What Drives Variation in the U.S. Debt/Output Ratio? The Dogs that Didn't Bark*

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Abstract

Higher U.S. government debt/output ratios do not forecast higher future surpluses or lower real returns on Treasurys. Neither future cash flows nor discount rates account for the variation in the current debt/output ratio. The market valuation of Treasurys is surprisingly insensitive to the macro fundamentals. Instead, the future debt/output ratio accounts for most of the variation. Systematic surplus forecast errors may help to account for these findings. Since the start of the GFC, surplus projections have anticipated a large fiscal correction that failed to materialize.

Key words: fiscal policy, debt sustainability, government budget constraint, mispricing
JEL codes: E6, H6, G1

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In light of large increases in the debt/GDP ratio, there is an ongoing debate in the U.S. and other advanced economies about fiscal sustainability (see, e.g., Croce, Nguyen, and Schmid, 2012; Chernov, Schmid, and Schneider, 2020; Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2019; Brunnermeier, Merkel, and Sannikov, 2020; Reis, 2021; Mian, Straub, and Sufi, 2021a). Some economists have pointed out that lower real growth-adjusted returns on government debt (lower nominal returns, higher inflation, or higher growth) can rationalize the recent increase in the U.S. debt/output ratio (Blanchard, 2019; Furman and Summers, 2020; Cochrane, 2021a), while others have argued that the run-up in debt could reflect higher future surpluses (Bohn, 1998; Cochrane, 2020).

To analyze the empirical validity of these claims, we approach the valuation of the U.S. federal government debt using standard tools from asset pricing. We apply a Campbell and Shiller (1988) decomposition to the market value of the U.S. federal government's debt divided by the U.S. gross domestic product. A higher debt/output ratio has to be followed by lower real growth-adjusted returns, higher surplus/output ratios, or, absent adjustment in fundamentals, higher future debt/output ratios.

Our approach delivers a model-free test of fiscal sustainability defined as mean reversion in the debt/output ratio driven by adjustments in future returns or surpluses. At horizons of up to 10 years, our variance decomposition attributes only 8% of the variation in the current debt/output ratio in the post-war U.S. sample to variation in either future surpluses or future returns. Rather, the future debt/output ratio 10 years from now accounts for 92% of the variation in today's debt/output ratio.

First, higher debt/output does not predict lower real growth-adjusted returns on U.S. government debt. Discount rates do not explain variation in the U.S. debt/output ratio. Second, a higher debt/output ratio does not forecast higher surpluses. Cash flows do not explain variation in the debt/output ratio either. Instead, we cannot rule out that the U.S. debt/output ratio follows a unit-root process. Fundamentals fail to push the debt/output ratio back to the mean, which violates fiscal sustainability.[1] The confidence intervals for the discount rate component are narrow around zero, even at longer horizons, allowing us to rule out a significant role for discount rates. The confidence intervals for the cash flow component are wider at longer horizons, implying that we cannot rule out some role for cash flows, but the point estimates suggest a minor role.

Instead, using survey evidence and CBO projections, we find evidence of a role for cash flows under the investors' subjective measure. Investors seem to anticipate larger surpluses in the future as the debt/output ratio rises, but these surpluses do not subsequently materialize. As a result, investors impute more mean-reversion to the debt/output ratio under the subjective measure than

[1] The small role for fundamentals (cash flows and discount rates) is implied by the persistence of the debt/output ratio. The first-order auto-correlation of the debt/output ratio is 0.99 at annual frequencies. As one gets closer to a unit root, the fraction of the variance accounted for by fundamentals decreases to zero percent at all horizons.

the econometrician can detect.

We implement a forward-looking decomposition of the government debt/output ratio which splits the market value of debt into components due to future returns and future surpluses.[2] In earlier work, Hall and Sargent (2011); Berndt and Yeltekin (2015); Hall and Sargent (2021) implement a backward-looking decomposition of the variation in the U.S. debt/GDP ratio, imputing changes in the ratio to contemporaneously realized inflation, growth and returns. Our ex-ante approach provides a different and complementary perspective.

Our findings differ from those in the literature.[3] Studying a shorter sample that ends in the mid-1990s, Bohn (1998) finds evidence that the primary surplus increases when the debt/output ratio is high. Cochrane (2021a,b) finds evidence that the debt/output ratio predicts nominal returns on the government debt portfolio. We find no evidence that the debt/output ratio predicts surpluses nor real growth-adjusted returns over the full post-WW-II sample. Results from a longer sample that starts in 1842 confirm the post-WW-II results. Bond prices today are insufficiently responsive to news about future macro fundamentals, i.e., government surpluses and real growth-adjusted returns.

We reach a different conclusion because of a statistical inference challenge. This issue is well known in the asset pricing literature (Stambaugh, 1999), and at least as relevant in our context. Specifically, there is a small-sample bias in the slope coefficients of the return and surplus predictability regressions, leading one to overstate the degree of predictability. The issue arises because of the combination of (i) the high persistence of the debt/output ratio, the predictor, and (ii), the high correlation between the innovations to the predictor and the predicted variables (surpluses and returns). An increase in bond risk premia will tend to lower realized returns, the dependent variable, and simultaneously lower the market value of debt to output ratio, the predictor. As a result, the expectation of the residual in the return predictability regression conditional on the debt/output ratio this year and last year cannot be zero, violating the classical OLS assumptions. The OLS estimator of the slope coefficient will tend to be too high as a result. Because of this mechanical link between realized returns and the predictor, OLS will find evidence of return predictability, even where there is none. The same small-sample bias also affects the surplus predictability regressions. Governments must issue more debt when they run large deficits. The negative correlation between the innovation in the debt/output ratio and the surplus similarly produces spurious evidence of surplus predictability. The unbiased coefficients

[2] See also Berndt, Lustig, and Yeltekin (2012); Cochrane (2021a). In earlier work, Gourinchas and Rey (2007) decompose the variation in the U.S. net foreign asset position using the same approach.

[3] There is a large literature in macro-economics that addresses the question of government debt sustainability (Hamilton and Flavin, 1986; Trehan and Walsh, 1988, 1991; Bohn, 1998, 2007; Croce, Nguyen, and Schmid, 2012; Croce, Kung, Nguyen, and Schmid, 2012; Croce, Nguyen, and Raymond, 2021; Mian, Straub, and Sufi, 2021b), starting with the seminal work by Hansen and Sargent (1980); Hansen, Roberds, and Sargent (1991). Sargent (2012) and D'Erasmo, Mendoza, and Zhang (2016) provide comprehensive reviews of the literature. This literature largely sidesteps the issue of discount rate variation by assuming constant discount rates.

display substantially weaker surplus and return predictability than what is indicated by the OLS estimates.

Despite the clear evidence in terms of the bias-adjusted point estimates, the standard errors point to the need to interpret the results cautiously. Large standard errors result from the low power of unit root tests in short samples against close-to-unit-root alternatives (Schwert, 1987; Lo and MacKinlay, 1989, in finance). Confidence intervals shrink for a longer sample that starts in 1842, and strengthen the evidence against debt adjustment via growth-adjusted returns. The statistical evidence against the discount rate channel is strong at all horizons. Even for this long sample, standard errors on the surplus predictability coefficients remain substantial. Hence, we cannot rule out some role for the cash flow channel at long horizons. Given the higher short-run volatility of surpluses than of Treasury returns, what we can say is that, if there is mean reversion in the debt/output ratio, then debt adjustment is more likely to occur via adjustment of surpluses than via adjustment of real growth-adjusted returns. There is no statistical evidence that the recent increase in the U.S. debt output ratio can be ascribed to lower future cost of capital for the government or to higher future growth.

Taken together, the evidence indicates that the debt/output ratio may be subject to permanent innovations, inconsistent with fiscal sustainability. As corroborating evidence, we simulate data from a model under the null hypothesis that the debt/output ratio has a unit root, i.e., a model where fundamentals do not adjust to the debt/output ratio. This model exactly replicates our empirical findings. We find spurious evidence of surplus and return predictability using unadjusted OLS slope estimates. The bias-corrected estimates recover the true values that of no predictability.[4]

One type of permanent shocks to the debt/GDP ratio are structural breaks. There may have been a structural break in the relation between the debt/output ratio, growth-adjusted returns, and surpluses. In stock markets, there also is evidence of structural breaks in the relation between the dividend yield, stock returns, and dividend growth (see Lettau and Van Nieuwerburgh, 2008; Smith and Timmermann, 2020).[5] We estimate a structural break in the debt/output ratio in 2007. The mean debt/output ratio is 0.78 log points higher after 2007 than before 2007. We define the transitory component of the debt/GDP ratio as the raw debt/GDP series minus the different subsample mean before and after 2007. Using this transitory debt/GDP series, we find stronger evidence for surplus predictability but not return predictability. Fundamentals now account for about 50% of the variation in the transitory component of the debt/output ratio at the 10-year horizon. Removing a structural break removes a low-frequency component in the debt/output ratio and creates more room for fundamentals in explaining the–now more transitory nature of the—variation in the debt/output ratio. The resulting transitory debt/GDP ratio is less persis-

[4]Croce, Nguyen, and Schmid (2012); Croce, Nguyen, and Raymond (2021) warn about the high persistence of the U.S. debt/output ratio and explore the effects on fiscal uncertainty in an equilibrium model.

[5]Such structural breaks can lead to instability in the predictive coefficients (Lettau and Van Nieuwerburgh, 2008) and may help to account for the poor out-of-sample performance of these predictors (Goyal and Welch, 2008).

tent; predictive coefficients have smaller small-sample biases. Importantly, this analysis leaves the large, permanent increase in the debt/output ratio (as well as its timing) unexplained.

We identify surplus forecast errors as a potential contributor the low-frequency change in the relationship between debt and fundamentals around 2007. An econometrician with access to the U.S. sample does not predict higher surpluses or lower returns when the debt/output ratio rises, but a bond investor may under her subjective beliefs. If bond investors systematically over-predict surpluses and/or under-predict returns as the debt/output ratio increases, this could impute a unit root the debt/output ratio under the actual measure, while generating a stationary debt/output ratio and surplus and/or return predictability under the subjective measure. We show evidence from Congressional Budget Office (CBO) projections that the structural break in the debt/output ratio has been partly fueled by persistently biased forecasts of surpluses. These projections impute more mean-reversion to the predicted debt/output ratio under the subjective than under the actual measure.[6] After 2007, the CBO consistently overestimated future surpluses and underestimated the government's effective cost of funding. For example, in 2010, which is after the GFC, the CBO projections of 10-year cumulative surpluses were overshooting realized surpluses by roughly 3.6% of GDP per annum. Under the investors' subjective measure, more of the variation in the debt/output ratio can be attributed to perceived future surpluses.

Another candidate explanation for the low-frequency change in the relationship between debt and fundamentals is the rise in foreign and Central Bank purchases of U.S. Treasurys. Removing foreign and Federal Reserve holdings of Treasuries results in a particularly large adjustment after 2007. If we use the remaining domestic private sector holdings of debt scaled by GDP as the predictor, we find larger surplus predictability, similar to the evidence that uses the transitory component of the debt/output ratio as predictor. This suggests that inelastic demand by the Fed and the rest of the world may be a contributing factor to the insensitivity of government bond prices to fundamentals. We leave this question for future research.

Our paper builds on the growing literature that examines the predictability of returns in asset markets. There is substantial evidence that stock returns are predictable (see Koijen and Nieuwerburgh, 2011, for a survey of the literature on stock return predictability). When the valuations are high in stock markets, they revert back to the mean mostly through lower subsequent returns (Campbell and Thompson, 2007; Cochrane, 2008), partly through higher dividend growth (Binsbergen and Koijen, 2010; Golez and Koudijs, 2018). Overall, the discount rates on stocks are remarkably volatile (Hansen and Jagannathan, 1991), and the valuation of stocks seems excessively volatile compared to its fundamentals (LeRoy and Porter, 1981; Shiller, 1981). This evidence that high valuations revert back to the mean through low subsequent returns is common across most

[6] The CBO projections do not take into account any future legislative action that could implement a fiscal correction. The source of bond market investor optimism about future surpluses may be the expectation of future fiscal rectitude through such legislation. A comparison of the average private-sector forecast of one-year ahead surpluses from Consensus Economics to the CBO forecast shows that the two forecasts are close in both levels and dynamics.

asset classes. Not so for the entire U.S. government bond portfolio. Even though there is evidence of return predictability for individual bonds (see Fama and Bliss, 1987; Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005; Ludvigson and Ng, 2009; Cochrane, 2011), we find no evidence that lower returns on the entire bond portfolio push the debt/GDP ratio back to its long-run mean following a high value. The fluctuations in the market value of the entire U.S. government bond portfolio is too small compared to the fluctuations in its fundamentals; there is excess smoothness as opposed to excess volatility in the stock market. The cash flows and the discount rates are the dogs that did not bark when there is news about future surpluses or returns. Instead, we only find a statistically significant role for the future debt in accounting for the entire value of debt today.

Our evidence on forecast errors in the surplus/GDP ratio connects to the growing literature on the role of subjective beliefs in asset pricing (Barberis, 2018; Bordalo, Gennaioli, LaPorta, and Shleifer, 2019; Bordalo, Gennaioli, Ma, and Shleifer, 2020). For the stock market, De La O and Myers (2021); Wang (2020) impute a much larger share of variation in the price/dividend ratio to the cash flow component under subjective than under objective beliefs. Mistakes in interest rate and growth forecasts also contribute to our findings. Piazzesi, Salomao, and Schneider (2015); Cieslak (2018) document evidence on the gap between statistical risk premia measured by econometricians and subjective risk premia expected by investors in the bond market.

Our paper does not impose no-arbitrage restrictions, but only uses accounting identities to understand variation in the debt/output ratio, rather than the level. In other work, Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019) price a claim to future surpluses in a no-arbitrage model, and they conclude that the government bond portfolio is overpriced. In no-arbitrage models, all of the information for forecasting bond returns is embedded in the yield curve, except when the underlying factors cannot be inverted from the yields (see Duffee, 2011). Some authors report evidence that macro factors have incremental forecasting ability for bond returns (Cooper and Priestley, 2008; Ludvigson and Ng, 2009; Joslin, Priebsch, and Singleton, 2014). Bauer and Hamilton (2017) conclude that the evidence for macro variables predicting bond returns, after controlling for bond yields, against the spanning hypothesis, is weaker than previously thought, citing similar small-sample biases in the presence of persistent predictors.

There is a large literature on the pitfalls of predicting returns with persistent predictors, going back to Nelson and Kim (1993); Stambaugh (1999); Lewellen (2004); Torous, Valkanov, and Yan (2004); Campbell and Yogo (2006); Boudoukh, Israel, and Richardson (2020). The Stambaugh correction can be refined if the econometrician rules out ex ante that the predictor variable has a true autocorrelation larger than one (Lewellen, 2004).[7] We do not rule out non-stationary behavior of the debt/output ratio, because we are interested in testing fiscal sustainability. We implement the Campbell and Yogo (2006) testing procedure, valid under general assumptions, which confirms

[7] If the econometrician imposes that the true autocorrelation of the debt/output ratio is smaller than one, that effectively implies that either surpluses or returns are predictable.

that there is neither evidence of surplus nor return predictability.

Section 1 derives and estimates the decomposition of the debt/output ratio, with and without small-sample bias adjustment. Section 2 allows for permanent shocks to the debt/output ratio. Finally, section 3 studies two sources of permanent shocks to the debt/output dynamics: foreign and Fed holdings and systematic forecast errors. Section 4 concludes.

1 What Drives Variation in the U.S. Debt/Output Ratio?

We use the log-linear Campbell and Shiller (1988) decomposition of the government's debt/output ratio in Cochrane (2021a,b). This approach decomposes the variation in the debt/output ratio into three components: expected future government surpluses (cash flows), expected future real growth-adjusted bond returns (discount rates), and the expected future value of the debt/output ratio. After we correct for the Stambaugh (1999) bias, we find that most of the variation in the debt/output ratio cannot be attributed to cash flows or discount rates over the next 10 years.

1.1 Realized Treasury Returns and Surpluses

We compute the market value of all marketable U.S. Treasurys using data from CRSP Treasurys. Following Hall and Sargent (2011), we compute the return on government debt as the sum of the principal and coupon payments less new issuance, plus the market value at the end of the period, divided by the market value at the end of the previous period. We exclude non-marketable debt which is mostly held in intra-governmental accounts. The inflation rate is the annual log change in the CPI, taken from the BLS. Nominal GDP is from the Bureau of Economic Analysis. Our sample of annual data comprises the period from 1947 to 2020.

Let r_{t+j} denote the nominal return on the government debt portfolio (in logs), x_{t+j} real GDP growth (in logs), and π_{t+j} log inflation. Let $\widetilde{r}_{t+j} = r_{t+j} - x_{t+j} - \pi_{t+j}$ denote the adjusted log bond return, which is the nominal return on the government bond portfolio minus output growth and inflation. We refer to $\widetilde{r}$ as the "real growth-adjusted return" or simply the "return" when there is no scope for confusion. Let $s_{t+j} = sy_{t+j}/exp(v)$ denote the adjusted government surplus/output ratio, where sy_t denotes the actual surplus/output ratio and v denotes the average log debt/output ratio.

We start from the log-linearized return equation implied by the government budget constraint:

$$\widetilde{r}_{t+1} = r_{t+1} - \pi_{t+1} - x_{t+1} = \rho v_{t+1} - v_t + s_{t+1},$$

where ρ is a constant of linearization. We set $\rho = \exp(-(r - x - \pi)) = 1$. This decomposition expands the debt/output ratio around the unconditional average $r = x + \pi$.[8] We back out the

[8] The details of the derivation are in the appendix of Cochrane (2021a) on pages 3 and 4. We do robustness to different

surplus variable s_t from this equation:

$$s_{t+1} \equiv \tilde{r}_{t+1} - \Delta v_{t+1}. \quad (1)$$

Table 1: Summary Stats: Decade-by-Decade Averages

	$\tilde{r}$	r	x	π	$x+\pi$	sy
1950-1959	-3.8%	2.7%	4.1%	2.4%	6.5%	1.6%
1960-1969	-2.8%	3.9%	4.4%	2.3%	6.7%	2.0%
1970-1979	-2.5%	7.0%	3.2%	6.3%	9.5%	-1.4%
1947-1979	-3.5%	3.9%	3.6%	3.8%	7.4%	0.3%
1980-1989	4.1%	11.8%	3.0%	4.6%	7.6%	-2.2%
1990-1999	1.6%	6.9%	3.2%	2.2%	5.3%	1.9%
2000-2009	0.8%	4.9%	1.9%	2.2%	4.1%	-3.3%
2010-2019	-1.3%	2.6%	2.2%	1.7%	3.9%	-5.5%
2010-2020	-0.4%	2.9%	1.7%	1.6%	3.3%	-5.5%
1980-2019	1.3%	6.5%	2.6%	2.7%	5.2%	-2.3%
1980-2020	1.5%	6.5%	2.4%	2.6%	5.1%	-2.7%
1947-2019	-0.9%	5.4%	3.0%	3.2%	6.2%	-1.1%
1947-2020	-0.7%	5.4%	3.0%	3.2%	6.1%	-1.3%

Table 1 reports the decade-by-decade averages for these variables. The evidence for $r < x + \pi$ is concentrated in the first half of the sample. The real growth-adjusted returns on the entire bond portfolio, reported in the first column, are strongly negative in the 1950s (-3.8%), due to low real returns and strong economic growth. This pattern continues in the 1960s, with real growth-adjusted returns of -2.6%. In the 1970s, bondholders were surprised by persistently high inflation, delivering growth-adjusted returns of -2.5%. Overall, in the first three decades after WW-II, the average gap between returns and growth $\tilde{r}$ is -3.5%; r was much lower than $x + \pi$. In the next four decades, the average real growth-adjusted return $\tilde{r}$ is positive (1.5%); r was higher than $x + \pi$. The 1980s represents a radical departure from what came before. Bondholders earned high real returns despite high inflation (4.1%). Even in the 1990s and 2000s, bondholders earned high real growth-adjusted returns of 1.6% and 0.8%, respectively.

Surpluses, reported in the last column, are modestly positive in the 1950s, 1960s, and 1990s, and negative in the five other decades. Surpluses are most negative over the past twenty years. For the entire post-war sample, we see that the "debt deflation" from negative real growth-adjusted returns $\tilde{r}$ of less than -1% per year in absolute value is more than offset by the deficits in excess of 1% per year in the last column. In this paper, we take a forward-looking perspective and study how debt dynamics affect the variation in long-run future surpluses and returns shown in Table 1.

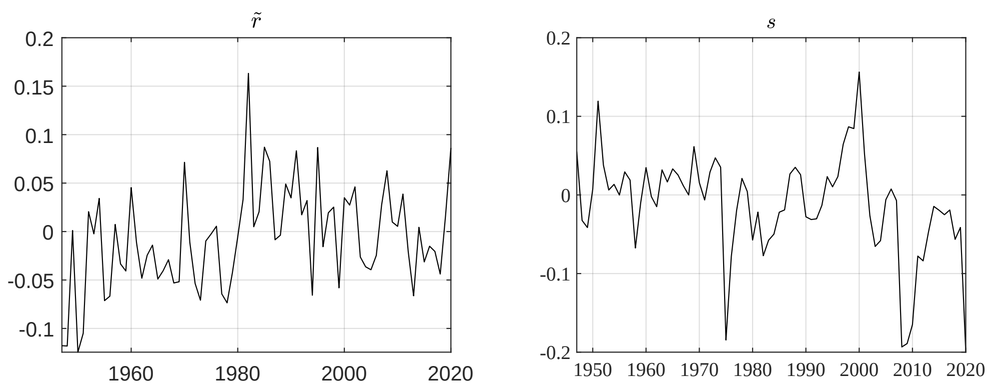
Figure 1 plots the variables we want to forecast: the real growth-adjusted returns $\tilde{r}_t$ in the left

values of ρ in section 1.6.3.

panel and the surplus s_t in the right panel. As discussed above, returns were low from the start of the sample until the 1980s. After 2000, real growth-adjusted returns declined again. The surpluses s are high in the 1950s and 1990s and decline precipitously in the last two decades. We note that the surplus s_t, which is backed out from equation (1), has a high correlation of 83% with the raw surplus/output ratio. The implied surplus s_t is more volatile (standard deviation of 6.47%) than the actual surplus/output ratios (standard deviation of 2.6%).

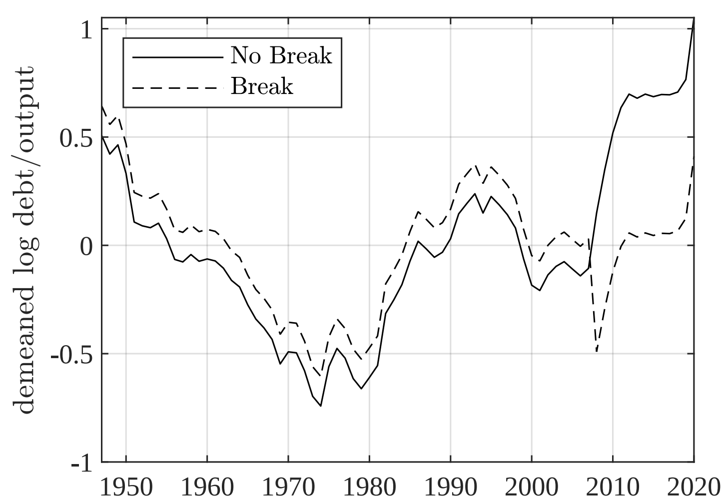
The solid line in Figure 2 is the predictor variable, the demeaned log debt/output ratio. There was a marked decline in the debt/output ratio between 1948 and 1974, followed by a gradual

Figure 1: Debt Returns and Government Surpluses



This figure plots the inflation-and-growth-adjusted log debt returns $\tilde{r}_t$ and the surplus/output ratio s_t.

Figure 2: Debt/Output Ratio



The full line is the demeaned log debt/output ratio. The dashed line is the demeaned log debt/output ratio, demeaned by two different sub-sample means before and after 2007.

increase in the eighties and early nineties, and after a brief period of decline, a new surge in the wake of the GFC (after 2007).

1.2 Campbell-Shiller Decomposition of the Debt/Output Ratio

By iterating forward on equation (1) and taking expectations, we obtain the following expression for the debt/output ratio:

$$v_t = \mathbb{E}_t \sum_{j=1}^{T} \left(s_{t+j} - \widetilde{r}_{t+j} \right) + \mathbb{E}_t v_{t+T}. \tag{2}$$

The debt/output ratio reflects either expected future surpluses, expected future real growth-adjusted returns, or the expected future debt/output ratio. Since we only iterate forward on the budget constraint T times, we do not impose the transversality condition (TVC).[9]

The decomposition in equation (2) allows for the presence of convenience yields on government debt.[10] Convenience yields would lower $\widetilde{r}$.

Taking covariances with v_t on both sides of the previous equation, we obtain the following expression for the variance of the debt/output ratio:

$$var(v_t) = cov\left(\sum_{j=1}^{T} s_{t+j}, v_t\right) - cov\left(\sum_{j=1}^{T} \widetilde{r}_{t+j}, v_t\right) + cov(v_t, v_{t+T}). \tag{3}$$

The log debt/output ratio varies because it either predicts future surpluses, future returns, or the future debt/output ratios. In other words, the adjustment to an increase in the debt/output ratio either happens through an increase in future surpluses or a decrease in real growth-adjusted returns, or the debt is simply rolled over to T periods from now. If v_t is non-stationary, then we replace the unconditional population moments with the finite-sample moments in the variance decomposition:[11]

$$\hat{var}_N(v_t) = \hat{cov}_N\left(\sum_{j=1}^{T} s_{t+j}, v_t\right) - \hat{cov}_N\left(\sum_{j=1}^{T} \widetilde{r}_{t+j}, v_t\right) + \hat{cov}_N(v_t, v_{t+T}). \tag{4}$$

[9] Imposing a TVC requires taking a stand on the right discount rate is to eliminate the terminal value v_{t+T} term (see Giglio, Maggiori, and Stroebel, 2016; Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2019). There are equilibrium models that generate violations of the TVC including Samuelson (1958); Diamond (1965); Blanchard and Watson (1982); Hellwig and Lorenzoni (2009). Most of these models abstract from aggregate risk premia which would be priced into the terminal value and are likely to enforce the TVC (Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2020; Barro, 2020). If the TVC is satisfied and if v_t is stationary, then $\mathbb{E}_t[v_{t+T}]$ converges to its unconditional mean.

[10] In models developed by Bassetto and Cui (2018); Brunnermeier, Merkel, and Sannikov (2020); Reis (2021); Chien and Wen (2019, 2020); Kocherlakota (2021); Mian, Straub, and Sufi (2021a), government debt allows agents to self-insure against idiosyncratic risk and provides liquidity services. The resulting convenience yield contributes a component to the valuation of public debt.

[11] If v_t is non-stationary, the unconditional variance is infinite. However, in any finite sample, we can still compute the sample moments and report the in-sample variance decomposition using sample moments.

To compute the variance decomposition, we directly estimate a system of univariate forecasting regressions for $\sum_{j=1}^{T} s_{t+j}$, $\sum_{j=1}^{T} \widetilde{r}_{t+j}$, and v_{t+T} using the lagged debt/output ratio v_t as a predictor:

$$\begin{aligned} \sum_{j=1}^{T} s_{t+j} &= a_s + b_T^s v_t + \varepsilon_{t+T}^s, \\ \sum_{j=1}^{T} \widetilde{r}_{t+j} &= a_r + b_T^r v_t + \varepsilon_{t+T}^r, \\ v_{t+T} &= \phi_0 + \phi_T v_t + \varepsilon_{t+T}^v. \end{aligned} \tag{5}$$

Cochrane (2008); Lettau and Van Nieuwerburgh (2008) adopt the same approach to implementing a Campbell-Shiller decomposition of the price/dividend ratio for stocks. Our claim is not that the debt/output ratio is the only predictor of bond returns or surpluses. We simply want to compute the covariance between the debt/output ratio and future returns and surpluses. We estimate separate univariate regressions over longer horizons to infer the long-run dynamics rather than constraining the predictability across horizons through a VAR.[12]

Just as their counterparts from predictability regressions using stock returns, these regression coefficients measure the fraction of the variance in the debt/output ratio at each horizon T that is attributable to each component: future surpluses, future growth-adjusted returns, and future debt/output ratios:

$$\begin{aligned} \frac{cov(\sum_{j=1}^{T} s_{t+j}, v_t)}{var(v_t)} &= b_T^s, \\ \frac{cov(-\sum_{j=1}^{T} \widetilde{r}_{t+j}, v_t)}{var(v_t)} &= -b_T^r, \\ \frac{cov(v_{t+T}, v_t)}{var(v_t)} &= \phi_T. \end{aligned}$$

Note that the three predictability coefficients sum to one at each horizon. The cross-equation restriction $b_T^s - b_T^r + \phi_T = 1$ is automatically satisfied, so that these three components jointly explain 100% of the variation in the debt/output ratio v_t.

To develop intuition for how the persistence in the debt/output ratio impacts the variance decomposition, consider the case of an AR(1) process for the debt/output ratio with persistence parameter ϕ_1. The variance decomposition is then given by $b_1^s(1-\phi_1)^T/(1-\phi_1)$, $-b_1^r(1-\phi_1)^T/(1-\phi_1)$, and ϕ_1^T. If the debt/output approaches a unit-root process ($\phi_1 \to 1$), v_{t+T} accounts for all of the variation in v_t at horizon T.

We define fiscal sustainability over a finite horizon T as $\phi_T < 1$. We define fiscal sustainability

[12]Long-run VAR forecasts are more susceptible to misspecification of the dynamics (Jordà, 2005).

over long horizons as $\lim_{T\to\infty}\phi_T = 0$. If this condition is not satisfied, then the surplus/output ratio will inherit a unit root, which means that surpluses can drift arbitrarily far apart from output, which is not fiscally sustainable.

What do we expect to find in this variance decomposition? To anchor ideas, consider a world where the debt/output ratio is stationary ($\phi_T < 1$). In this world, the entire variance of ex-post realized returns is attributable to cash flows in the long run (see Appendix B):

$$\lim_{k\to\infty}\frac{1}{k}var[\tilde{r}_{t\to t+k}] = \lim_{k\to\infty}\frac{1}{k}var[s_{t\to t+k}]. \tag{6}$$

A similar equation for stocks implies that, if the dividend-price ratio is stationary, the variance of stock returns is driven only by dividend growth in the long run. Consider two polar cases in this world with a stationary debt/gdp ratio.

In the first case, growth-adjusted returns are unpredictable. If growth-adjusted returns are i.i.d., with a variance ratio of one at all horizons (including $\lim_{k\to\infty}\frac{1}{k}var[\tilde{r}_{t\to t+k}] = var[\tilde{r}_t]$), then equation (6) dictates that there is mean reversion in the surpluses, i.e., the variance ratio of surpluses is smaller than one at long horizons,

$$\lim_{k\to\infty}\frac{1}{k}var[s_{t\to t+k}] < var[s_t]$$

provided that the returns are less volatile than the surpluses in the short run ($var[\tilde{r}_t] < var[s_t]$). The only channel pushing the debt/output ratio back to the mean following an increase in the debt/output ratio is through larger surpluses: $b^s_T = 1 - \phi_T > 0$. In the 1947-2020 sample, the standard deviation of s_t is 6.47% which exceeds the standard deviation of $\tilde{r}_t$ of 5.30%. The small difference in short-run variances of surpluses and returns suggests only weak mean reversion.

In the second case, surpluses are unpredictable. If the surpluses are i.i.d., then returns must do the adjustment to bring the debt/output ratio back down after an increase: $-b^r_T = 1 - \phi_T > 0$. However, given the empirical constellation of short-run variances of surpluses and returns, the stationarity of the debt/output ratio would imply long-run mean *aversion* in returns, which is at odds with the data.

This back-of-the-envelope evidence on the relative volatility of surpluses versus returns seems to rule out a world with a stationary debt/output ratio and no surplus predictability, but potentially allows for a world with a stationary debt/output ratio and no return predictability.[13]

If there is no predictability of surpluses nor of real growth-adjusted returns, then the debt/output ratio inherits a unit root, which violates fiscal sustainability.

[13] Treasurys are different from stocks. In the case of stocks, the short run variance of returns exceeds that of dividend growth. I.i.d. dividend growth leads to mean reversion in returns, while i.i.d. stock returns would lead to mean aversion in dividend growth. In the case of stocks, the prima facie evidence rules out a world without return predictability, because the short run volatility of returns is so high.

1.3 Variance Decomposition of the U.S. Debt/Output Ratio

Bond investors care about the government's ability to roll over the debt. Given that the average maturity of the U.S. government debt portfolio has hovered around 5-7 years, and that only about 13% of outstanding government debt has maturities longer than 10 years,[14] the U.S. Treasury will typically roll over the entire debt in less than a decade, and, hence, studying predictability at horizons up to 10 years is natural.

Figure 3 plots the baseline OLS regression results. Each panel plots the OLS slope coefficients in (5), where the dependent variable is future surpluses (top left), future real growth-adjusted returns (top right), and future debt/output ratios (bottom left) for horizons from $T = 1$ to $T = 10$ years. The bottom right panel has surpluses minus growth-adjusted returns as the dependent variable. We will refer to this dependent variable as "fundamentals." Table 2 reports these same predictability results in table format.

At the one-year horizon, 101% of the variance is attributed to the next year's debt/output ratio. That is to say, the log debt/output ratio is highly persistent. The first-order autocorrelation is 1.01.[15] Even at the five-year horizon, 83% of the debt/output ratio fluctuations can be attributed to the future debt/output ratio. The R^2 in this debt/output predictability regression exceeds 50% at the five-year horizon. At the ten-year horizon, the other two channels start to matter: 39% of the variation is attributed to fluctuations in expected future surpluses and 25% of the variation is attributed to fluctuations in expected future returns. The terminal debt/output ratio in the ten-year horizon accounts for the remaining 36% of the variation.

Why is the discount rate channel so weak? The debt/output ratio does predict nominal bond returns with the right sign, as emphasized by Cochrane (2021a): higher debt/output predicts lower nominal returns. This effect is mechanical because bond prices are determined by the nominal log returns over the maturity of the bond: $p_t^N = -\sum_{i=0}^{N-1} hpr_{t+i+1}^{N-i}$. But the predictability effect on nominal returns is almost completely offset by a similar-sized predictability effect on inflation, as shown in the third panel of Table 3. The net effect of the debt/output ratio on future real growth-adjusted returns, shown in the bottom panel, is small and statistically insignificant.

To generate confidence intervals for these predictability coefficients, we construct standard errors by bootstrapping 10,000 samples from the joint residuals in a regression of (r_t, v_t) on two lags of v_t. We draw the surplus such that the budget constraint identity is enforced. Here is the full system of equations for the bootstrap:

$$\begin{aligned} v_{t+1} &= \psi_0 + \psi_1 v_t + \psi_2 v_{t-1} + u_{t+1}^v, \\ \widetilde{r}_{t+1} &= a_r + b_1^r v_t + b_2^r v_{t-1} + u_{t+1}^r, \end{aligned}$$

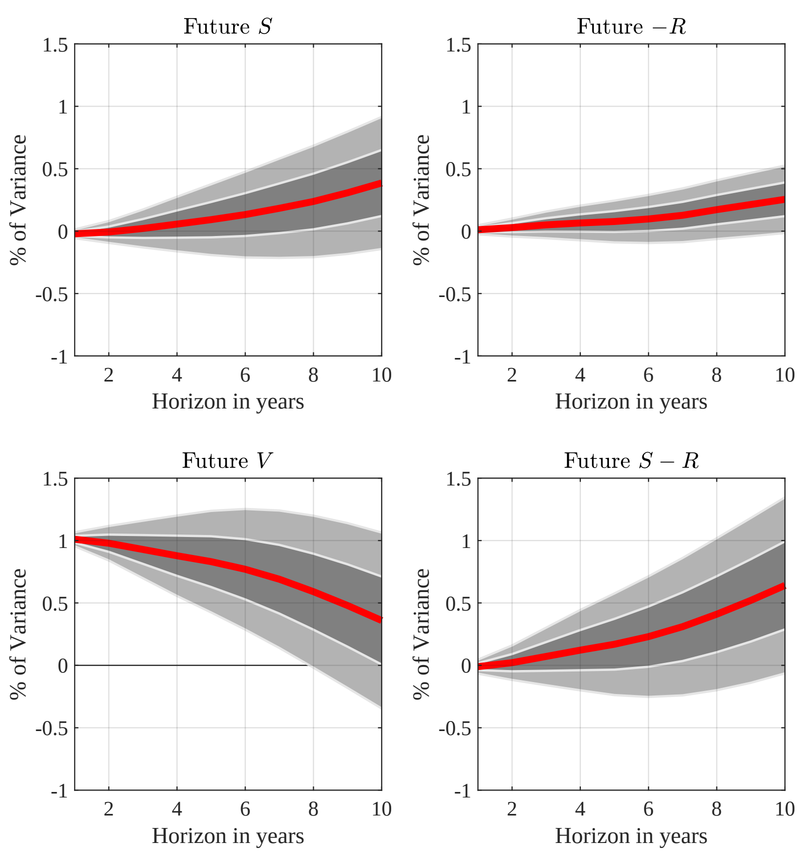
[14] Average for the (longest available) sample from 2000–20 of data on the maturity distribution of U.S. Treasury Debt.

[15] Using the Augmented Dickey-Fuller test, we cannot reject the null hypothesis of the presence of the unit root in the log debt/output ratio in our sample period. However, we need to acknowledge the low power of such unit root tests.

$$s_{t+1} = \widetilde{r}_{t+1} - \Delta v_{t+1} \tag{7}$$

The bootstrapped standard errors are under the null that the debt/output ratio is mean-reverting and there is no role for cash flow or discount rates to affect the debt dynamics. For bootstrapping purposes, we use two lags because the debt/output ratio in the data fits an AR(2) structure well, and delivers white noise estimated residuals to bootstrap from. The confidence intervals (CIs) imply considerable uncertainty about the point estimate. Even at the ten-year horizon, the two-standard-deviation CIs for the b_j^s and b_j^r coefficients contain zero, so that the explanatory power of future surpluses and growth-adjusted returns are indistinguishable from zero. The confidence intervals for the coefficients in the growth-adjusted return predictability equation are substantially smaller than for the coefficients in the surplus predictability equation.

Figure 3: Variance Decomposition of Log Debt/Output Ratio



This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future government surpluses $\sum_{j=1}^{T} s_{t+j}$, future real growth-adjusted returns (with a minus sign) $\sum_{j=1}^{T} -\widetilde{r}_{t+k}$, future log debt/output ratio v_{t+T}, and the difference of the future surpluses and returns. Sample is annual, 1947—2020. We plot 1 s.e. (dark) and 2 s.e. (light) confidence intervals. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t.

Table 2: Forecasting Returns and Surpluses with log Debt/Output ratio

OLS Regression of $\sum_{j=1}^{T} s_{t+j}, \sum_{j=1}^{T} \widetilde{r}_{t+j}, v_{t+T}$ on v_t. Annual data. Sample: 1947—2020. Standard errors generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t, and we set $s_{t+1} = \widetilde{r}_{t+1} - \Delta v_{t+1}$.

Horizon	1	2	3	4	5	6	7	8	9	10
					Forecasting $\sum_{j=1}^{T} -\widetilde{r}_{t+j}$					
$-b_T^r$	0.01	0.03	0.05	0.07	0.08	0.1	0.13	0.17	0.21	0.25
s.e.	0.02	0.04	0.05	0.07	0.08	0.09	0.11	0.12	0.13	0.13
R^2	0.01	0.02	0.03	0.04	0.04	0.05	0.06	0.08	0.1	0.12
unbiased	-0.01	-0.02	-0.02	-0.03	-0.04	-0.04	-0.04	-0.01	0	0.02
					Forecasting $\sum_{j=1}^{T} s_{t+j}$					
b_T^s	-0.02	-0.01	0.02	0.06	0.09	0.13	0.18	0.24	0.31	0.39
s.e.	0.02	0.04	0.08	0.11	0.14	0.17	0.2	0.22	0.24	0.26
R^2	0.02	0	0	0.01	0.02	0.03	0.05	0.06	0.09	0.11
unbiased	-0.05	-0.07	-0.08	-0.07	-0.07	-0.06	-0.05	-0.03	0.01	0.05
					Forecasting v_{t+T}					
ϕ	1.01	0.98	0.93	0.88	0.83	0.77	0.69	0.59	0.48	0.36
s.e.	0.03	0.07	0.11	0.16	0.2	0.24	0.27	0.3	0.33	0.35
R^2	0.95	0.85	0.74	0.64	0.54	0.43	0.32	0.22	0.13	0.07
unbiased	1.07	1.09	1.1	1.1	1.11	1.11	1.08	1.04	0.99	0.92

Table 3: Forecasting Nominal Returns and Inflation with Log Debt/Output Ratio

This table further decomposes the adjusted bond return $\widetilde{r}_{t+j}$ into its three components: nominal bond return r_{t+j}, GDP growth x_{t+j}, and inflation π_{t+j}. The adjusted bond return $\widetilde{r}_{t+j}$ is equal to $r_{t+j} - x_{t+j} - \pi_{t+j}$. We run OLS Regression of $\sum_{j=1}^{T} r_{t+j}, \sum_{j=1}^{T} x_{t+j}, \sum_{j=1}^{T} \pi_{t+j}$ on $\widetilde{v}_t$. Annual data. Sample: 1947—2020. Standard errors generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, x_t, π_t, v_t) on two lags of v_t.

Horizon	1	2	3	4	5	6	7	8	9	10
					Forecasting $\sum_{j=1}^{T} r_{t+j}$					
b_T^r	-0.05	-0.1	-0.16	-0.22	-0.28	-0.34	-0.42	-0.5	-0.59	-0.67
s.e.	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.08	0.09	0.11
R^2	0.19	0.28	0.34	0.38	0.41	0.44	0.47	0.5	0.53	0.55
unbiased	-0.04	-0.08	-0.12	-0.17	-0.21	-0.26	-0.32	-0.4	-0.47	-0.54
					Forecasting $\sum_{j=1}^{T} x_{t+j}$					
b_T^x	0	0	0	0	0	0	0	0.01	0.01	0.02
s.e.	0.01	0.02	0.02	0.03	0.04	0.05	0.05	0.06	0.07	0.07
R^2	0	0	0	0	0	0	0	0	0	0.01
unbiased	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	-0.07	-0.07	-0.07	-0.08
					Forecasting $\sum_{j=1}^{T} \pi_{t+j}$					
b_T^π	-0.04	-0.07	-0.11	-0.15	-0.2	-0.24	-0.29	-0.34	-0.39	-0.44
s.e.	0.01	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
R^2	0.44	0.51	0.54	0.55	0.59	0.62	0.63	0.63	0.63	0.63
unbiased	-0.04	-0.07	-0.11	-0.15	-0.2	-0.24	-0.29	-0.34	-0.39	-0.44
					Forecasting $\sum_{j=1}^{T} \widetilde{r}_{t+j}$					
$b_T^{\tilde{r}}$	-0.01	-0.03	-0.05	-0.07	-0.08	-0.1	-0.13	-0.17	-0.21	-0.25
s.e.	0.02	0.04	0.06	0.07	0.09	0.1	0.11	0.12	0.13	0.14
R^2	0.01	0.02	0.03	0.04	0.04	0.05	0.06	0.08	0.1	0.12
unbiased	0.01	0.02	0.02	0.03	0.04	0.04	0.04	0.01	0	-0.02

1.4 Small-Sample Bias Correction

The high persistence in the explanatory variable v_t raises concern of small-sample bias. The OLS slope coefficients in the return predictability relationship, $|b_j^r|$, are biased upwards in absolute value, because the innovations to the returns ε^r are positively correlated with the regressor innovations ε^v, and the regressor is highly persistent (Stambaugh, 1999). Intuitively, an increase in bond risk premia will tend to induce lower realized returns and lower the ratio of the market value of debt to output. That gives rise to positive correlation between the regressor innovations and the return innovations. The positive biases tend to increase for long-horizon predictability regressions (Boudoukh, Israel, and Richardson, 2020).

We find a similar positive bias for the surplus predictability regression. An increase (decrease) in debt issuance tends to coincide with the government running large deficits (surpluses). As a result, there is a strong, negative correlation between ε^s and the regressor innovations ε^v. This, combined with the persistence of the debt/output ratio, induces a large upward bias in b_T^s as well.

In both case, the bias leads us to find too much predictability in small samples. One can construct unbiased coefficients by applying the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in the predictability regression with horizon T:

$$
\begin{aligned}
bias_T^r &= \mathbb{E}\left(\widehat{b}_T^r - b_T^r\right) = \frac{1}{N}\left[T(1+\phi) + 2\phi\frac{1-\phi^T}{1-\phi}\right] \times -\frac{cov(\varepsilon^v, \varepsilon^r)}{var(\varepsilon^v)}, \\
bias_T^s &= \mathbb{E}\left(\widehat{b}_T^s - b_T^s\right) = \frac{1}{N}\left[T(1+\phi) + 2\phi\frac{1-\phi^T}{1-\phi}\right] \times -\frac{cov(\varepsilon^v, \varepsilon^s)}{var(\varepsilon^v)},
\end{aligned}
$$

where ϕ denotes the first-order autocorrelation of v_t , T denotes the predictability horizon, and N denotes the size of the sample. We note that $corr(\varepsilon^v, -\varepsilon^r) = -0.75$ and $corr(\varepsilon^v, \varepsilon^s) = -0.85$, so the implied biases for the coefficient b_T^s associated with surplus and for the coefficient $-b_T^r$ associated negative returns are both positive. OLS will tend to overstate the importance of both the cash flow and the discount rate component in accounting for the variation in the debt/output ratio. Holding fixed the volatilities of these innovations, the small-sample bias grows as the true autocorrelation $\phi \to 1$, and as $corr(\varepsilon^v, -\varepsilon^r) \to -1$ and $corr(\varepsilon^v, \varepsilon^s) \to -1$. Given the persistence of the debt/output ratio, and the size of the residual correlations, the bias is close to its upper bound.

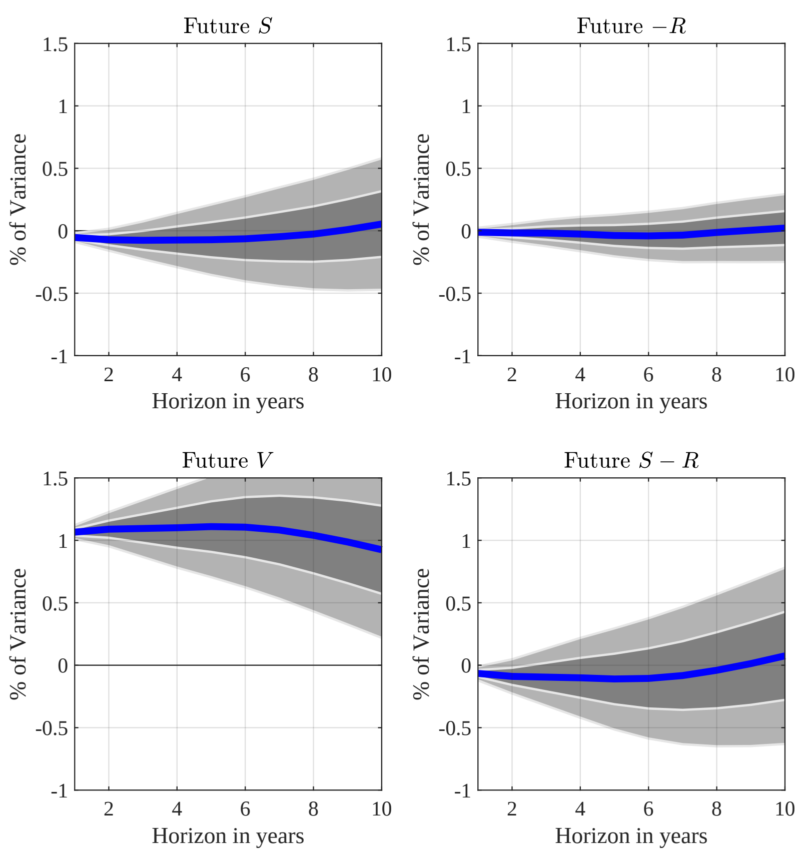
To better understand the bias, we can restate the bias of the coefficients at horizon $T = 1$ as follows :

$$
\begin{aligned}
bias_1^r &= \mathbb{E}[\widehat{\phi} - \phi] \times -\frac{cov(\varepsilon^v, \varepsilon^r)}{var(\varepsilon^v)}, \\
bias_1^s &= \mathbb{E}[\widehat{\phi} - \phi] \times -\frac{cov(\varepsilon^v, \varepsilon^s)}{var(\varepsilon^v)},
\end{aligned}
$$

where $\mathbb{E}[\widehat{\phi} - \phi]$ is roughly $-(1 + 3\phi)/N$. This expectation is taken over all possible values of true autocorrelation ϕ. If instead we are willing to restrict $\phi \leq 1$, then the bias attains an upper bound at $\phi = 1$ (Lewellen, 2004). In the case of stock return predictability, Lewellen (2004) shows that the null of no predictability can be rejected more often if this stationarity restriction is imposed. Imposing that $\phi = 1$ is equivalent to imposing $b_1^r + b_1^s = 0$, i.e., assuming that there is return or cash flow predictability. We do not impose any ex-ante restrictions on the true autocorrelation of the debt/output ratio. We do not want to rule out that the U.S. is on a fiscally unsustainable path.

Figure 4 reports the bias-adjusted regression coefficients. Table 2 reports the bias-corrected coefficient estimates in the rows labeled *unbiased*. The three predictability coefficients still sum to one. The bias-corrected variance decomposition attributes −4% and −7% of the debt/output ratio variance to the discount rate and cash flow channel respectively at the five-year horizon. As

Figure 4: Variance Decomposition of Log Debt/Output Ratio after Bias Correction



This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future government surpluses $\sum_{j=1}^{T} s_{t+j}$, future discount rates $\sum_{j=1}^{T} \tilde{r}_{t+k}$, future log debt/output ratio v_{t+T}, and the combination of the future government surpluses and discount rates. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1947—2020. We plot 1 s.e. (dark) and 2 s.e. (light) CIs. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t.

a result, 111% is accounted for by the future debt/output ratio. At the ten-year horizon, we still attribute 92% of the variance to the future debt/output ratio, after correcting for the small-sample bias. Variation in future surpluses over the next ten years accounts for 5% and future returns for 2% of the variation in the debt/output ratio. The latter two numbers are far below the 39% and 25% OLS point estimates, showing a very large small-sample bias. The bias-corrected point estimates combined with the standard errors imply that there is no evidence that the debt/output ratio predicts either future government surpluses or future bond returns. The null hypothesis that there is no predictability in future government surpluses or future bond returns cannot be rejected at any horizon. At the ten-year horizon, we cannot even reject the joint null that the debt/output ratio does not predict fundamentals, the difference between surpluses and growth-adjusted returns. This is the main result in the paper.

The confidence intervals for the discount rate contribution in the top right panel are quite narrow. We can rule out that discount rates play a quantitatively significant role in imputing mean reversion to the debt/output ratio, if there is any role at all. However, the confidence intervals for the cash flow channel in top left panel are wider at longer horizons. Even though the point estimates are close to zero, the 95%-confidence interval includes values of 50% at the ten-year horizon. Similarly, we cannot definitively rule out that there is significant mean-reversion at the ten-year horizon, even though the point estimate for the future V contribution, shown in the bottom left panel, is close 100%. We have low power at longer horizons, a well-documented feature of unit root tests.

Our surplus variable s_t, as defined in equation (1), is defined based on the log-linearization that enforces the linearized budget constraint exactly. Under this definition, the cross-equation restriction on the three predictability coefficients always holds. As noted earlier, s_t has a correlation with the actual surplus/GDP ratio S_t^{Raw} of 0.83, but it is more volatile. We now check whether the log debt/GDP ratio predicts the sum of actual government surplus/GDP ratios:

$$\sum_{j=1}^{T} \rho^{j-1} S_{t+j}^{Raw} = a_s + \hat{b}_T^s v_t + \varepsilon_{t+T}^s.$$

We report the regression coefficients in Appendix Figure A1. Consistent with our main result, the log debt/GDP ratio does not predict the actual government surplus/GDP ratios either.[16]

[16] Müller, Storesletten, and Zilibotti (2016) study taxes and spending separately and show that a higher debt/output ratio is associated with lower non-defense spending but also with lower taxes. They trace these patterns to the political parties that are in control. Their results are consistent with ours. They find that the level of debt/output has an insignificant effect on debt growth.

1.5 Tests of Predictability Under Local-to-Unity Asymptotics

Standard return predictability tests, such as the t-test of the slope of the OLS predictability coefficient may be inappropriate when the predictor is persistent and its innovations are highly correlated with returns. In such cases, large-sample theory provides a poor approximation to the finite-sample distribution of tests statistics. Campbell and Yogo (2006) offer an alternative test that is valid under general assumptions about the predictor dynamics, even when the largest root is larger than one, and general assumptions on the distribution of innovations. Campbell and Yogo develop a pre-test to diagnose whether the conventional t-test leads to valid inference. Their Dickey-Fueller Generalized Least Squares test is based on the CI for the largest autoregressive root of the predictor variable. Applying their method to our context, we obtain a 95% CI of $[0.958, 1.061]$ for the persistence of the debt/output variable v_t. Since this CI contains the unit root, this indicates that standard t-tests are not valid.

Campbell and Yogo go on to develop an asymptotically valid and efficient Q-test, which results in a Bonferroni CI for the predictive coefficient of interest. When we apply their procedure to the surplus predictability regression, we find a 90% CI of $[-0.051, 0.019]$. This CI includes zero, so that we fail to reject the null hypothesis that the lagged debt/gdp ratio v_t does not forecast the surplus/gdp ratio s_{t+1} ($H_0 : b_1^s = 0$). We repeat the analysis for the adjusted return on the debt portfolio, $\widetilde{r}_{t+1}$ and find a 90% Bonferroni CI of $[-0.013, 0.044]$. We fail to reject the null hypothesis that the lagged debt/gdp ratio does not forecast the government debt return ($H_0 : b_1^r = 0$). This analysis confirms our findings: once the high degree of persistence of the debt/gdp ratio is taken into account, the evidence for predictability of future surpluses or future returns by the lagged debt/gdp ratio is very weak.

1.6 Sample Period and Linearization Constant

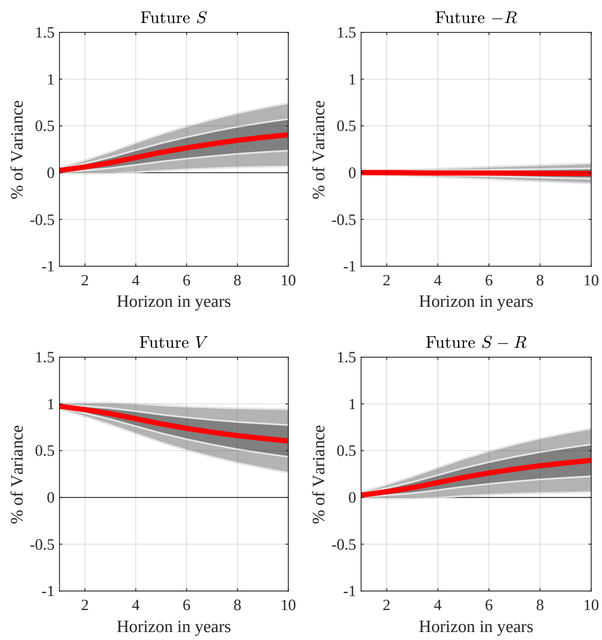
We consider the robustness of our results to considering a longer sample, revisiting the shorter Bohn (1998) sample, and considering different steady-state values for the adjusted returns on the debt portfolio.

1.6.1 Longer Sample

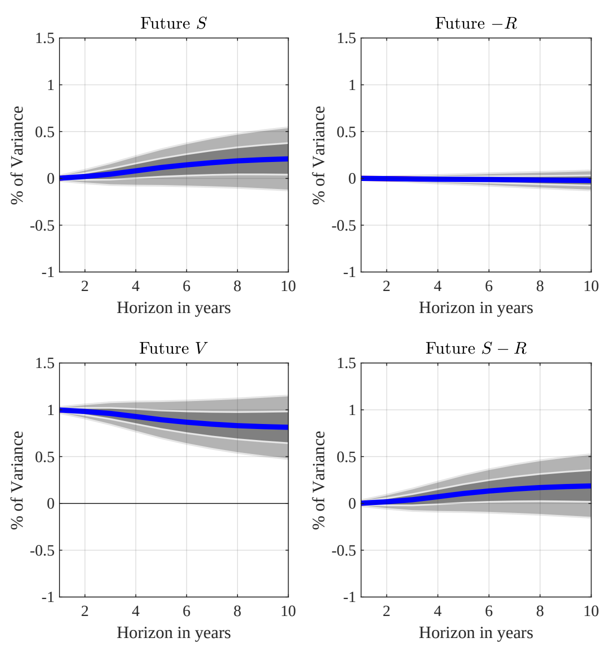
We extend the sample in an effort to improve the power of our tests. Using the Hall, Payne, and Sargent (2018) historical data, we construct a longer U.S. sample that starts in 1842. The variance decomposition with and without bias adjustment is plotted in Figure 5. The bias-corrected point estimates in Panel B are similar to those obtained for the post-WW-II sample. The longer sample helps to shrink the CIs around the point estimates. The shrinkage in CIs is pronounced for the growth-adjusted return predictability coefficients, but only modest for the surplus predictability coefficients. We can strongly reject that growth-adjusted returns contribute non-trivially to debt

Figure 5: Variance Decomposition of Log Debt/Output Ratio: Longer Sample 1842—2020

Panel A: Before Bias Correction



Panel B: After Bias Correction



This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future log debt/output ratio $\rho^T v_{t+T}$, and the combination of the future government surpluses and discount rates. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1842—2020. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t. We plot 1 s.e. (dark) and 2 s.e. (light) CIs.

adjustment. Surplus adjustment accounts for about 20% of the variability in the debt/GDP ratio, according to the bias-adjusted point estimate. While this is higher than in the post-war sample, we still cannot reject the null hypothesis that there is no surplus predictability. At the ten-year horizon, we can now definitively rule out that future v accounts for less than 50% of the variation. Conversely, we can rule out that fundamentals account for more than 50% of the variation in the debt/output ratio. After the bias correction, future v ten years out explains about 85% of the variation in today's v based on the point estimates.

The longer data sample allows us to extend the predictability horizon further. Since we only have 9 independent observations of 20-year samples in the 180-year long time series, confidence intervals naturally grow with the horizon. Appendix Figure A2 shows results for up to 20 years. The point estimates indicate no role for growth-adjusted returns and a larger role for surpluses, but we still cannot reject the null hypothesis that surpluses are not predictable. In short, we continue to find strong evidence against return predictability and little evidence for surplus predictability.

1.6.2 Shorter Sample

In a classic paper on this topic, Bohn (1998) finds evidence that the debt/GDP ratio predicts surpluses and interprets this as evidence against the unit root hypothesis. Bohn considers several samples that all end in 1995, and his main results are robust across samples. We re-estimate our results for the two main Bohn samples: 1916–1995 and 1948–1995. Before correcting for the small-sample bias, our methodology also identifies a larger role for fundamentals in the samples ending in 1995, as shown in Figure A3 and Figure A4, consistent with Bohn's findings. However, this evidence mostly disappears after the Stambaugh bias correction.

1.6.3 Approximation around Different Steady-State Values

Recall that we back out the linearized government surplus s_{t+1} from the following relation,

$$\tilde{r}_{t+1} = \rho v_{t+1} - v_t + s_{t+1},$$

where $\rho = \exp(-(r - x - \pi))$ is a constant of linearization. In the benchmark case, we chose $\rho = 1$ to linearize the equation system at $r = x + \pi$. Here, we re-derive our equations for general $\rho = \exp(-(r - x - \pi))$, and report a robustness result using $r - (x + \pi) = -1\%$ so that the risk-free rate is below the output growth rate by 1% per annum, in the Blanchard (2019) region of the parameter space.

For the case of an arbitrary ρ, equation (2) becomes:

$$v_t = \mathbb{E}_t \sum_{j=1}^{T} \rho^{j-1} \left(s_{t+j} - \widetilde{r}_{t+j} \right) + \mathbb{E}_t \rho^T v_{t+T}. \tag{8}$$

The corresponding regression equations become:

$$\begin{aligned}\sum_{j=1}^{T}\rho^{j-1}s_{t+j} &= a_s + b_T^s v_t + \varepsilon_{t+T}^s, \\ \sum_{j=1}^{T}\rho^{j-1}\widetilde{r}_{t+j} &= a_r + b_T^r v_t + \varepsilon_{t+T}^r, \\ \rho^T v_{t+T} &= \phi_0 + \phi_T v_t + \varepsilon_{t+T}^v. \end{aligned} \tag{9}$$

so that the cross-equation restriction $b_T^s - b_T^r + \phi_T = 1$ is still satisfied.

Since we use the same persistent predictor v_t on the right-hand side, we apply the same Stambaugh-bias adjustment formula as in the main text. Figure A5 reports the estimates with and without bias adjustment for $\rho = \exp(1\%)$. Figure A6 reports the estimates for $\rho = \exp(-1\%)$. The results after the bias correction for both cases are similar to the ones obtained for the benchmark $\rho = 0$ case.

2 Permanent Shocks to the Debt/Output Ratio

2.1 Simulation Evidence

Our evidence is consistent with a unit root in the debt/output ratio. Next, we evaluate the accuracy of the small-sample bias correction term by simulating 10,000 samples with the same length as the actual sample. We simulate under the null that there is no mean reversion in the debt/output ratio. Under this null, there is no contribution from return or surplus predictability (the fundamentals) either: $b_T^s - b_T^r = 0 = 1 - \phi_T$ at all horizons T.

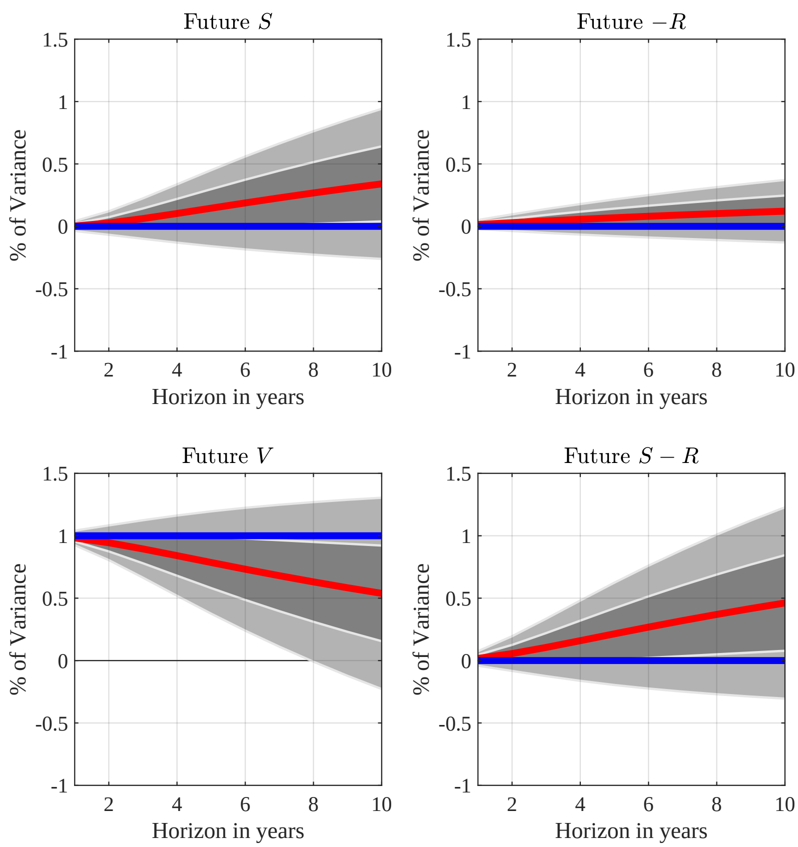
We assume that returns are i.i.d. and that the debt/output ratio follows a unit root process. We estimate an ARIMA(1,1,0) process for the debt/output dynamics. The system is given by the following equations:

$$\begin{aligned} v_{t+1} &= v_t + \Delta v_{t+1}, \\ \Delta v_{t+1} &= \psi_0 + \psi_1 \Delta v_t + \varepsilon_{t+1}^v, \\ \tilde{r}_{t+1} &= r_0 + \varepsilon_{t+1}^r. \end{aligned}$$

As always, we infer the surplus from the budget constraint: $s_{t+1} = \widetilde{r}_{t+1} - \Delta v_{t+1}$. In each round of simulation, we draw with replacement from the joint distribution of residuals $(\varepsilon_{t+1}^v, \varepsilon_{t+1}^r)$. Then, we estimate the forecasting regressions (5) by OLS in each simulated sample. We plot the mean slope coefficients and the two-standard-deviation CIs around these mean slope coefficient estimates.

In the simulated data, we find that future government surpluses and discount rates appear

Figure 6: Variance Decomposition of Log Debt/Output Ratio under Unit Root



This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future surpluses $\sum_{j=1}^{T} s_{t+j}$, future real growth-adjusted returns (with a minus sign) $\sum_{j=1}^{T} -\widetilde{r}_{t+k}$, the future log debt/output ratio v_{t+T}, and the difference between the future surpluses and returns. The samples are generated by simulation under the null that the debt/output ratio has a unit root. We plot the mean of the small-sample OLS slope coefficients in red. We plot 1 s.e. (dark) and 2 s.e. (light) CIs. We also plot the long-sample slope coefficients in blue from a single simulation of 100,000 periods.

to explain variations in the debt/output ratio, even though they do not by assumption. Figure 6 reports the variance decomposition implied by the simulated samples.[17] The average OLS slope coefficients generated by the unit-root model imply variance decompositions that are very close to our point estimates from the historical sample in Figure 3. The OLS estimates for the autoregressive coefficient are severely biased downwards in small samples when the true model has a unit root (Hamilton, 1994, p. 217). As a result, we find spurious evidence of mean reversion that creates a large role for fundamentals over longer horizons, in cases where there is no mean-reversion. The true slope coefficients ϕ_T are one at all horizons T.

Quantitatively, the simulation suggests that the downward bias in the variance explained by the future log debt/output ratio v_{t+T} is about 50% at the ten-year horizon. If we adjust the vari-

[17]The variance decomposition itself is not well defined for a unit root process, because the unconditional variance is not well defined. However, we can still run the estimation in the simulated small samples.

ance decomposition result in Figure 3, the bias-adjusted variance explained by the future log debt/output ratio is about 36% + 50% = 86% at the ten-year horizon. This number is close to the 92% bias-corrected number we obtained in Figure 4 and Table 2. In sum, the simulation exercise provides corroborating evidence for the presence of a unit root in the log debt/output ratio.

2.2 Structural Breaks

One way of allowing for permanent shocks to the debt/output ratio, consistent with the unit-root evidence, is to allow for a structural break. There may have been structural shifts in the relation between the valuation of debt and the fundamentals. A major contributor to the small role of fundamentals is the large run-up in government debt during the GFC which was not followed by commensurate increases in surpluses or decreases in returns. Consequently, we consider a structural break in the log debt/output ratio in 2007. A Chow test for structural breakpoints rejects the null hypothesis of no structural break at the 1% level in 2007 and at no other date.

Following Lettau and Van Nieuwerburgh (2008)'s work on stock return predictability, we allow for a structural break in the log debt/output ratio in 2007. The structural break introduces a 0.78 (in log scale) permanent increase in the debt/output ratio. Obviously, when we allow for this break, we introduce permanent innovations in the debt/output ratio. We then demean the log debt/output ratio $\widetilde{v}_t = v_t - \bar{v}_t$ with a lower pre-2007 sample mean ($\bar{v}_t, t < 2007$) and a higher post-2007 sample mean ($\bar{v}_t, t \geq 2007$). Figure 2 plots the resulting demeaned series $\widetilde{v}_t$ as the dashed line. This approach removes a low-frequency component from the debt/output ratio, keeping the transitory variation. The resulting series is much less persistent than the original series (lower ϕ_T).

We re-estimate the forecasting regressions using this new predictor $\widetilde{v}_t$. We recompute the surpluses by feeding $\widetilde{v}_t$ into equation (1) so that the cross-equation restriction still holds. The slope coefficients now provide a variance decomposition of the transitory variation in debt/output. Taking covariances with $\widetilde{v}_t$ on both sides of the previous equation, we obtain the following expression for the variance of the transitory component of debt/output ratio $\widetilde{v}_t$:

$$var(\widetilde{v}_t) = cov\left(\sum_{j=1}^{T} s_{t+j}, \widetilde{v}_t\right) - cov\left(\sum_{j=1}^{T} \widetilde{r}_{t+j}, \widetilde{v}_t\right) + cov(\widetilde{v}_t, \widetilde{v}_{t+T}). \tag{10}$$

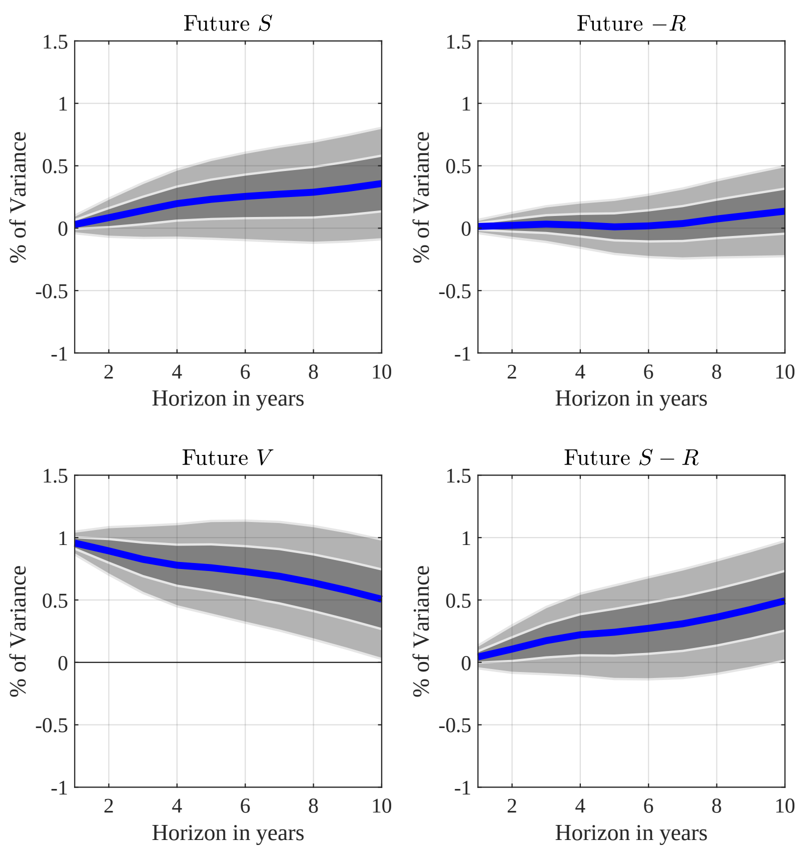
Because the cross-equation restrictions $b_T^s - b_T^r + \phi_T = 1$ continues to hold, the lower value for ϕ_T directly implies a larger role to fundamentals in explaining the variation in the transitory component of debt/output.

Figure 7 decomposes the variance of the transitory component of the debt/output ratio. The transitory component of the debt/output ratio explains $R^2 = 36\%$ of the variation in five-year cumulative surpluses. When the transitory component of the debt/output ratio is high, surpluses tend to increase to push the debt/output ratio back down. Table 4 shows that future surpluses and returns combine to explain 24% (50%) of the variation in transitory component of debt/output at the five-year (ten-year) horizon after bias correction. We can reject the null that fundamentals do not play a role at the ten-year horizon. This approach restores a role for fundamentals after accounting for the small-sample bias, but only in explaining the transitory variation in the debt/output ratio.[18]

This evidence shows that our main results, which find little evidence for surplus or return predictability, are to a substantial extent due to the permanent component in the debt/GDP ratio.

[18] An alternative approach to separate persistent and transitory components is to consider the budget constraint (1) in first-differences and apply the variance decomposition to those first-differences. We thank Howard Kung (discussant) for showing that this produces similar results.

Figure 7: Variance Decomposition of Log Debt/Output Ratio with Break



This figure reports the variance decomposition of the transitory component of the log debt/output ratio $v_t - \bar{v}_t$ with a lower pre-2007 sample mean ($\bar{v}_t, t < 2007$) and a higher post-2007 sample mean ($\bar{v}_t, t \geq 2007$) into components due to future government surpluses $\sum_{j=1}^{T} s_{t+j}$, future discount rates $\sum_{j=1}^{T} \tilde{r}_{t+k}$, future log debt/output ratio $v_{t+T} - \bar{v}_{t+T}$, and the combination of the future government surpluses and discount rates. Sample is annual, 1947—2020. We impose a structural break in the log debt/output ratio by shifting its average level before and after 2007. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of $(r_t, \tilde{v}_t)$ on two lags of $\tilde{v}_t$. We plot 1 s.e. (dark) and 2 s.e. (light) CIs.

Table 4: Forecasting Returns and Surpluses with Break-Adjusted log Debt/Output ratio

OLS Regression of $\sum_{j=1}^{T} s_{t+j}$, $\sum_{j=1}^{T} \widetilde{r}_{t+j}$, and $\widetilde{v}_{t+T}$ on $\widetilde{v}_t$. The predictor $\widetilde{v}_t$ is the break-adjusted debt/output series which removes a structural break in 2007. Annual data. Sample: 1947—2020. Standard errors generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t, and we set $s_{t+1} = \widetilde{r}_{t+1} - \Delta v_{t+1}$.

Horizon	1	2	3	4	5	6	7	8	9	10
					Forecasting $\sum_{j=1}^{T} -\widetilde{r}_{t+j}$					
$-b_T^r$	0.03	0.05	0.07	0.07	0.07	0.08	0.11	0.16	0.2	0.24
s.e.	0.03	0.05	0.07	0.09	0.11	0.12	0.14	0.15	0.17	0.18
R^2	0.02	0.03	0.04	0.03	0.02	0.02	0.04	0.06	0.08	0.1
unbiased	0.01	0.02	0.03	0.02	0.01	0.02	0.04	0.07	0.11	0.14
					Forecasting $\sum_{j=1}^{T} s_{t+j}$					
b_T^s	0.07	0.16	0.25	0.34	0.41	0.46	0.51	0.56	0.62	0.68
s.e.	0.04	0.08	0.11	0.14	0.16	0.18	0.19	0.2	0.21	0.22
R^2	0.04	0.12	0.2	0.29	0.36	0.42	0.47	0.5	0.53	0.57
unbiased	0.03	0.08	0.14	0.2	0.23	0.25	0.27	0.29	0.32	0.36
					Forecasting v_{t+T}					
ϕ	0.91	0.79	0.68	0.59	0.53	0.45	0.38	0.29	0.19	0.08
s.e.	0.05	0.1	0.13	0.16	0.19	0.2	0.22	0.23	0.23	0.24
R^2	0.86	0.7	0.55	0.44	0.35	0.27	0.19	0.11	0.05	0.01
unbiased	0.96	0.89	0.83	0.78	0.76	0.73	0.69	0.64	0.58	0.51

3 Sources of Permanent Shocks to the Debt/Output Ratio

The variance decomposition of the previous section punts on the explanation for the large and permanent increase in the debt/output ratio in the post-2007 sample. In the last part of the paper, we consider two candidate explanations for the structural break in the relationship between debt/GDP and fundamentals. A first explanation for the structural break is biased beliefs about future surpluses, discussed in Section 3.1. We also explore the rise in foreign and Central Bank holdings of U.S. Treasurys. The bulk of the rise in these holdings comes after 2007, so the timing fits with the estimated time of the structural break.

3.1 Biased Beliefs

We find a large Stambaugh bias in the fiscal predictability regressions. After we correct this bias, future government surpluses and discount rates do not explain variation in the level of outstanding debt/output ratio. One possible explanation for these results we explore now is that the U.S. government debt portfolio is mispriced, perhaps because Treasury bond investors have been overly optimistic about the U.S. fiscal situation.

The Evolution of Debt Under Subjective versus Objective Expectations. Suppose investors evaluate equation (2) using their own subjective expectation, denoted by the operator $\mathbb{F}$, which differs from the true conditional expectation operator $\mathbb{E}$:

$$v_t = \mathbb{F}_t \sum_{j=1}^{T} \left(s_{t+j} - \widetilde{r}_{t+j}\right) + \mathbb{F}_t v_{t+T}.$$

When the econometrician parses out the variations in the debt/GDP ratio under the objective measure, she obtains:

$$v_t = \mathbb{E}_t \sum_{j=1}^{T} \left(s_{t+j} - \widetilde{r}_{t+j}\right) + \mathbb{E}_t v_{t+T}.$$

Comparing the two equations, we obtain:

$$\mathbb{E}_t v_{t+T} = \mathbb{F}_t v_{t+T} + (\mathbb{F}_t - \mathbb{E}_t) \sum_{j=1}^{T} (s_{t+j} - \widetilde{r}_{t+j}),$$

where the last term denotes the investors' forecast errors about fundamentals. This setup allows for the possibility that the debt/GDP ratio is explained fully by fundamentals under the subjective measure, i.e., $cov(v_t, \mathbb{F}_t v_{t+T}) = 0$, while the econometricians finds the opposite, i.e., $cov(v_t, \mathbb{E}_t v_{t+T}) \approx var[v_t]$. In this setting, what the econometrician interprets as a unit root in the debt/GDP process reflects investors' forecast errors that covary with the debt/output ratio.

Measuring Subjective Beliefs To assess this possibility empirically, we ideally would observe a time-series of private-sector forecasts of future surpluses and future GDP over various horizons ranging from one to ten years. Unfortunately, such data are unavailable. Private sector survey evidence on the future surplus and GDP is available starting in 1994 from Consensus Economics, but only at the one-year-ahead horizon. However, the Congressional Budget Office (CBO) predicts future surpluses ten years out. Figure 8 shows that the one-year ahead surplus forecast in both data sets is highly positively correlated (98.9%). If anything, the CBO's forecast is slightly more conservative.[19] This evidence motivates us to use the CBO forecast as a proxy for the private-sector forecast. Given the complex nature of surplus forecasts and the CBO's decades-long expertise, this strikes us as a reasonable assumption.

Systemic Forecast Errors. The top panel of Figure 9 plots the CBO projections $\mathbb{F}_t v_{t+T}$ for the debt/GDP ratio against the actual time series from 1980 to 2020 (black line). The CBO reports five-year projections from 1980 to 1995 and ten-year projections starting in 1996. The projections for each projection date are indicated by a different color.

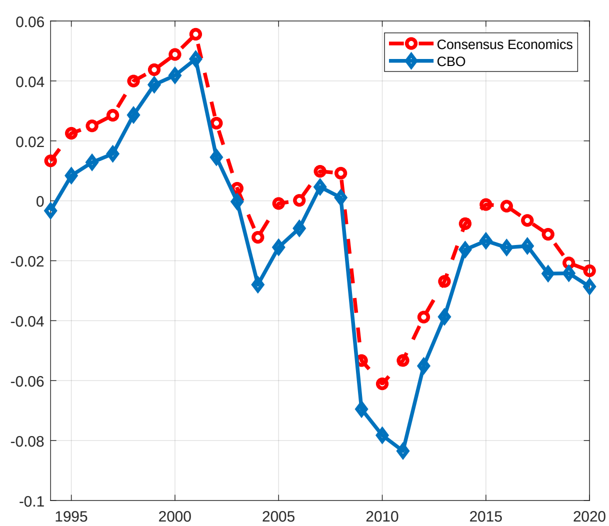
[19] While the CBO forecasts GDP, inflation, and interest rates in unrestricted fashion, it makes projections of future surpluses based on current law.

The CBO has been consistently under-predicting the debt/GDP ratio since the GFC. This was much less the case prior to the GFC. This bias arises mainly because the CBO has been over-predicting future surpluses as a fraction of GDP since the GFC, as shown in the bottom panel. The surplus projections (colored lines) are systematically above the realized values (black line). The GFC and the COVID-19 pandemic are large unanticipated shocks with a huge fiscal impact. But the projections remain biased well after the GFC. As an aside, if the CBO had assumed future fiscal corrections –beyond those already stipulated in current law– when making its debt/output projections, its projections would have been even farther from the realized debt/output ratios.

Figure 10 plots annualized forecast errors $(\mathbb{F}_t - \mathbb{E}_t) \sum_{j=1}^{T} s_{t+j}$, obtained by comparing the CBO projected to the realized future surpluses. The forecast errors were close to zero from 1980 to 1997, -0.4% per year on average, but rose to 10.48% per annum from 1998 to 2020. The size of these forecast errors in the latter part of the sample is surprisingly large in light of the standard deviation of realized surpluses s_t of 6.52% over the post-war period. Even well after the GFC, in 2010, forecast errors are 3.6% per annum over the next ten years.

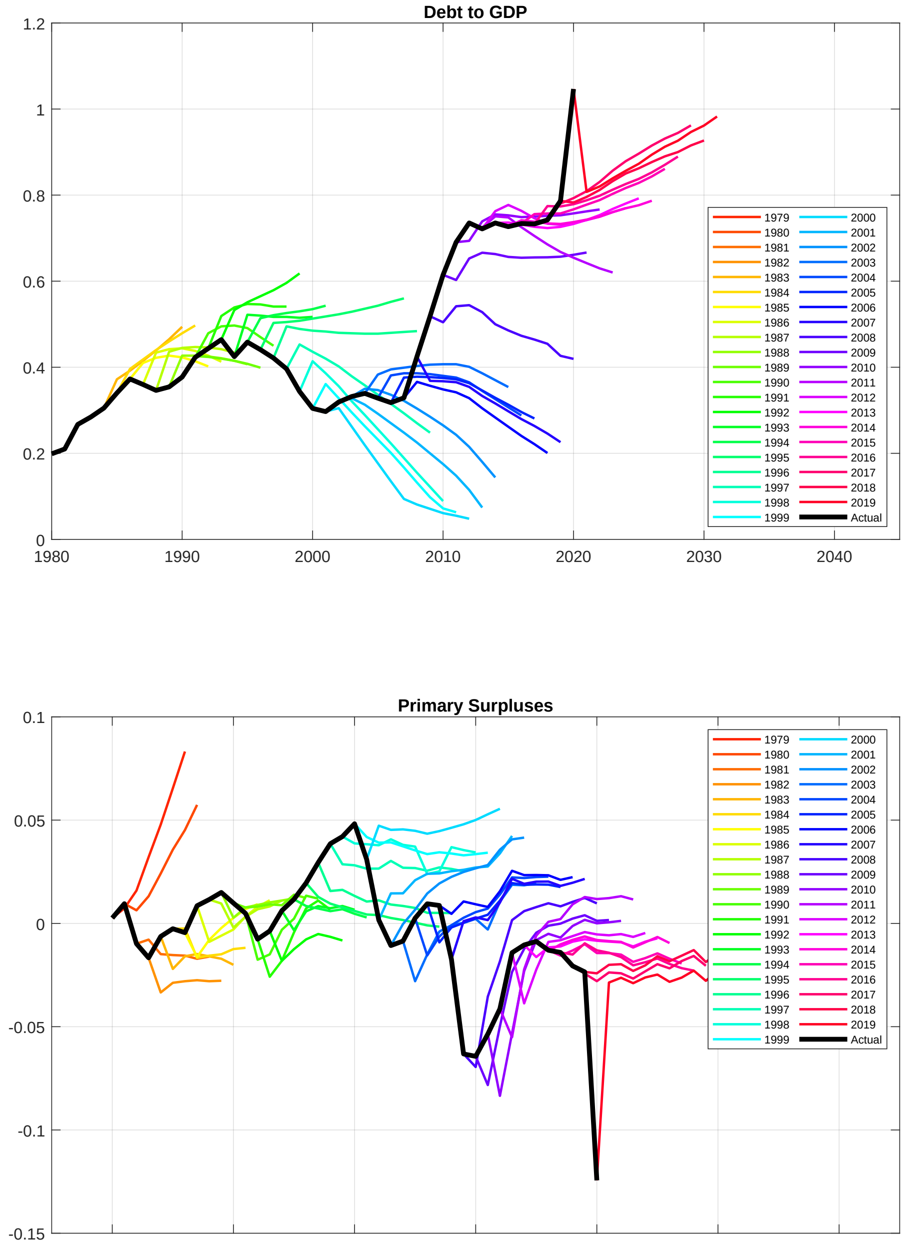
In sum, the CBO projections consistently impute too much mean-reversion to the debt/output ratio in the later part of the sample. Given the size and persistence of the CBO projection errors, and based on the evidence from one-year surplus forecast in Figure 8, it is conceivable that private investors held similarly-biased forecasts of future surpluses.

Figure 8: Comparing CBO and Private-Sector Surplus Forecasts



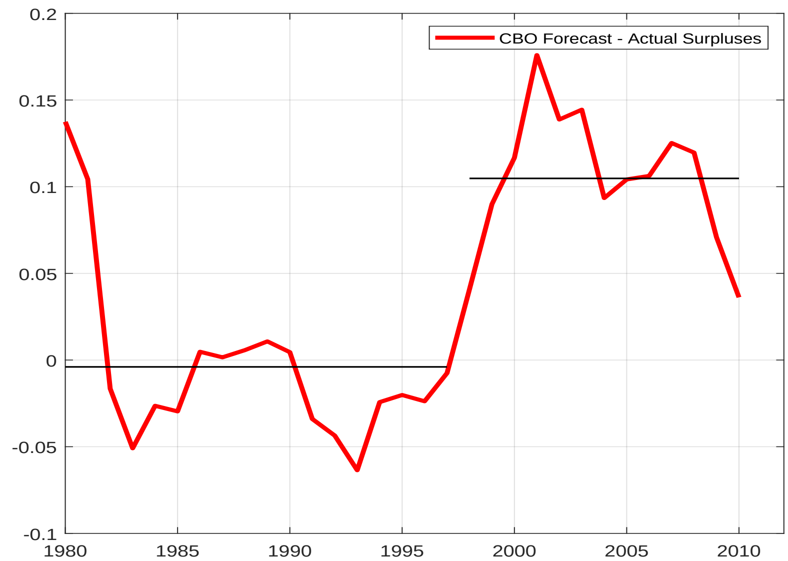
This figure plots the forecast, made at the start of the year (in January), for the current calendar year surplus/GDP ratio. The solid blue line with diamonds is for the Congressional Budget Office forecast (CBO) while the dashed red line with circles line is the mean forecast from Consensus Economics, a dataset of private sector forecasters. Each line combines forecasts for the government surplus, debt service, real GDP growth, and inflation from the respective data sets. Since the Consensus Economics data starts in 1994, we use the longest overlapping sample of 1994—2020 for this graph.

Figure 9: CBO Projections for Debt/GDP and Surplus/GDP



This figure reports the five(ten)-year U.S. federal government debt/GDP (top panel) and total federal government primary surplus/GDP (bottom panel) projections made by the CBO at the start of each year from 1980–2020. For the period from 1980 to 1995, the five-year projections are reported. For the period from 1996 to 2020, the ten-year projections are reported. The solid black line is the ex-post realized debt/GDP (top panel) and surplus GDP/ratio (bottom panel).

Figure 10: CBO Projection Errors



The plot shows the difference between the annualized cumulative future surpluses $\frac{\sum_{j=1}^{T} s_{t+j}}{T}$ using CBO projection and the annualized cumulative realized future surpluses, where $s_{t+j} = sy_{t+j}/e^{v}$ and v is the average log debt/output ratio between 1980 and 2020. The CBO projections go out five years ($T = 5$) from 1980 to 1995 and ten years ($T = 10$) from 1996 to 2020.

Predictability Revisited. To fix ideas, consider the case where there is no return predictability under either the objective or the subjective measure.[20] To explain our findings that the future debt/output ratio v_T accounts for most of the variation without any surplus predictability under the objective measure, we need a large positive sample covariance between surplus forecast errors and the debt/output ratio under the subjective measure:

$$\hat{cov}_N(v_t, v_{t+T}) = \hat{cov}_N\left(v_t, \mathbb{F}_t v_{t+T}\right) + \hat{cov}_N\left(v_t, \left(\mathbb{F}_t - \mathbb{E}_t\right) \sum_{j=1}^{T} s_{t+j}\right) = \hat{var}_N(v_t),$$

That is, if the first covariance term is not as large under the subjective than under the objective measure, then the second covariance term must be positive and make up the difference. Figure 9 indeed shows that, as the fiscal situation worsened during and after the GFC and the debt/output ratio increased, the CBO persistently over-projected surpluses. Likewise, bond market investors may have been pricing in larger cumulative surpluses than materialized, particularly after 2007.

To test this more formally, we re-estimate the Campbell-Shiller decompositions of the debt/output ratio under subjective beliefs. We construct a time series for the subjective cumulative surplus/GDP

[20] This can be justified by the tighter confidence intervals around a zero point estimate we obtained for the return predictability coefficient. Also, Figure A7 studies subjective future returns using CBO projections and shows that they are less biased than surplus forecasts.

ratio, which is equal to the observed series from 1947 until 2006 but replaces the observed series with the CBO forecast after 2007. Given the average forecast error near zero in the first part of the sample and the large forecast error in the last part (recall Figure 10), our procedure only adjusts the surplus when there is in fact a non-zero forecast error. Similarly, we construct a subjective expected future debt value $\mathbb{F}_t v_{t+T}$ from the observed data before 2007 and from the CBO projections after 2007. The subjective growth-adjusted returns are implied from the government budget constraint. We then re-estimate the system of forecasting regressions in (5).

Figure 11 plots the OLS regression results after bias correction. When using the subjective surpluses (red line), we find substantially more surplus predictability than we do using the actual surpluses throughout (blue line). At the ten-year horizon, 24% of the variance of the debt/output ratio is attributed to variation in subjective future surpluses, compared to 5% for objective future surpluses (top panel). This difference is large given that only the last 14 observations of the independent variable differ between the red and blue lines. The bottom panel shows that, at the ten-year horizon, future subjective debt/GDP ratios explain 73% of the variation in the current debt/GDP ratio, 19% points less than when using the objective future debt/output ratio of our benchmark estimation. Figure A8 shows that the results are similar when we use subjective surplus/output and debt/output ratios starting from 1996 rather than from 2007.

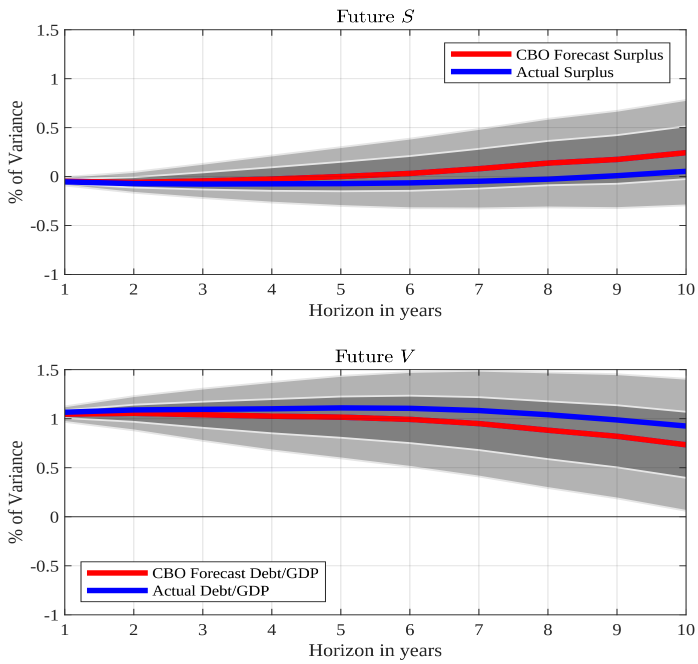
In sum, there is a stronger covariance between debt and future surpluses under subjective than under objective beliefs. This is due to forecast errors about the surplus that covary systematically and positively with the debt/output ratio. There is a structural break in the wedge between objective and subjective beliefs in the last part of the sample, which imputes a persistent component in the debt/GDP ratio. If investors' subjective expectations are aligned with CBO projections, an assumption supported in the data, they think they live in a world without permanent shocks to debt/output. But their systematic forecast errors induce permanent shocks in the debt/output ratio from the perspective of an econometrician given the historical sample.

3.2 Foreign and Central Bank Holdings

Foreign investors and the Federal Reserve Bank are considered to be relatively price-inelastic investors (Jiang, Krishnamurthy, and Lustig, 2022). Foreign holdings rise starting in the mid 1980s while the holdings of the Federal Reserve Bank increase dramatically following the GFC and Covid-19 crises, when the Fed embarks on multiple rounds of quantitative easing.

To examine the extent to which our results are driven by these holdings, we construct a measure of the debt/output ratio that excludes the holdings of the Fed and the rest of the world. We denote the resulting ratio of the debt held by the domestic private sector and output by v_t^d. Figure 12 shows that there has indeed been an increasing gap between domestic private sector holdings of Treasury debt and the total amount of Treasury debt, especially since 2007. The re-

Figure 11: Variance Decomposition of Log Debt/Output Ratio: CBO Projection 2007 - 2020



This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future log debt/output ratio $\rho^T v_{t+T}$, and the combination of the future government surpluses and discount rates. All estimates are bias-corrected. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1947—2020. From 2007 to 2020, we use CBO projected values for $\sum_{j=1}^{T} s_{t+j}$, $\sum_{j=1}^{T} \tilde{r}_{t+j}$ and v_{t+T}. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of $(r_t, \tilde{v}_t)$ on two lags of $\tilde{v}_t$. We plot 1 s.e. (dark) and 2 s.e. (light) CIs for the sample using CBO projected values.

sulting series v_t^d bears similarity to the transitory component of the debt/GDP ratio $\tilde{v}_t$ from the previous section in that it removes a persistent component of different magnitude before and after 2007.

Do domestic private holdings of Treasury debt have stronger predictive power for future surpluses or discount rates? We regress future surpluses and future adjusted debt returns on v_t^d:

$$\sum_{j=1}^{T} \rho^{j-1} s_{t+j} = a_s + \hat{b}_T^s v_t^d + \varepsilon_{t+T}^s,$$
$$\sum_{j=1}^{T} \rho^{j-1} \tilde{r}_{t+j} = a_r + \hat{b}_T^r v_t^d + \varepsilon_{t+T}^r,.$$

Since the domestically held debt/output ratio is not equal to future government surpluses minus adjusted debt returns, the adding-up constraint between $\hat{b}_T^s$, $\hat{b}_T^r$ and the autocorrelation of v_t^d does not hold.[21] Nevertheless, we can still study these regression coefficients and apply small-sample

[21]When the Fed buys long-term government debt as part of its Quantitative Easing operations, it issues short-term reserves to the banking sector in equal measure. One could consolidate the activities of the Fed and the Treasury and

bias adjustments.

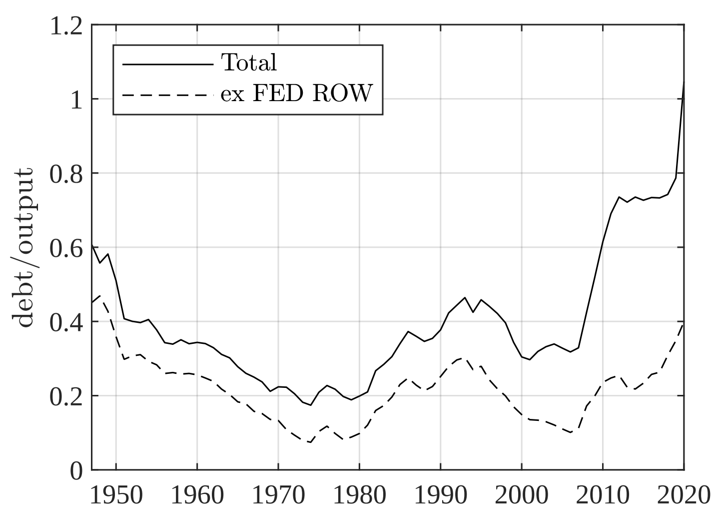
We report the regression coefficients in Figure A9. The predictor v_t^d explains around 40% of the variation in the future surpluses $\sum_{j=1}^{T} \rho^{j-1} s_{t+j}$ at the ten-year horizon after bias adjustment, similar to the evidence in Figure 7 from the structural break model. This result suggests that future surpluses may respond more to the debt that has to be absorbed by domestic investors, who have been found to be more price-elastic. However, after the bias correction, we still cannot reject the null that surpluses are not predictable.

4 Conclusion and Discussion

The bond market's valuation of a claim to surpluses is surprisingly insensitive to news about future surpluses or returns from the perspective of an econometrician looking at the U.S. historical sample. This is a direct result of the debt/output ratio's persistence. This persistence plagues small-sample predictability regressions that aim to uncover the extent to which high debt/output episodes are followed by higher government surpluses, higher growth, higher inflation, or lower bond returns. After correcting the small-sample bias there is no conclusive evidence in favor of such adjustments. These results, which show that there is no statistical evidence to conclude that the U.S. is on a fiscally sustainable path, stand in contrast with the existing literature.

adjusts the returns on the combined debt and reserve portfolio held by the private sector. We thank Lukas Schmid (discussant) for pointing this out.

Figure 12: Debt/Output Ratio: The Role of the Fed and the Foreign Sector



The full line is the debt/output ratio. The dashed line is the domestically and privately held debt/output ratio.

The evidence instead suggests that there is a large permanent, or at least highly persistent, component in the debt/GDP ratio. We identify biased beliefs about future surpluses as a potential source. Even though the econometrician may not forecast larger surpluses or lower discount rates when the debt/output ratio rises, the bond market investor may. Evidence from fiscal projections suggests that bond investors may have been over-predicting future surpluses in the last two decades, and hence may have anticipated mean reversion in the debt/output ratio that failed to materialize. Systematic forecast errors could be a source of persistence in the debt/output ratio. The portfolio of U.S. government debt may be persistently mispriced from the perspective of rational investors. Limits to arbitrage would plausibly prevent arbitrageurs from taking advantage of overpricing. Treasurys benefit from safe asset demand, appreciating in high marginal utility states of the world when arbitrageurs are more likely to be constrained.

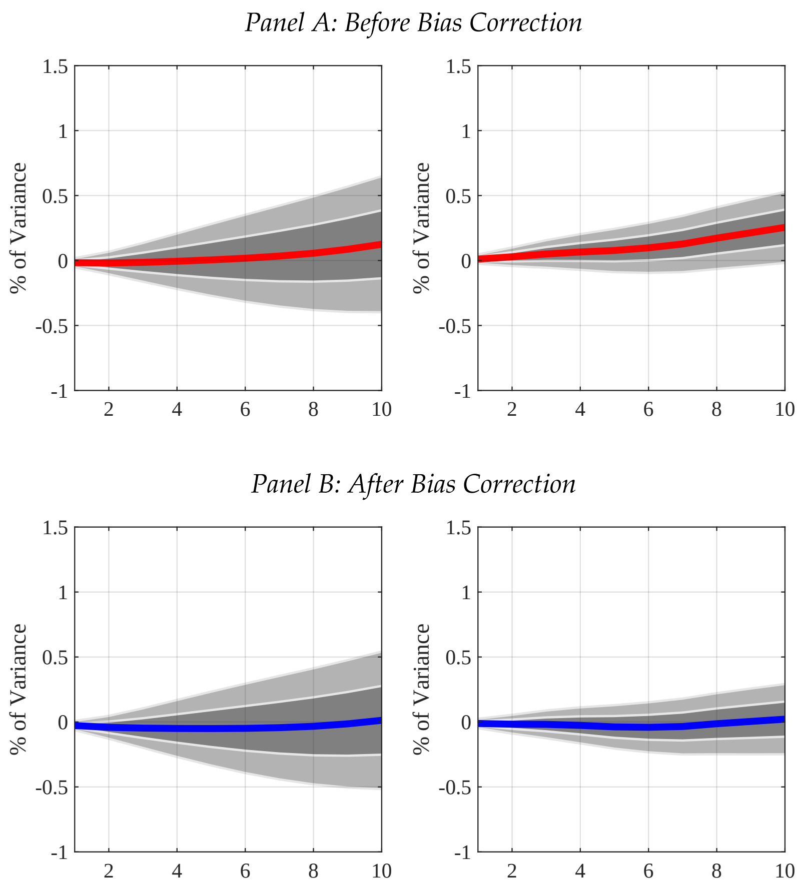
We also present evidence for a structural break in the debt/GDP ratio in 2007. One potential source such a structural break is foreign and especially Central Bank holdings of U.S Treaurys, which ramp up after 2007.

Finally, we cannot rule out the possibility of a peso-event, a large fiscal correction not observed in our sample, but anticipated and priced in by investors. Such fiscal adjustments might take place, for example, when the debt/GDP ratio becomes very high (as assumed in Elenev, Landvoigt, Shultz, and Van Nieuwerburgh, 2021). However, there are a few reasons to be skeptical about this explanation. First, we have considered a long data sample spanning nearly two centuries and a diverse set of economic and political circumstances. Second, even the most sophisticated budget projections (which abstract from future fiscal corrections) have proven too optimistic hitherto. Third, fiscal adjustments when debt is high are politically difficult because they occur in high marginal-utility states of the world.

Appendix

A Robustness Tables

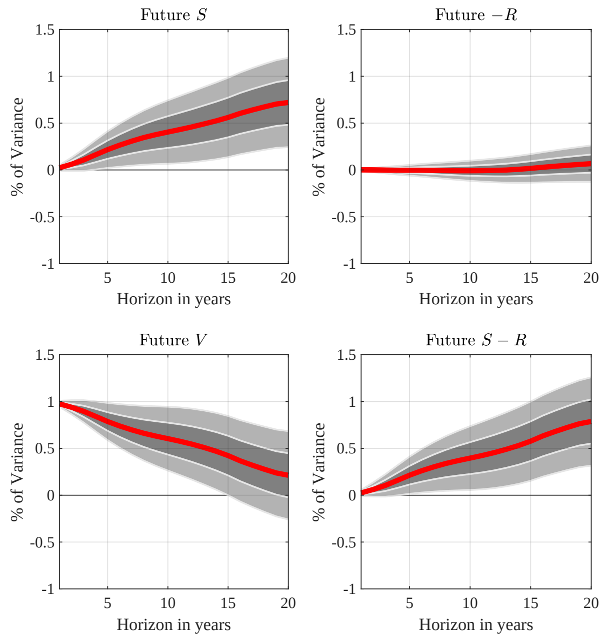
Figure A1: Forecasting Power of Log Debt/Output Ratio for Actual Government Surplus



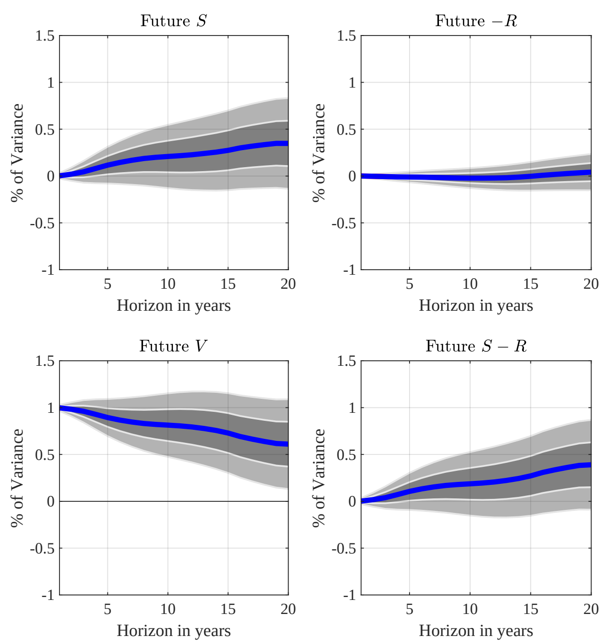
This figure reports regression coefficients associated with the log debt/output ratio. S^{raw} is based on the actual government surplus/GDP ratio. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1947—2020. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t. We plot 1 s.e. (dark) and 2 s.e. (light) CIs.

Figure A2: Variance Decomposition of Log Debt/Output Ratio: Longer Sample 1842—2020

Panel A: Before Bias Correction



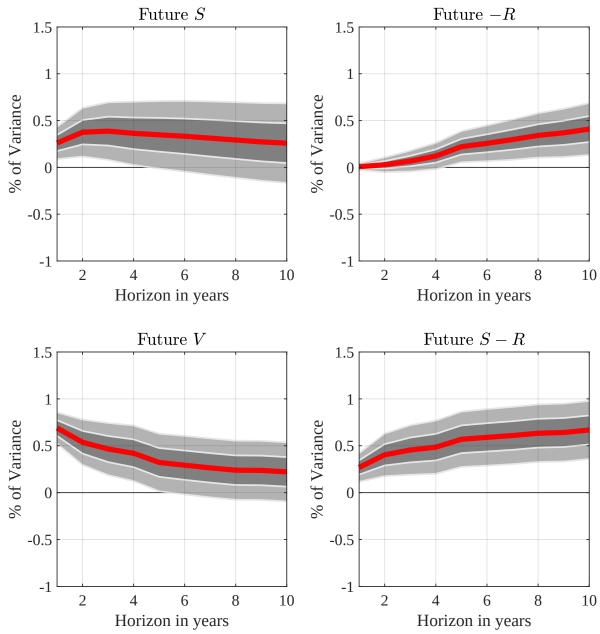
Panel B: After Bias Correction



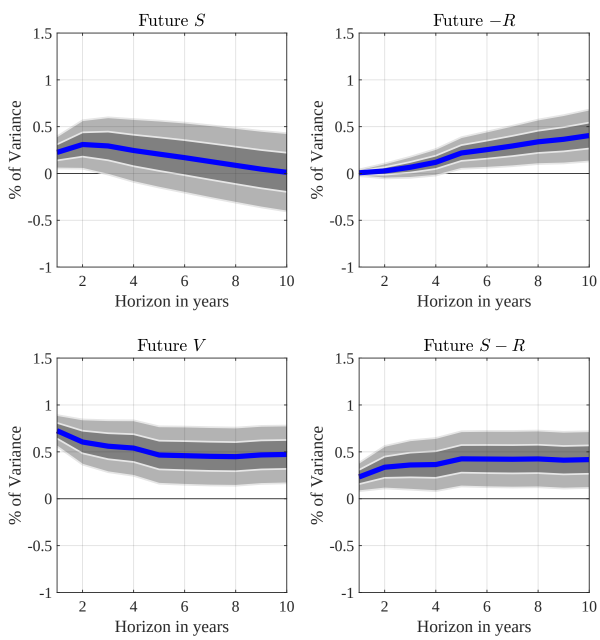
This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future log debt/output ratio $\rho^T v_{t+T}$, and the combination of the future government surpluses and discount rates. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1842—2020. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t. We plot 1 s.e. (dark) and 2 s.e. (light) CIs.

Figure A3: Variance Decomposition of Log Debt/Output Ratio: Longer Bohn Sample 1916—1995

Panel A: Before Bias Correction



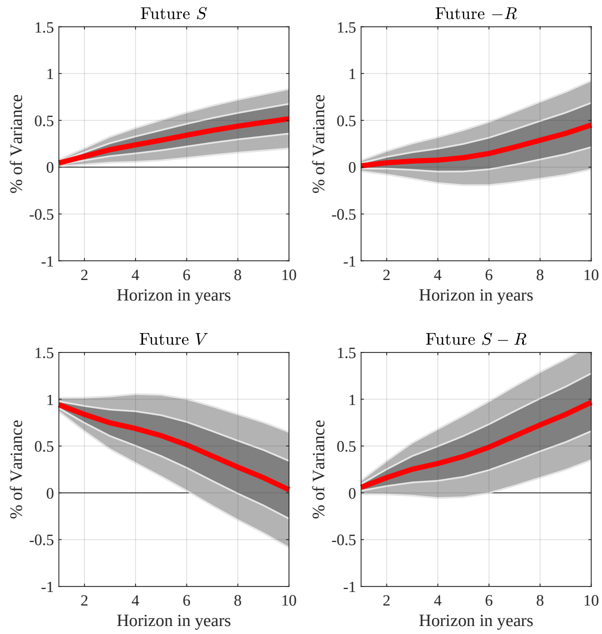
Panel B: After Bias Correction



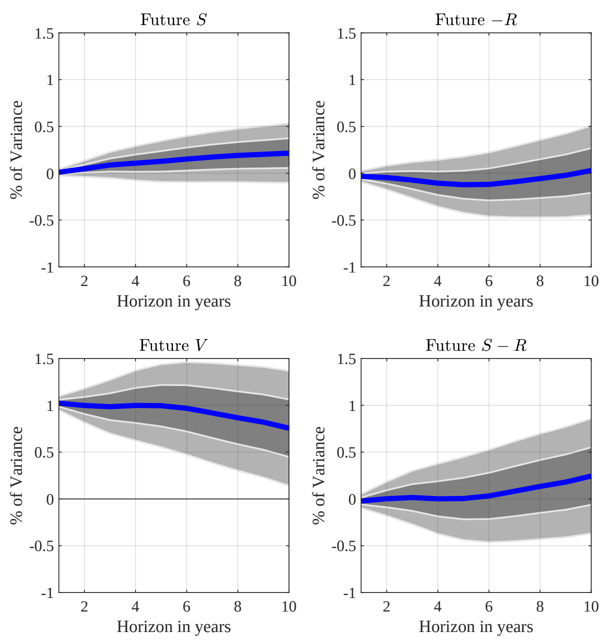
This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future log debt/output ratio v_{t+T} and the combination of the future government surpluses and discount rates. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1916—1995. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t. We plot 1 s.e. (dark) and 2 s.e. (light) CIs.

Figure A4: Variance Decomposition of Log Debt/Output Ratio: Shorter Bohn Sample 1948—1995

Panel A: Before Bias Correction



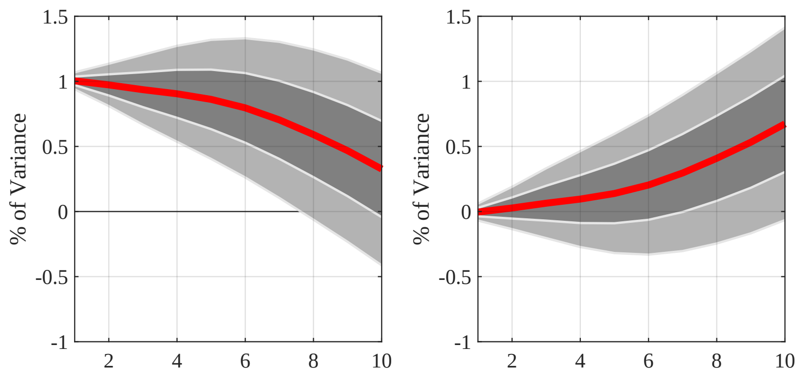
Panel B: After Bias Correction



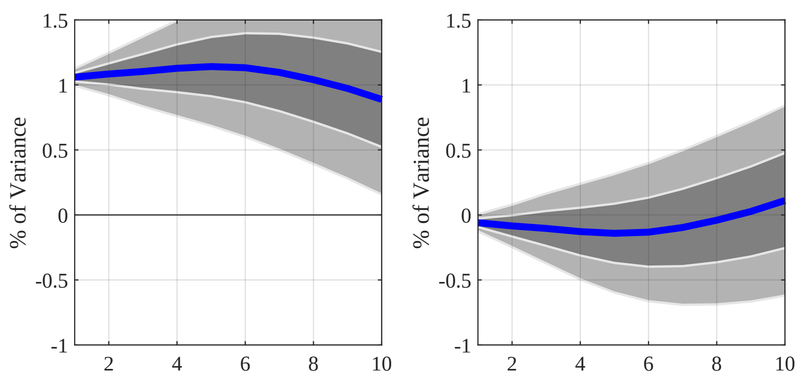
This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future log debt/output ratio v_{t+T} and the combination of the future government surpluses and discount rates. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1948—1995. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t. We plot 1 s.e. (dark) and 2 s.e. (light) CIs.

Figure A5: Variance Decomposition of Log Debt/Output Ratio: with $\rho = \exp(1\%)$

Panel A: Before Bias Correction

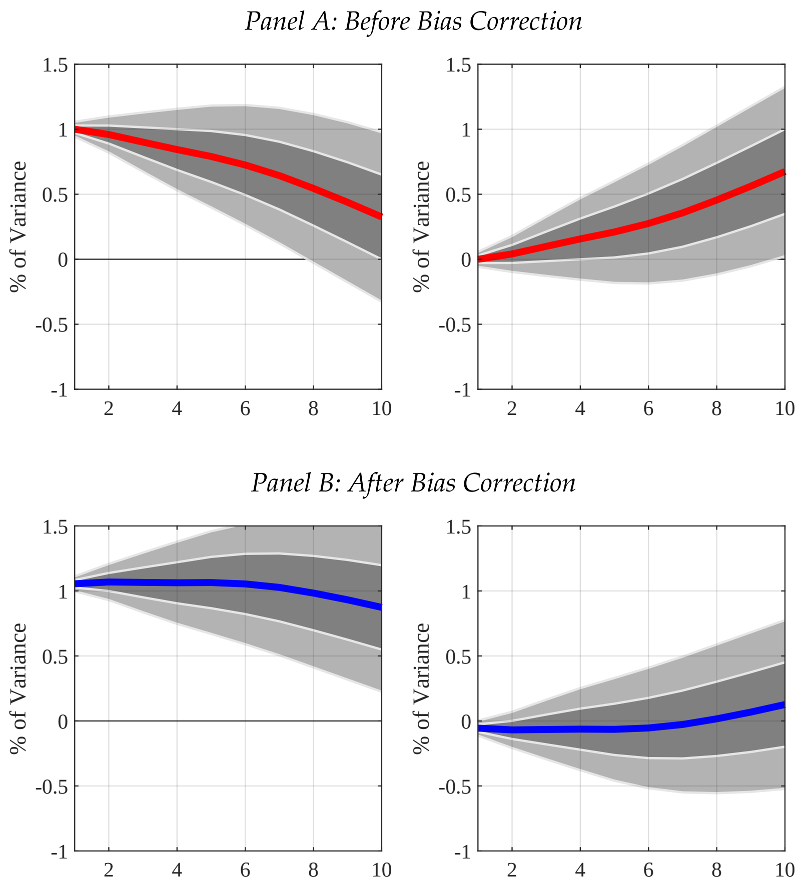


Panel B: After Bias Correction



This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future log debt/output ratio $\rho^T v_{t+T}$, and the combination of the future government surpluses and discount rates. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1947—2020. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t. We plot 1 s.e. (dark) and 2 s.e. (light) CIs.

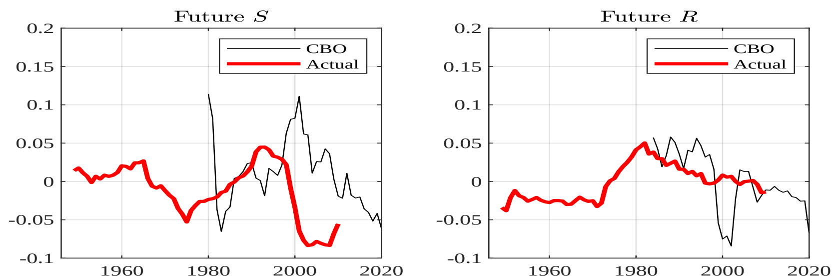
Figure A6: Variance Decomposition of Log Debt/Output Ratio: with $\rho = \exp(-1\%)$



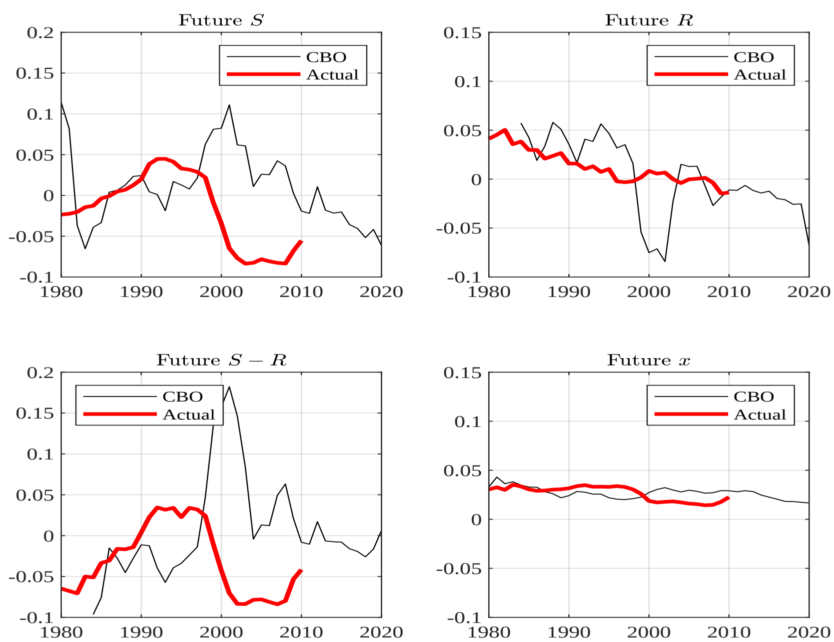
This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future log debt/output ratio $\rho^T v_{t+T}$, and the combination of the future government surpluses and discount rates. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1947—2020. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t. We plot 1 s.e. (dark) and 2 s.e. (light) CIs.

Figure A7: Decomposition for CBO Projections

Panel A: 1947—2020

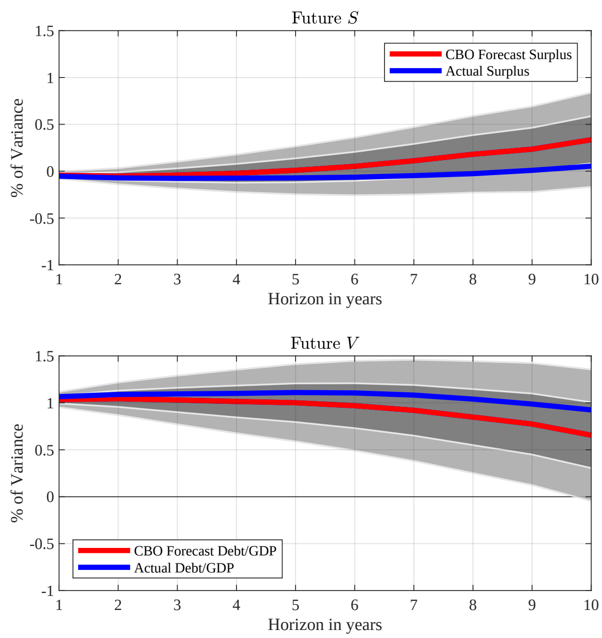


Panel B: 2007—2020



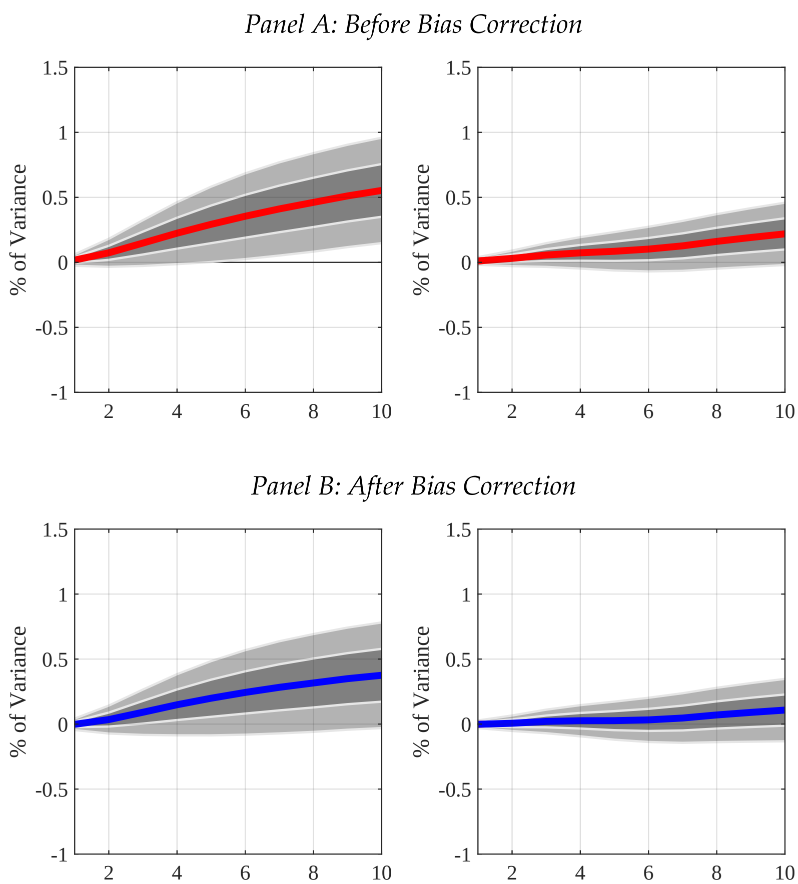
The plot shows the decomposition of the log debt/output ratio v_t into components due to CBO-projected (and realized) future government surpluses $\sum_{j=1}^{T} s_{t+j}$, future discount rates $\sum_{j=1}^{T} \widetilde{r}_{t+k}$, for $T = 10$. We also report future real growth $\sum_{j=1}^{T} \widetilde{x}_{t+k}$. All numbers are annualized.

Figure A8: Variance Decomposition of Log Debt/Output Ratio: CBO Projection 1996 - 2020



This figure reports the variance decomposition of the log debt/output ratio v_t into components due to future log debt/output ratio $\rho^T v_{t+T}$, and the combination of the future government surpluses and discount rates. All estimates are bias-corrected. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1947—2020. From 1996 to 2020, we use CBO projected values for $\sum_{j=1}^{T} s_{t+j}$, $\sum_{j=1}^{T} \tilde{r}_{t+j}$ and v_{t+T}. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of $(r_t, \widetilde{v}_t)$ on two lags of $\widetilde{v}_t$. We plot 1 s.e. (dark) and 2 s.e. (light) CIs for the sample using CBO projected values.

Figure A9: Forecasting Power of Log Debt/Output Ratio Held by the Domestic Sector



This figure reports regression coefficients associated with the log domesticly held debt/output ratio. We generate bias-corrected slope coefficients using the Stambaugh (1999); Boudoukh, Israel, and Richardson (2020) small-sample bias correction for the OLS coefficients in long-horizon predictability regressions. Sample is annual, 1947—2020. Standard errors are generated by bootstrapping 10,000 draws from the joint residuals in a regression of (r_t, v_t) on two lags of v_t. We plot 1 s.e. (dark) and 2 s.e. (light) CIs.

B Mean Reversion of Returns and Surpluses

We start from the log-linearized return equation implied by the government budget constraint:

$$r_{t+1} - \pi_{t+1} - x_{t+1} = \rho v_{t+1} - v_t + s_{t+1},$$

where ρ is a constant of linearization. We choose $\rho = \exp(-(r - x - \pi)) = 1$.

Over longer horizons k, the cumulative log return equals the change in the log of the debt/output ratio less the cumulative surplus over this horizon:

$$\tilde{r}_{t\to t+k} = \Delta v_{t\to t+k} - s_{t\to t+k}.$$

By taking variances on both sides, dividing by k and then taking the limit of the horizon to ∞, we obtain the following expression for the per period variance of the cumulative log returns:

$$\lim_{k\to\infty}\frac{1}{k}var[\tilde{r}_{t\to t+k}] = \lim_{k\to\infty}\frac{1}{k}var[\Delta v_{t\to t+k}] + \lim_{k\to\infty}\frac{1}{k}var[s_{t\to t+k}] - 2\lim_{k\to\infty}\frac{1}{k}cov(\Delta v_{t\to t+k}, s_{t\to t+k}).$$

If we impose that debt/output ratio s_t is stationary, then we end up with the implication that the variance of cumulative returns converges to the variance of cumulative surpluses:

$$\lim_{k\to\infty}\frac{1}{k}var[\tilde{r}_{t\to t+k}] = \lim_{k\to\infty}\frac{1}{k}var[s_{t\to t+k}], \tag{A1}$$

where we have used that $\lim_{k\to\infty}\frac{1}{k}var[\Delta v_{t\to t+k}] = 0$ and $\lim_{k\to\infty}\frac{1}{k}cov(\Delta v_{t\to t+k}, s_{t\to t+k}) = 0$.

Consider a first case in which the returns are i.i.d.. Then the long-horizon variance ratio of returns is one:

$$\lim_{k\to\infty}\frac{1}{k}var[\tilde{r}_{t\to t+k}] = var[\tilde{r}_t]. \tag{A2}$$

The variance of the cumulative surpluses per period converges to the one-period variance of returns:

$$var[\tilde{r}_t] = \lim_{k\to\infty}\frac{1}{k}var[s_{t\to t+k}].$$

If $var[\tilde{r}_t] < var[s_t]$ then it follows from the fact that the variance ratio of long-horizon returns is one (see equation A2) and from the fact that the variance of returns and surpluses converge over long horizons (see equation A1) that the variance ratio of surpluses is smaller than one:

$$\lim_{k\to\infty}\frac{1}{kvar[\tilde{r}_t]}var[\tilde{r}_{t\to t+k}] = 1 > \lim_{k\to\infty}\frac{1}{kvar[s_t]}var[s_{t\to t+k}].$$

There is mean reversion in the surpluses when the debt/output ratio is stationary and the surpluses are more volatile than the returns. Surpluses are predictable.

Now consider a second case in which the surpluses are i.i.d.. Then the long-horizon variance ratio of surpluses is one:

$$\lim_{k\to\infty}\frac{1}{k}var[s_{t\to t+k}] = var[s_t], \tag{A3}$$

which implies the following equality:

$$var[s_t] = \lim_{k\to\infty}\frac{1}{k}var[\tilde{r}_{t\to t+k}].$$

If $var[\tilde{r}_t] < var[s_t]$ then we know from the fact that the variance ratio of surpluses converges to one (see equation A3) and from the fact that the variance of returns and surpluses converge over long horizons (see equation A1) that the variance ratio of surpluses is smaller than one:

$$\lim_{k\to\infty}\frac{1}{kvar[\tilde{r}_t]}var[\tilde{r}_{t\to t+k}] > 1 = \lim_{k\to\infty}\frac{1}{kvar[s_t]}var[s_{t\to t+k}].$$

This implies that there is mean aversion in the returns when the debt/output ratio is stationary, and the surpluses are i.i.d.

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