varepsilon_{nt} \sim N(\mu_n, \tau_n^{-1}) \)

- Additive noise + linear asset demands
Correlated payoff and noise

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where \( \varepsilon_{nt} \sim N(\mu_n, \tau_n^{-1}) \)

- Additive noise + linear asset demands

- Equilibrium price

\[ p_t = \frac{\psi}{\bar{\alpha}_p} + \frac{\bar{\alpha}_\theta}{\bar{\alpha}_p} \theta_t + \frac{\bar{\alpha}_s}{\bar{\alpha}_p} \eta_t + \frac{\bar{\alpha}_n}{\bar{\alpha}_p} n_t \]
Correlated payoff and noise

- Aggregate noise component given by
  \[ n_t = \omega \eta_t + \varepsilon_{nt}, \]
  where \( \varepsilon_{nt} \sim N(\mu_n, \tau_n^{-1}) \)

- Additive noise + linear asset demands

- Equilibrium price
  \[
  p_t = \frac{\psi}{\alpha_p} + \left( \frac{\alpha_{\theta}}{\alpha_p} - \rho \frac{\alpha_s + \omega \alpha_n}{\alpha_p} \right) \theta_t + \frac{\alpha_s + \omega \alpha_n}{\alpha_p} \theta_{t+1} + \frac{\alpha_n}{\alpha_p} \varepsilon_{nt}
  \]

- Price informativeness
  \[
  \tau_p \equiv (\text{Var}[\hat{p}_t|\theta_{t+1}, \theta_t])^{-1} = \frac{\beta_2^2}{\sigma^2_{\varepsilon}}.
  \]
Correlated payoff and noise

- Aggregate noise component given by

\[ n_t = \omega \eta_t + \varepsilon_{nt}, \]

where \( \varepsilon_{nt} \sim N(\mu_n, \tau_n^{-1}) \)

- Additive noise + linear asset demands

- Equilibrium price

\[
p_t = \frac{\psi}{\alpha_p} + \left( \frac{\alpha \theta}{\alpha_p} - \rho \frac{\alpha_s + \omega \alpha_n}{\alpha_p} \right) \theta_t + \frac{\alpha_s + \omega \alpha_n}{\alpha_p} \theta_{t+1} + \frac{\alpha_n}{\alpha_p} \varepsilon_{nt}
\]

- Price informativeness

\[
\tau_{\hat{p}} \equiv (\text{Var} [\hat{p}_t | \theta_{t+1}, \theta_t])^{-1} = \frac{\beta^2}{\sigma^2}.
\]

- Identification results remain valid, interpretation of noise differs
  - Orthogonal component of aggregate noise \( n_t \)
Multiple risky assets

- $N$ risky assets with correlated payoffs

\[ \theta_{t+1} = \mu_\theta + C\theta_t + \eta_t, \]

where $\mu_\theta$, $\theta_t$ and $\eta_t$ are $N \times 1$ vectors and $\theta_{t+1}$ is stationary.
Multiple risky assets

- \( N \) risky assets with correlated payoffs

\[
\theta_{t+1} = \mu_\theta + C\theta_t + \eta_t,
\]

where \( \mu_\theta, \theta_t \) and \( \eta_t \) are \( N \times 1 \) vectors and \( \theta_{t+1} \) is stationary.

- Additive noise + linear asset demands
Multiple risky assets

- \( N \) risky assets with correlated payoffs

\[
\theta_{t+1} = \mu_\theta + C\theta_t + \eta_t,
\]

where \( \mu_\theta, \theta_t \) and \( \eta_t \) are \( N \times 1 \) vectors and \( \theta_{t+1} \) is stationary.

- Additive noise + linear asset demands

- Equilibrium price

\[
p_{j,t} = \Psi_0^j + \Psi_1^j \theta_{j,t} + \Psi_2^j \theta_{j,t+1} + \Gamma_j u_{j,t}
\]

where

\[
u_{j,t} = \begin{bmatrix} \omega_{t+1}^j, \omega_t^j, n_t^j \end{bmatrix}' \quad \text{and} \quad \omega_{h,t}^j = \theta_{h,t} - \frac{\text{Cov} [\theta_{h,t}, \theta_{j,t}]}{\text{Var} [\theta_{h,t}]} \theta_{j,t}
\]
Multiple risky assets

- $N$ risky assets with correlated payoffs
  \[ \theta_{t+1} = \mu_\theta + C\theta_t + \eta_t, \]
  where $\mu_\theta$, $\theta_t$ and $\eta_t$ are $N \times 1$ vectors and $\theta_{t+1}$ is stationary.

- Additive noise + linear asset demands

- Equilibrium price
  \[ p_{j,t} = \Psi^j_0 + \Psi^j_1 \theta_{j,t} + \Psi^j_2 \theta_{j,t+1} + \Gamma_j u_{j,t} \]
  where
  \[ u_{j,t} = \begin{bmatrix} \omega^{j'}_{t+1} \\ \omega^j_t \\ n_t \end{bmatrix} \quad \text{and} \quad \omega^j_{h,t} = \theta_{h,t} - \frac{\text{Cov} [\theta_{h,t}, \theta_{j,t}]}{\text{Var} [\theta_{h,t}]} \theta_{j,t} \]

- Price informativeness
  \[ \tau^j_p = \text{Var} [\hat{p}_j | \theta_{j,t+1}, \theta_{j,t}]^{-1} = \left( \Psi^j_2 \right)^2 \text{Var} [\Gamma_j u_{j,t}]^{-1} \]
Multiple risky assets

- \( N \) risky assets with correlated payoffs

\[
\theta_{t+1} = \mu_\theta + C\theta_t + \eta_t,
\]

where \( \mu_\theta, \theta_t \) and \( \eta_t \) are \( N \times 1 \) vectors and \( \theta_{t+1} \) is stationary.

- Additive noise + linear asset demands

- Equilibrium price

\[
p_{j,t} = \Psi_j^0 + \Psi_j^1 \theta_{j,t} + \Psi_j^2 \theta_{j,t+1} + \Gamma_j u_{j,t}
\]

where

\[
u_{j,t} = [\omega_{t+1}^j, \omega_t^j, n_t^j]' \quad \text{and} \quad \omega_{j,t}^j = \theta_{h,t} - \frac{\text{Cov} [\theta_{h,t}, \theta_{j,t}]}{\text{Var} [\theta_{h,t}]} \theta_{j,t}
\]

- Price informativeness

\[
\tau_j^p = \text{Var} \left[ \hat{p}_j | \theta_{j,t+1}, \theta_{j,t} \right]^{-1} = \left( \Psi_j^2 \right)^2 \text{Var} \left[ \Gamma_j u_{j,t} \right]^{-1}
\]

- Identification results remain valid, interpretation of noise differs