Discount-Rate Risk in Private Equity: Evidence from Secondary Market Transactions

Brian Boyer, Taylor Nadauld, Keith Vorkink, Mike Weisbach

Q-Group Conference

May 2023

Private Markets

- Private Equity has become an important asset class.

- Total Private Markets AUM totaled $11.7 trillion June 2022.
  - McKinsey Global Private Markets Review, March 2023.

- Is investing in PE utility enhancing?
  - For example, does tilting toward PE improve my Sharpe ratio? (Alpha)

- Data on secondary market prices not has not been publicly available.
- Researchers have had to rely on cash flows or NAVs to measure performance.

Private vs Public Markets

- Variation in discount rates for public markets
  - About half of the variation in *public* equity returns is driven by news about discount rates Campbell (1991).
  - All variation in dividend price ratios is driven by news about discount rates (Campbell and Shiller (1988), Cochrane (2011)).

- Two questions:
  1) Why and how might discount rate risk create disparity between cash-flow based measures of investment performance and standard measures (alpha).

  2) To what extent does discount-rate risk in PE actually generate a meaningful empirical difference between alpha and measures of performance developed for PE?

What We Do

1) Derive the theoretical relation between the GPME of Korteweg and Nagel (2016) and alpha

2) Use data obtained from a large intermediary to create a secondary market PE index
   - Compare estimates of GPME for funds in the index and standard alpha.
   - Compare betas/alphas/volatilities of our index with those of NAV-based indices and the S&P listed PE index.
   - Estimate time-series variation in implied PE discount rates (Campbell and Shiller (1998)) and compare with variation in implied NAV discount rates.

3) Create a series of synthetic "PE funds" that invest in small-cap public companies and compare estimates of GPME and standard alpha for these funds.

Summary of Results

1) Theoretical connection between GPME and alpha

- Alpha accounts for a discount-rate risk premium ignored by GPME.
- Risk is characterized differently: total return beta vs cash-flow-yield betas

2) Secondary market index

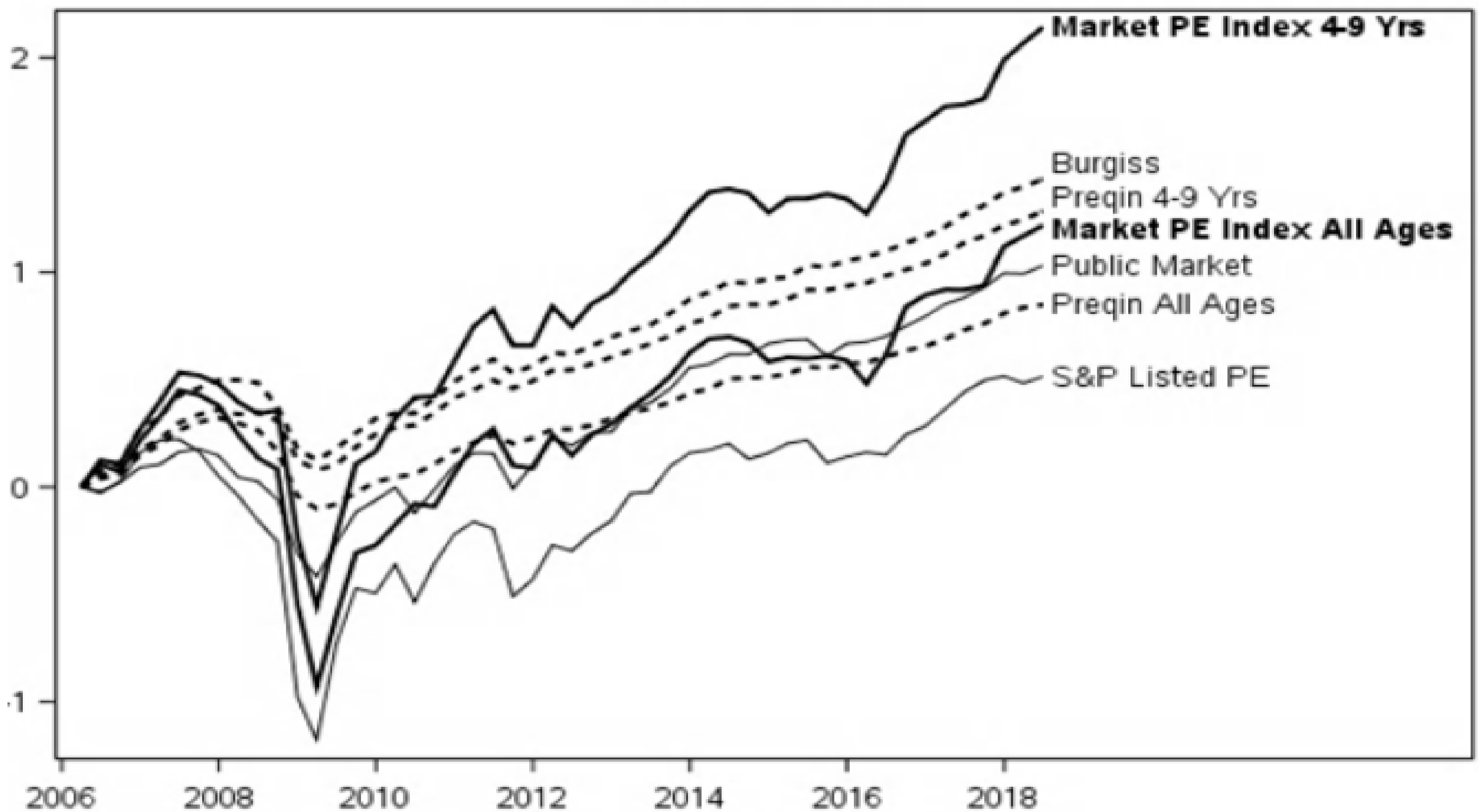
- GPME is large: 0.27 cents of present value per $1 total commitment.
- Alpha is slightly negative and insignificant.
- Beta = 1.79
  - Double leverage of portfolio firms with betas of 1.0.
  - Total return betas, not betas based on cash flows.

Summary of Results

2) Secondary market index results: variation in discount rates

- Volatility of 5-year expected return for PE is 0.37.
- Volatility of five-year expected return of public market portfolio with similar leverage as PE is 0.52 (Cochrane (2011)).
- Volatility of 5-year expected NAV return is 0.10.
- Variation in PE discount rates is considerably greater than that in NAVs, but may be somewhat less than that in the public market.
  - Variation in risk-aversion and sentiment of the marginal investor in PE may be somewhat muted relative to that of the marginal investor in public markets.

Summary of Results



Summary of Results

3) Synthetic Funds that invest in small-cap public equity

- GPME is large: 20-30 cents of present value per $1 of total commitment
- CAPM alphas of these funds are pinned very close to zero, insignificant, and in most cases, slightly negative.

- It appears that discount rate risk might create a significant wedge between alpha and standard PE measures of performance.

Point #1: Who Cares About Discount Rate Risk?

- Many PE investors are buy and hold, and just collect the cash flows.

- Same is true for corporate bond holders.
  - Corporate bond markets are illiquid because they are highly customized.
  - But bond investors care about discount rates.
  - Bond investors dynamically optimize their portfolios using credit and interest rate swaps.

- PE Markets are also illiquid.
  - Difficult to vaule.
  - Different “sleeves” of the same fund

- Just as the factor exposure of coupon payments does not capture the full risk exposure of bond investors, the factor exposures of PE cash flows do not capture the full risk exposure of PE investors.

- There exist states of the world in which even self proclaimed “buy and hold” investors will find it optimal to engage in Secondary PE markets, as observed during the financial crisis.

Point #2: Sample Selection

- The PE Secondary market is a seller driven market.
- Funds that come to market, and the funds they sell are not random.
- We define our index for a defined universe – all buyout funds in the Preqin database.
- Estimate the index return using a Heckman (1979) sample selection model.

Point #3: Liquidity Discount

- Buyers earn a small premium: about 5%. (Nadauld, Vorkink, Weisbach (2019)).

- Our PE Index quarterly return reflects returns from buying *and* selling at secondary market prices.
    - To the extent buyers earn a premium, this premium is largely canceled out when the position is sold three months later.

- A shrinking discount leads to higher index returns over time
    - If anything we are likely overstating the performance of PE in buying and selling at the discount each quarter.

Prior Studies

- NAV-based performance, Dimson adjust to account for “staleness”
- PME/GPME based on cash flows to and from LPs
- Regressions of IRRs on IRRs of market over same period
- Mimicking Portfolios using public securities.
- Approaches that impose models on cash flows
- Listed PE securities

- Vast majority of studies find that buyout funds outperform on a risk-adjusted basis.

GPME vs PME

$$\text{PME}= E\left[\frac{\sum_i D^i_{0:t_i}/R_{m,0:t_i}}{\sum_j C^j_{0:t_j}/R_{m,0:t_j}}\right]$$

- GPME (Korteweg and Nagel (2016))
  - Let $M_{0:t}$ denote an SDF. For example, $M_{0:t} = 1/R_{m,0:t_i}$.

$$\text{GPME}= E\left[\sum_i D^i_{0:t_i} M_{0:t_i} - \sum_j C^j_{0:t_j} M_{0:t_i}\right]$$
$$= E\left[\sum_i X_{i,t} M_{0:t}\right]$$

  - $M_{0:t} = 1/R_{m,0:t_i}$ is too restrictive. There are better choices.

GPME vs PME

- Korteweg and Nagel (2016)
  - Think of $M_{0:t}$ as an investor-specific SDF that describes marginal utility across various states.
  - Standard finance theory tells us that if $E\left[\sum_i X_{i,t} M_{0:t}\right] > 0$ then tilting toward asset i will increase expected utility of a buy and hold investor.
  - In contrast, finance theory does not have much to say about the ratio of stochastically discounted cash flows (Jensen's Inequality)

GPME vs Alpha

$$GPME_i - \sum_{t=0}^{T-1} Cov(M_{0:t}\, p_{i,t},\, \alpha_{i,t}) = \alpha_i \sum_{t=0}^{T-1} E[M_{0:t} p_{i,t}]$$

- $M_{0:t} p_{i,t}$ can be interpreted as the marginal benefit of selling and consuming a unit of the PE stake
- $\alpha_{i,t}$ is the conditional alpha relative to the investor-specific $M_{0:t}$
  - depends on how the market discounts PE stake relative to investor.
- α_i is the expected conditional alpha

GPME vs. Alpha

- GPME is present value using a *stochastic* discount factor.

$$GPME_i = E\left[\sum_t X_{i,t} M_{0:t}\right]$$

- Can be recast in terms of a "pre-determined" discount factor

$$GPME_i = \sum_{t=0}^{T} \frac{E_t(X_{i,t})}{\mathcal{K}_{i,t}}$$

- Where $\mathcal{K}_{i,t}$ is a linear function of $\mathcal{B}_{i,t}$

$$\mathcal{B}_{i,t} = \frac{Cov_0[M_{0:t}, Y_{i,t}]}{Var_0(M_{0:t})}$$

$Y_{i,t}$ is the cash-flow yield (cash flow over price)

Index Construction

- Sample Selection Model (Heckman 1979)
- Two first-order approximations

- Presentation
  - Intuition of sample selection model
  - Two first order approximations
  - Other Details

Intuition of Index Construction

- Return on an equally-weighted portfolio for a **population** of N funds.

$$\tilde{r}_p = \frac{1}{N}\sum_{i=1}^{N}\tilde{r}_i \qquad (1)$$

- Project returns onto a set of characteristics in the cross section

$$\tilde{r}_i = \boldsymbol{x}_{\boldsymbol{i}}'\boldsymbol{b} + e_i$$

- We can measure $\tilde{r}_p$ as

$$\tilde{r}_p = \overline{\boldsymbol{x}}\boldsymbol{b} \qquad (2)$$

- Holds for any $\boldsymbol{x}_{\boldsymbol{i}}$ we choose provided it contains an intercept.
  - $\boldsymbol{e}_{\boldsymbol{i}}$ averages exactly to zero across **full population** of **N** funds.

Intuition of Index Construction

- We do not observe r_i for the full population of N funds but only for those funds that transact.

- OLS estimate of b based on observed funds will be inconsistent unless the subset of funds that transacts is chosen randomly – not likely.

- Problem: probability of transaction is correlated with $\boldsymbol{e_i}$.
  - True e_i across sample of observed funds will not average out to zero.
  - Results in inconsistent estimate of portfolio return.

$$\tilde{r}_p = \overline{\boldsymbol{x}}\boldsymbol{b} + \overline{\boldsymbol{e}}_{\boldsymbol{i}}$$

Intuition of Index Construction

- We use a sample selection model to consistently estimate $\boldsymbol{b}$.

- Two step Heckman (1979) approach
  (1) Estimate probit model for selection as a function of fund characteristics.
    - Estimate the inverse mills ratio, $\hat{\lambda}_{it}$
  (2) Estimate the "pricing" equation

$$r_i = \boldsymbol{x_i'b} + d\hat{\lambda}_i + e_i$$

  - Provides a consistent estimate of $\boldsymbol{b}$ under standard assumptions.
    - e_i is uncorrelated with variables that determine selection.
    - We include selection variables in $\boldsymbol{x_i}$ for return regression long with an intercept.
    - Selection model also includes an exclusion restriction

  - Estimate index return as average predicted value of r_i.
    - We do not observe $\tilde{r}_i$ for every fund, but we do observe $\boldsymbol{x}_i$ for every fund in our population.

Two First-Order Approximations

- To compute $\tilde{r}_i$ we need transactions in subsequent quarters for the same fund.
- The same funds often do not transact in subsequent quarters.
- We infer index returns from observed log book-to-market ratios using the Campbell-Shiller (1988) identity.

$$r_{i,t+1} = n_{i,t+1} - \rho_{i,t+1} \; \theta_{i,t+1} + \theta_{i,t}$$

- $r_{i,t+1}$ is the log return
- $n_{i,t+1}$ is the log NAV return (observed for every fund in our universe)
- $\theta_{i,t+1}$ is the log book-to-market ratio (not observed for every fund in our universe)
- $\rho_{i,t+1}$ is a constant of approximation

Two First-Order Approximations

- Perform Heckman procedure to estimate average log BM ratios across *all* funds in our universe at t and $t+1$.

$$r_{i,t+1} = n_{i,t+1} - \rho_{i,t+1} \ \theta_{i,t+1} + \theta_{i,t}$$

- Estimate average log return.
- Average log return is log return of equally-weighted portfolio to a first order approximation.

- Exponentiate log return to obtain consistent estimate of simple return
  - Otherwise intercept in the usual regression using excess returns would can not be interpreted as alpha.
  - Alternatively: adjust intercept under assumption of normality.

Other Details

- Exclusion restriction for Heckman model.
  - A variable in the selection equation that is uncorrelated with BM ratio.
  - Fraction of LPs that are pension funds.
    - Pension funds are less likely to transact.

- Dimson adjust parameters.
  - Funds transact throughout any given quarter.

Data: 2006-2018

- Transaction data
  - From a large intermediary in the PE secondary market
  - Identifies fund that was sold, total capital committed by seller, purchase price, unfunded amount, transaction date.
  - We use the most recent transaction each quarter for each fund

- Fund Characteristics from Preqin
  - Calls, distributions, NAV, LP types, size.

- Explanatory variables:
  - Log fund size (total commitments)
  - Age dummies ($age < 4\ years,\ 4 \leq age < 9\ years$)
  - Fund PME
  - Fraction of LPs that are pension funds
  - Three state variables that proxy for quarter fixed effects.
    - Log value-weighted book-to-market ratio for small-cap stocks
    - TED spread
    - Total AUM scaled by number of firms in US with between 20 and 500 employees (US Census)

Summary Statistics

		Transactions	No Transaction
		(1)	(2)
$\theta_{i,t}$(dep variable)	Mean	0.19	
	Median	0.10	
	Stdev	0.42	
$\log(size_{i,t})$	Mean	21.95	21.28
	Median	21.99	21.13
	Stdev	0.96	0.87
$Age_{i,t}$	Mean	8.61	6.39
	Median	9.00	6.00
	Stdev	4.05	4.15
$PME_{i,t}$	Mean	1.15	1.13
	Median	1.10	1.06
	Stdev	0.36	0.37
PF_{it}	Mean	0.53	0.56
	Median	0.55	0.57
	Stdev	0.17	0.20
N per Quarter	Mean	17.48	290.56
	Q1	7	237
	Q3	27	353
Quarters		48	50
N		839	14,528

Sample Selection Model Estimates

	Heckman				OLS	
	Selection		Pricing		Pricing	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
	(1)	(2)	(3)	(4)	(5)	(6)
log(BM_t)	-0.03	-(0.1)	0.79	(3.8)***	0.79	(5.1)***
TED_t	0.31	(2.0)**	-0.05	-(0.4)	-0.06	-(0.9)
$IAUM_t$	0.20	(2.2)**	-0.20	-(2.4)**	-0.21	-(3.3)***
log($size_{i,t}$)	0.40	(13.4)***	0.00	(0.0)	-0.02	-(1.9)*
$I(Age<4)$	-0.50	-(4.1)***	0.15	(0.8)	0.18	(2.6)***
$I(Age>9)$	0.33	(4.6)***	0.20	(1.7)*	0.18	(4.0)***
$PME_{i,t}$	-0.13	-(2.0)**	-0.08	-(1.5)	-0.07	-(2.2)**
$PF_{i,t}$	-0.32	-(2.8)***				
Inv Mills Ratio			0.06	(0.3)		
Intercept	-10.66	-(14.7)***	1.05	(0.3)	1.65	(5.6)***
N (selected)	839		839		839	
N (not selected)	14,528					

Standard errors for Heckman model: Panel bootstrap clustered by time.
Standard errors for OLS equation: White (1980) clustered by time.

Index Parameters

	Transactions				S&P Listed		Preqin NAV				Burgiss	
	All Ages		4-9 Yrs Old				All Ages		4-9 Yrs Old			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
E[r]	0.14	(1.6)	0.22	(2.5)**	0.09	(1.0)	0.11	(3.4)***	0.14	(4.6)***	0.12	(3.2)***
β	1.79	(8.0)***	1.76	(8.7)***	1.74	(8.1)***	0.72	(5.7)***	0.64	(5.9)***	0.81	(5.7)***
α	-0.02	-(0.4)	0.06	(1.1)	-0.07	-(2.7)***	0.04	(1.6)	0.07	(3.3)***	0.04	(1.6)
σ	0.34	(2.8)***	0.31	(2.8)***	0.30	(4.2)***	0.17	(2.6)**	0.14	(2.5)***	0.20	(2.3)**
Sharpe	0.41	(1.3)	0.68	(1.9)*	0.27	(1.3)	0.67	(2.3)**	0.99	(2.7)***	0.60	(1.9)*
Corr w/ Mkt	0.83	(4.0)***	0.88	(3.7)***	0.90	(2.0)**	0.70	(1.7)*	0.76	(1.8)*	0.65	(1.6)
AC (unadjusted)	0.30	(0.8)	0.26	(0.7)	0.22	(0.5)	0.54	(1.4)	0.41	(0.8)	0.46	(1.3)
N (Pricing)	839		839									
N (Selection)	15,367		15,367									
N (Index)	49		49		49		49		49		49	

Results

- S&P Listed index has similar beta but much lower returns.
  - Returns to listed PE firms reflect returns to the GP, not the LP
    - GP returns largely driven by fees and carried interest.
  - Large PE firms hold assets other than PE
    - Real Estate, hedge funds, advisory services etc.
  - Some listed funds are funds of funds that charge an extra layer of fees.
  - There is sample selection in the types of funds that choose to list.

- NAV returns are too smooth, not only because they are "stale", but possibly because they overlook variation in market discount rates.
  - We Dimson adjust to account for staleness.

Table V Campbell-Shiller Regressions

This table reports the slope coefficient in Campbell-Shiller regressions as described in the paper,

$$\sum_{j=1}^{k} \bar{r}_{t+j} = a_r + \beta_r \bar{\theta}_t + w_{r,t+k}$$
$$-\sum_{j=1}^{k} \bar{n}_{t+j} = a_n + \beta_n \bar{\theta}_t + w_{n,t+k}$$
$$\varepsilon_{t+k} = a_\varepsilon + \beta_\varepsilon \bar{\theta}_t + w_{\varepsilon,t+k},$$

	Extra Control Variables: No			Extra Control Variables: $\bar{r}_t$, $\bar{n}_t$		
	slope coefficient on $\bar{\theta}_t$	st. err	R^2	slope coefficient on $\bar{\theta}_t$	st. err.	R^2
Panel A. k = 1 quarter						
β_r	0.06	(0.12)	0.01	0.31	(0.10)	0.45
β_n	0.03	(0.04)	0.03	-0.04	(0.03)	0.40
β_ε	0.89	(0.08)***	0.75	0.71	(0.08)	0.84
N	49			49		
Panel B. k = 4 quarters						
β_r	0.45	(0.43)	0.07	0.65	(0.44)	0.12
β_n	0.01	(0.17)	0.00	-0.10	(0.16)	0.09
β_ε	0.83	(0.26)***	0.41	0.56	(0.25)	0.54
N	46			46		
Panel C. k = 20 quarters						
β_r	1.65	(0.50)***	0.54	1.67	(0.51)	0.58
β_n	-0.44	(0.29)	0.22	-0.52	(0.28)	0.31
β_ε	0.05	(0.60)	0.00	-0.25	(0.58)	0.11
N	30			30		

Volatility of Expected Returns

- Expected 5-year return is $\beta_r \theta_t$
- Volatility of θ_t using annual data is 0.22
- Implied volatility of expected 5-year return is $1.65 \times 0.22 = 0.37$.

- Cochrane (2011) estimates volatility of 5-year public market return to be 0.29.

- To make comparable, volatility of levered market portfolio with beta of 1.79 is $1.79 \times 0.29 = 0.52$.

- In comparison, volatility of NAV-based expected return is 0.10.

Synthetic Funds

- For funds in index: GPME is about 0.27, though alpha of index is negative.

- Sort stocks with into deciles by size
- Use data from Prequin to create the cash flows of a "representative fund".
  - Average cash flows across fund in fund-inception time.

- For each decile portfolio
  - A new fund begins every six months from 1980 to 2008
  - Invests all capital in the assigned decile
  - Makes subsequent calls and distributions to mimic the representative fund.
  - Liquidates after 10 years.
  - Last fund liquidates in 2018.
- End result: overlapping cash flows for 590 funds, 59 for each decile.
- Same approach used by Korteweg and Nagle (2016) to create "benchmark funds" to estimate their specific SDF for GPME

Performance Metrics for Synthetic Funds

	Decile	Mean Mcap $MM Y2020	IRR	GPME	*p*-value	SDF alpha	*p*-value	CAPM alpha	*p*-value
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Public Equities 1980-2018	1	17.0	0.27	0.608	(0.00)***	0.099	(0.26)	0.024	(0.02)**
	2	40.7	0.10	0.201	(0.05)**	-0.009	(0.92)	-0.006	(0.46)
	3	81.6	0.10	0.193	(0.05)**	-0.009	(0.91)	-0.008	(0.24)
	4	147.0	0.10	0.308	(0.01)**	0.041	(0.71)	-0.006	(0.34)
	5	256.5	0.11	0.237	(0.02)**	0.059	(0.62)	-0.006	(0.26)
	6	446.9	0.12	0.209	(0.01)***	0.070	(0.54)	-0.003	(0.52)
	7	792.9	0.12	0.142	(0.01)***	0.071	(0.54)	-0.003	(0.38)
	8	1,511.3	0.13	0.113	(0.00)***	0.077	(0.49)	-0.001	(0.83)
	9	3,712.8	0.13	0.066	(0.00)***	0.062	(0.55)	0.000	(0.86)
	10	86,970.4	0.12	-0.040	(0.04)**	0.063	(0.55)	0.001	(0.42)
	N	Average of 538 Firms per Decile	59 Funds per Decile	59 Funds per Decile		157 Obs per Decile		157 Obs per Decile	

Conclusion

- Discount rates vary for PE.
- Discount rate risk premium creates a wedge between GPME and alpha.
- NAVs are too smooth not only because they are stale but because they overlook variation in PE discount rates.
- Listed PE index does not represent return to LPs, and underperforms
- Results using synthetic funds provide similar results.