Prospect Theory and Stock Market Anomalies

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Overview

- a decades-long effort has tried to make sense of "stock market anomalies"
  - stocks with particular characteristics have a higher (or lower) average return than predicted by the CAPM
- there are two broad approaches to understanding anomalies
- the rational finance approach
  - based on risk or frictions
- the behavioral finance approach
  - based on irrational beliefs
  - or preferences / risk attitudes

Overview

- in this paper, we try to make sense of stock market anomalies with a model based on psychologically-grounded assumptions about *risk attitudes*
- the leading psychological model of risk attitudes is prospect theory

  $\Rightarrow$ we try to make sense of anomalies with a model where risk attitudes are determined in part by prospect theory

  - we supplement investors' mean-variance preferences with a term that incorporates prospect theory
- despite years of effort, prospect theory's predictions for stock market anomalies have not been clearly laid out, until now

Overview

- in our model, an asset's price and average return are affected by three things:
  - the asset's return volatility
  - the asset's return skewness
  - the asset's capital gain overhang
- all else equal, these three quantities have a positive, negative, and positive impact, respectively, on the asset's average return
- for any anomaly decile, we compute empirical estimates of these quantities for the typical stock in this decile
  - our model then uses these inputs to make a quantitative prediction about the average return of the stocks in the decile

Overview

- we examine the model's ability to explain 23 prominent anomalies
- we find that the model is helpful for thinking about a majority of these anomalies
  - momentum, failure probability, idiosyncratic volatility, profitability
  - idiosyncratic skewness, return on equity, maximum daily return, Z-score
  - external finance, composite equity issuance, net stock issuance
  - post-earnings announcement drift, difference of opinion
- for these anomalies, the typical stock in the anomaly decile with the lowest average return is highly skewed and has a negative capital gain overhang
  - our model predicts a low average return for such a stock, in line with the data

Overview

- for some anomalies, the model makes counterfactual predictions
  - e.g., for the value anomaly
- we are able to make progress in understanding why, for a few anomalies, the model fails

Overview

Overall, the paper:

- answers a long-standing question: What does prospect theory predict for stock market anomalies?
- offers a psychological explanation for multiple stock market puzzles
- represents the first time a "behavioral" model of either beliefs or preferences has been used to make quantitative predictions about a wide range of anomalies

Prospect theory

Three components:

Gain-loss utility with loss aversion

- people derive utility from gains and losses
  - and are more sensitive to potential losses than potential gains

Diminishing sensitivity

- people are risk averse over moderate-probability gains
  - e.g., prefer a certain gain of \$500 to a 50% chance of \$1000
- but risk-seeking over moderate-probability losses
  - e.g., prefer a 50% chance of -\$1000 to a certain loss of -\$500

Prospect theory

Probability weighting

- when making decisions, people do not weight outcomes by their objective probabilities
- rather, they overweight low-probability tail outcomes
  - one motivation: the common preference for both lotteries and insurance

Prospect theory

- consider a gamble

$$(x_{-m}, p_{-m}; \ldots; x_{-1}, p_{-1}; x_0, p_0; x_1, p_1; \ldots; x_n, p_n)$$

- under Expected Utility, this is evaluated as

$$\sum_{i=-m}^{n} p_i U(W + x_i)$$

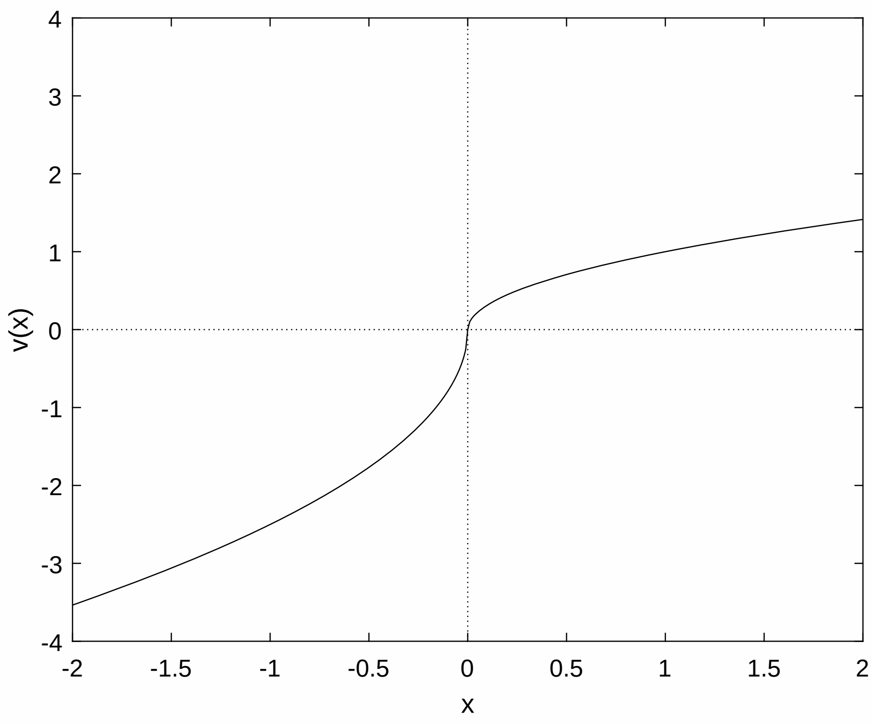
- under Prospect Theory, it is evaluated as

$$\sum_{i=-m}^{n} \pi_i v(x_i)$$

Prospect theory

- the value function $v(\cdot)$ is concave over gains, convex over losses, and has a kink at the origin

$$v(x) = \begin{cases} x^{\alpha} \\ -\lambda(-x)^{\alpha} \end{cases} \text{for} \begin{array}{l} x \geq 0 \\ x < 0 \end{array}$$



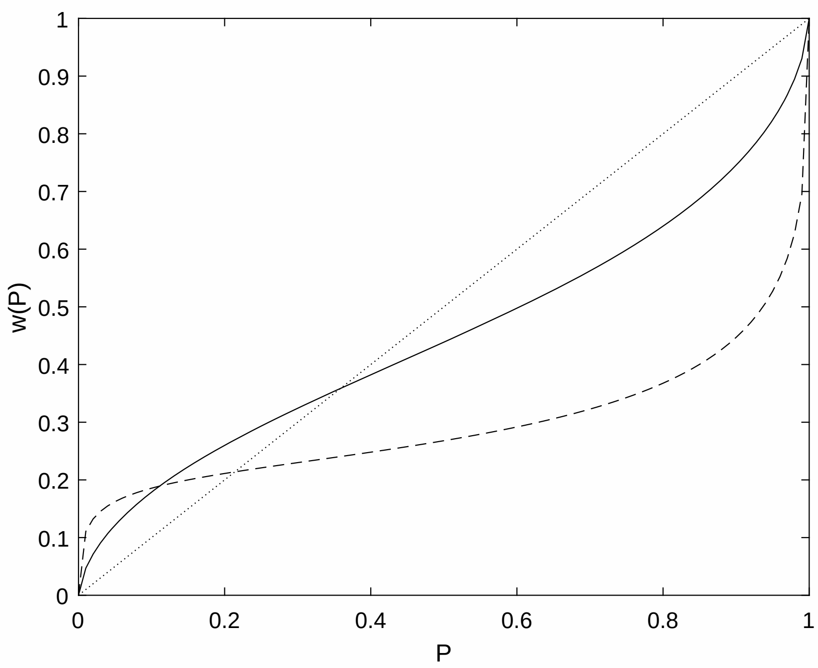
Prospect theory

- the probability weights are given by

$$\pi_i = \begin{cases} w(p_i + \ldots + p_n) - w(p_{i+1} + \ldots + p_n) & 0 \le i \le n \\ w(p_{-m} + \ldots + p_i) - w(p_{-m} + \ldots + p_{i-1}) & -m \le i < 0 \end{cases}$$

- where $w(\cdot)$ has the form

$$w(P) = \frac{P^\delta}{(P^\delta + (1-P)^\delta)^{1/\delta}}$$



Narrow framing

- in traditional models, people are assumed to take a portfolio view
  - when evaluating a new risk, they merge it with their pre-existing risks and check if the combination is an improvement
- in experiments, however, people often evaluate a new risk to some extent in isolation
  - "narrow framing"
- our model implements prospect theory in conjunction with narrow framing
  - investors derive utility, in part, from *stock*-level gains and losses

Intuition

- when investors have prospect theory risk attitudes, there are three forces that determine an asset's average return
  - each force corresponds to one of the elements of prospect theory

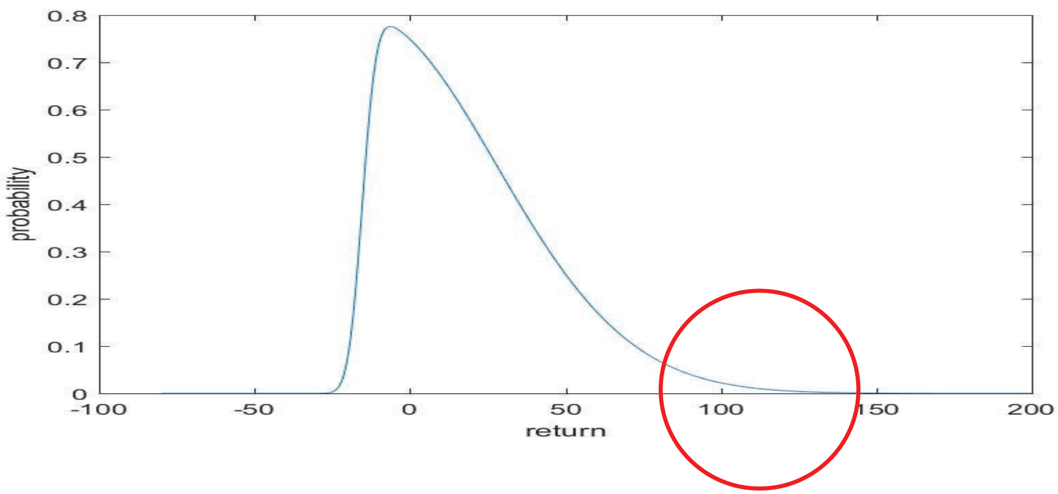
Loss aversion

- assets with more volatile returns should have *higher* average returns, all else equal

Intuition

Probability weighting

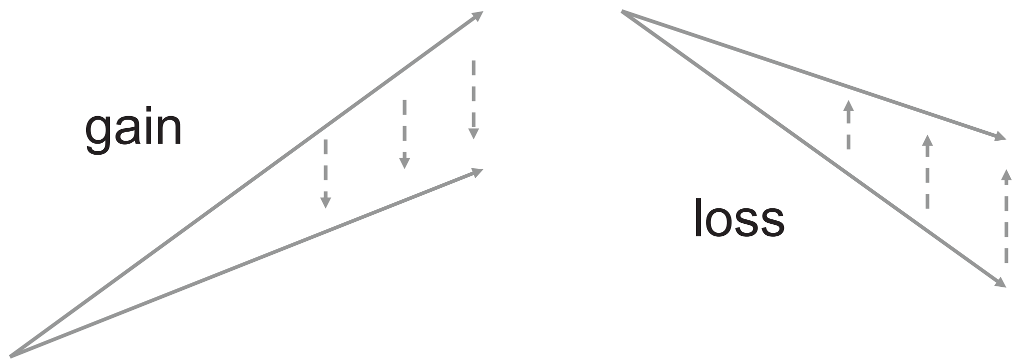
- assets with more positively-skewed returns should have *lower* average returns, all else equal



Intuition

Diminishing sensitivity

- assets with a higher capital gain overhang should have *higher* average returns, all else equal



Intuition

- these three forces indicate that, in an economy with prospect theory investors, three characteristics will be important in determining an asset's average return
  - the asset's return volatility
  - the asset's return skewness
  - the asset's capital gain overhang
- to compute prospect theory's prediction for an asset's average return, we need to quantitatively combine the three forces
  - this requires a model that incorporates all the elements of prospect theory, and takes account of investors' prior gains and losses
- no such model exists

  $\Rightarrow$ we construct one

Model

- three dates, $t = -1, 0, 1$

Assets:

- risk-free asset, gross interest rate R_f
- N risky assets
  - asset i has gross return $\widetilde{R}_i$
  - return vector $\widetilde{\boldsymbol{R}} = (\widetilde{R}_1, \widetilde{R}_2, \ldots, \widetilde{R}_N)$ has a multivariate distribution with cumulative distribution function $P(\widetilde{R})$
  - expected return of asset i is $\bar{R}_i$
  - covariance matrix is $\boldsymbol{\Sigma}$

Model

- at time 0, each of many identical investors solves

$$max_{\Theta_1,\ldots,\Theta_N} W_0(\boldsymbol{\Theta}'\bar{\boldsymbol{R}} + (1 - \mathbf{1}'\boldsymbol{\Theta})R_f) - \frac{\gamma}{2}W_0^2\boldsymbol{\Theta}'\boldsymbol{\Sigma}\boldsymbol{\Theta}$$
$$+b_0 \sum_{i=1}^{N} V(\widetilde{G}_i)$$

  – where

$$\widetilde{G}_i = W_0\Theta_i(\widetilde{R}_i - R_f) + W_{-1}\Theta_{-1,i}g_i$$

- $V(\widetilde{G}_i)$ is the prospect theory value of stock i's gain or loss $\widetilde{G}_i$
- the first component of $\widetilde{G}_i$ is the potential *future* gain or loss in stock i
  – the second component of $\widetilde{G}_i$ is the *prior* gain or loss coming into time 0
- the investor merges the potential future gain/loss with the prior gain/loss and computes the prospect theory value of the overall gamble

Model

- to model asset returns, we use the "generalized hyperbolic skewed t" distribution
  - location parameters $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_N)$
  - dispersion matrix $\boldsymbol{S}$
  - asymmetry parameters $\boldsymbol{\zeta} = (\zeta_1, \ldots, \zeta_N)$
  - degree of freedom parameter ν
- the mean of the distribution is given by

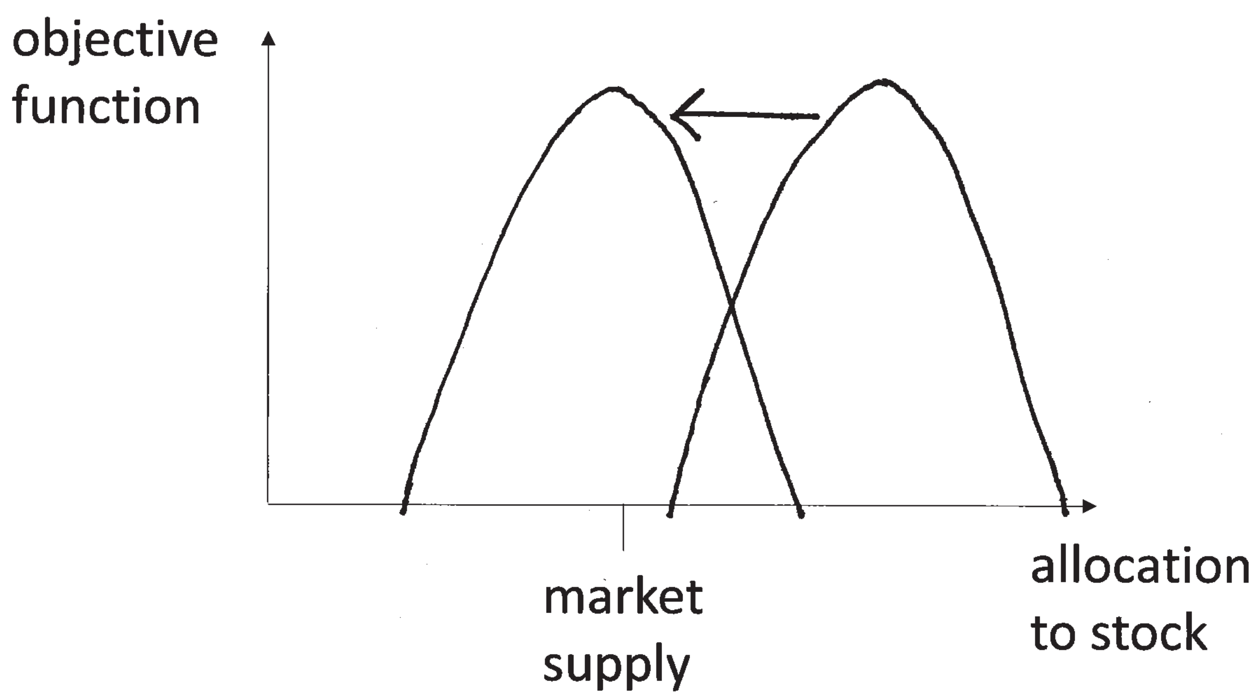
$$\boldsymbol{\mu} + \boldsymbol{\zeta} \frac{\nu}{\nu - 2}$$

- our approach is to set $\boldsymbol{S}$, $\boldsymbol{\zeta}$, and ν to match assets' empirical volatility and skewness levels
  - and to then search for $\boldsymbol{\mu}$ that leads to market clearing
  - the assets' expected returns are then given by

$$\boldsymbol{\mu} + \boldsymbol{\zeta} \frac{\nu}{\nu - 2}$$

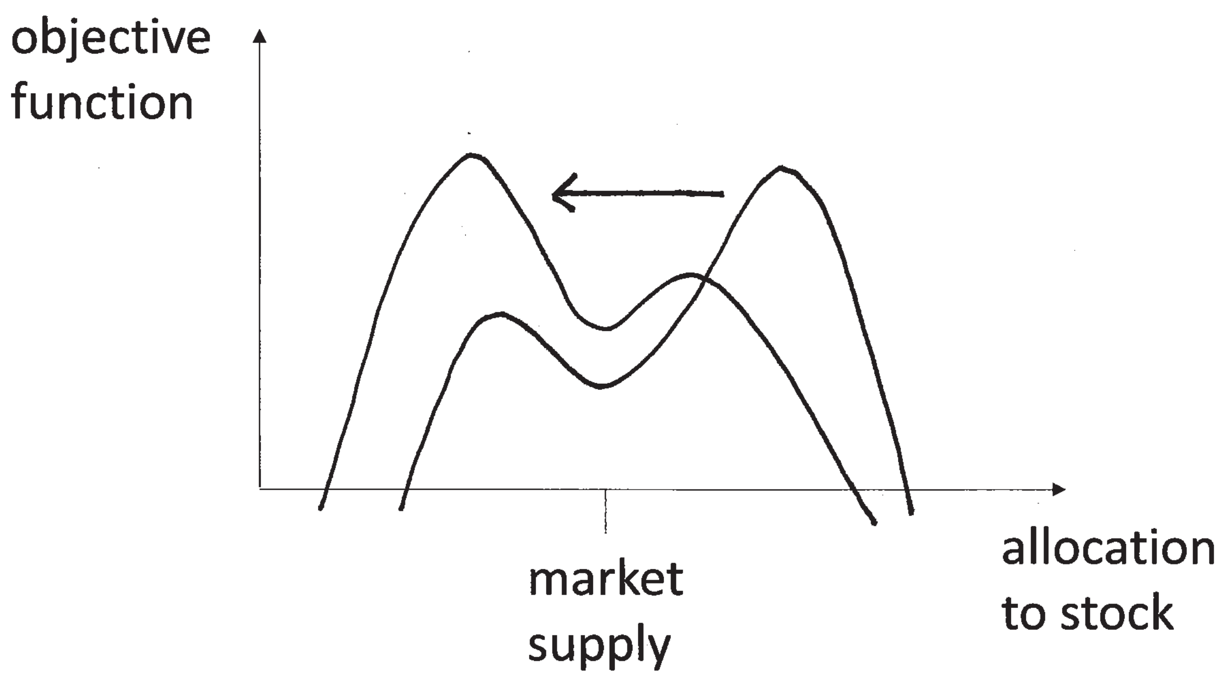
Equilibrium

- solving for equilibrium average returns is more challenging than in traditional models
- in a traditional Expected Utility model, the objective function is concave
  - by adjusting the asset's average return, we can equate the utility-maximizing demand to the market supply



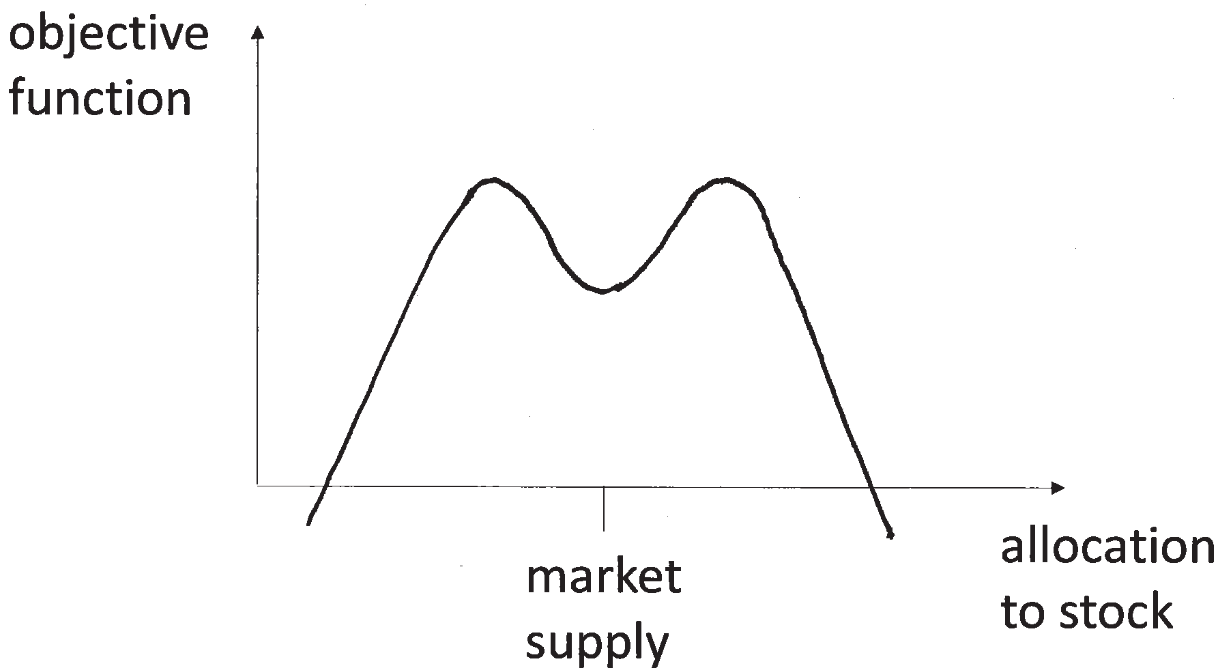
Equilibrium

- with prospect theory risk attitudes, the objective function is not globally concave
  - there may therefore be *no* average return for which the objective function has a unique maximum at the market supply



Equilibrium

- instead, the equilibrium may involve two global maxima



Equilibrium

- an asset's equilibrium expected return is determined by a vector $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_N)$ such that the investor's objective function

$$\theta_i(\mu_i + \frac{\nu\zeta_i}{\nu-2} - R_f) - \frac{\widehat{\gamma}}{2}(\theta_i^2\sigma_i^2 + 2\theta_i(\beta_i\sigma_M^2 - \theta_{M,i}\sigma_i^2))$$
$$-\lambda\hat{b}_0 \int_{-\infty}^{R_f-\theta_{i,-1}g_i/\theta_i}(\theta_i(R_f - R_i) - \theta_{i,-1}g_i)^\alpha dw(P(R_i))$$
$$-\hat{b}_0 \int_{R_f-\theta_{i,-1}g_i/\theta_i}^{\infty}(\theta_i(R_i - R_f) + \theta_{i,-1}g_i)^\alpha dw(1 - P(R_i))$$

  has a unique maximum at the market supply $\theta_{M,i}$ or two global maxima that straddle $\theta_{M,i}$

Anomalies

- we examine whether the model can shed light on 23 prominent anomalies
- the set is constructed to include the anomalies that have received the most attention from researchers and practitioners
- start with 11 anomalies from Stambaugh, Yu, and Yuan (2012)
  - and then pick, from among the 97 anomalies studied by McLean and Pontiff (2016), 11 more that are particularly prominent
  - and one more suggested by a referee

Anomalies

Anomaly	Abbreviation
Idiosyncratic volatility	VOL
Market capitalization	SIZE
Value	VAL
Expected idiosyncratic skewness	EISKEW
Momentum	MOM
Failure probability	FPROB
Z-Score	ZSC
Net stock issuance	NSI
Composite equity issuance	CEI
Accrual	ACC
Net operating assets	NOA
Gross profitability	PROF
Asset growth	AG
Return on equity	ROE
Investment	INV
Maximum daily return	MAX
Organizational capital	ORGCP
Long-term reversal	LTREV
External finance	XFIN
Short-term reversals	STREV
Difference of opinion	DOO
Post-earnings drift	PEAD
Capital gain overhang	CGO

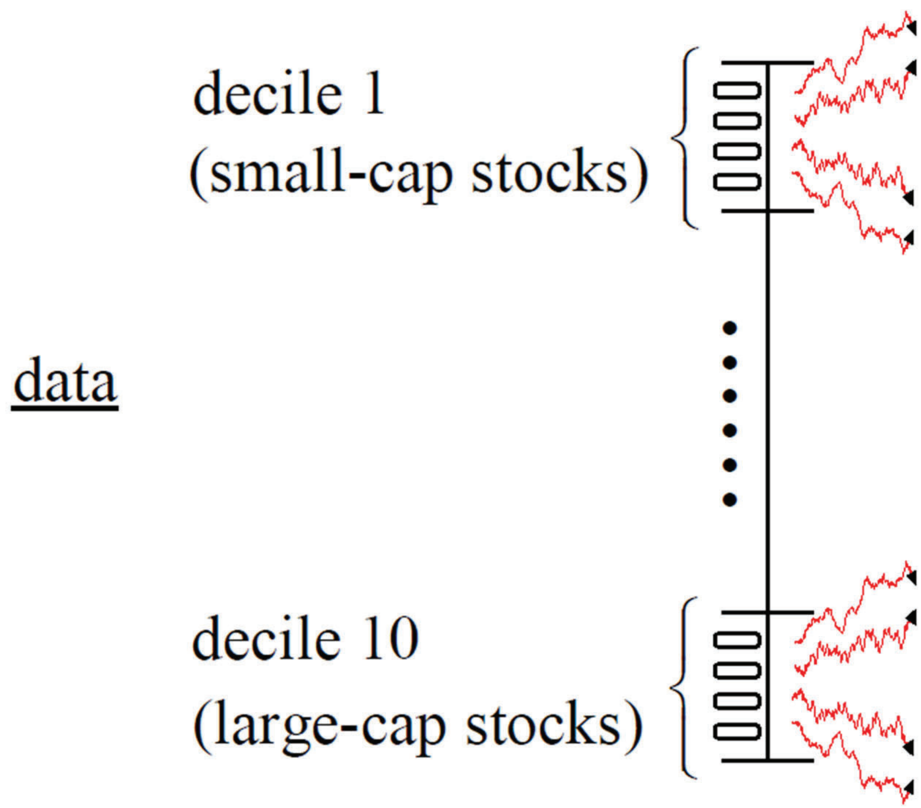
Anomalies

To see if our model can explain a stock market anomaly, we proceed as follows:

- for each anomaly, we sort stocks into ten deciles based on the anomaly characteristic
- we compute the model's predicted average return for the typical stock in each decile
- to do this, we need, for the typical stock in each decile, estimates of its
  - return volatility
  - return skewness
  - capital gain overhang
  - beta
- finally, we compare the predicted average return – and alpha – to the empirical ones

Anomalies

- to compute the return volatility (or skewness) of the typical stock in an anomaly decile
  - each month, we compute the cross-sectional standard deviation (or skewness) of the one-year subsequent returns of the stocks in the decile
  - we then average the monthly estimates



Anomalies

- to compute the gain overhang and beta of the typical stock in an anomaly decile
  - in each month, we compute the gain overhang and beta of each stock in the decile, and average across stocks
  - we then average the monthly estimates

Anomalies

What do the estimates look like?

- how do *small-cap* stocks (size decile 1) and *large-cap* stocks (size decile 10) compare in their volatility, skewness, and gain overhang?
- how do *value* stocks (book-to-market decile 10) and *growth* stocks (book-to-market decile 1) compare in their volatility, skewness, and gain overhang?
- how do *loser* stocks (momentum decile 1) and *winner* stocks (momentum decile 10) compare in their volatility, skewness, and gain overhang?

Anomalies

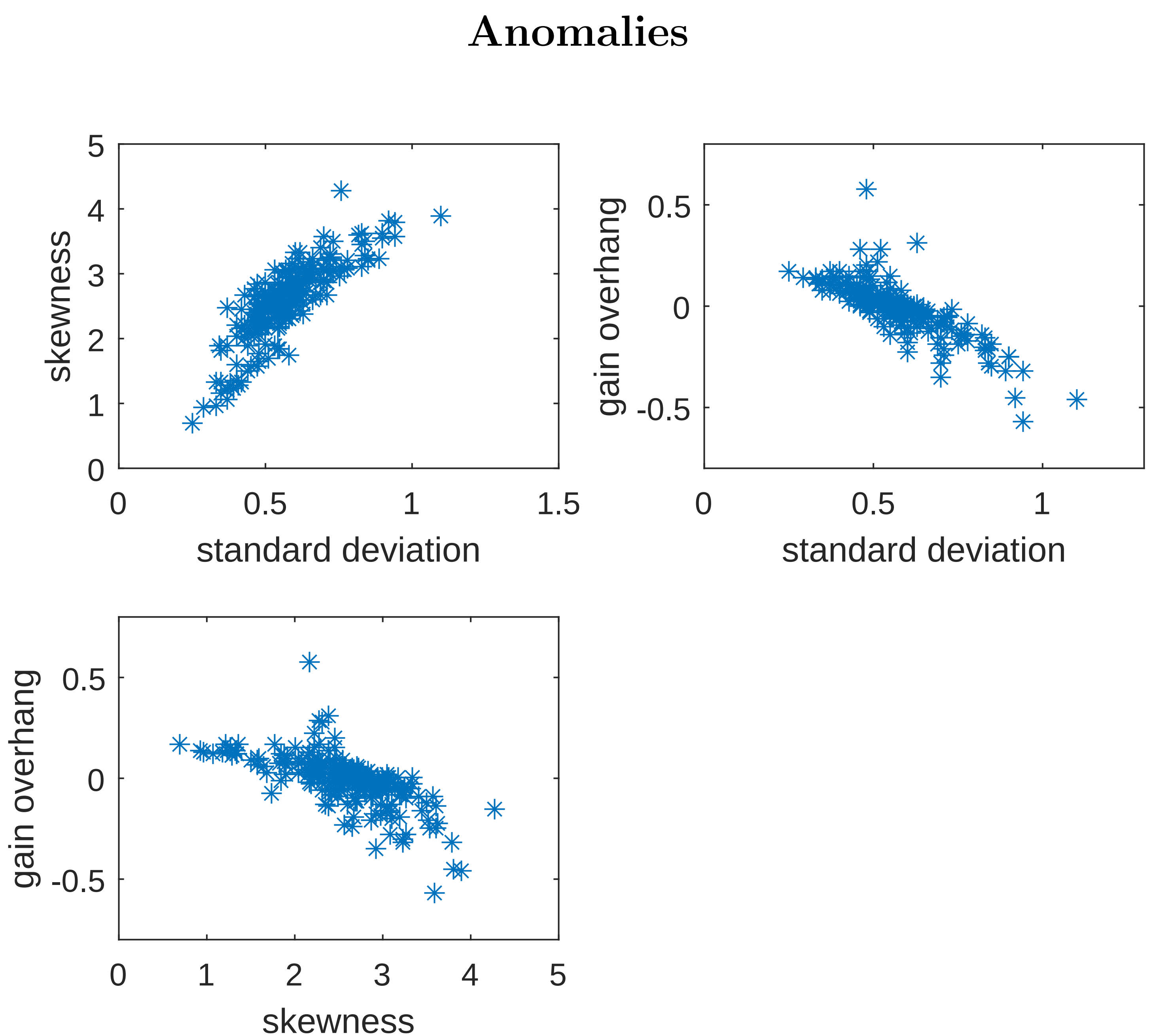
Anomaly	Average return Decile 1	Average return Decile 10	Skewness Decile 1	Skewness Decile 10
VOL	11.9	-3.2	2.46	3.79
SIZE	14.0	10.63	4.27	0.69
VAL	10.7	17.7	1.85	2.66
EISKEW	12.4	8.0	1.33	3.54
MOM	-2.4	20.6	3.84	2.46
FPROB	17.5	0.1	2.31	3.9
ZSC	3.8	13.9	3.5	2.56
NSI	16.0	6.8	2.71	3.2
CEI	14.0	6.8	2.46	2.68
ACC	15.4	7.4	3.2	3.0
NOA	14.8	6.9	3.13	2.95
PROF	8.8	14.4	3.49	2.68
AG	14.8	7.0	3.1	3.07
ROE	4.1	13.1	3.22	2.22
INV	15.9	8.0	3.6	3.31
MAX	10.9	1.2	2.76	3.61
ORGCP	10.9	15.8	2.76	3.38
LTREV	16.7	11.0	3.23	1.77
XFIN	13.0	4.2	3.17	3.44
STREV	14.4	7.6	3.63	3.03
DOO	15.5	10.0	1.36	1.74
PEAD	9.2	16.8	2.49	2.3
CGO	5.8	15.5	3.58	2.17

Anomalies

Anomaly	Standard deviation Decile 1	Standard deviation Decile 10	Gain overhang Decile 1	Gain overhang Decile 10
VOL	36.8	94.3	10.5	-31.6
SIZE	76.0	24.6	-15.4	17.0
VAL	54.1	71.0	12.1	-24.1
EISKEW	33.3	90.3	13.7	-24.8
MOM	91.4	63.1	-44.6	30.8
FPROB	51.5	110.4	28.2	-46.1
ZSC	84.4	51.6	-20.6	6.1
NSI	54.2	70.8	5.7	-8.4
CEI	42.1	66.5	7.4	-11.2
ACC	85.0	72.0	-18.7	-5.4
NOA	66.4	65.3	-2.6	-4.9
PROF	73.0	62.6	-12.0	1.1
AG	82.5	70.7	-20.6	-5.6
ROE	85.3	55.3	-29.9	15.1
INV	81.8	72.2	-14.0	-5.4
MAX	46.3	90.4	3.2	-25.1
ORGCP	63.3	77.5	-3.8	-7.9
LTREV	88.9	47.7	-32.1	17.1
XFIN	58.7	83.4	-0.5	-15.6
STREV	83.1	73.3	-22.2	-1.9
DOO	40.2	57.9	16.6	-7.8
PEAD	58.2	58.3	-9.4	7.5
CGO	93.9	47.8	-57.4	57.7

Anomalies

- the three key characteristics – volatility, skewness, and gain overhang – are strongly correlated across the anomaly deciles
  - if the typical decile 1 stock is more volatile than the typical decile 10 stock, then it is also more highly skewed
  - and has a more negative capital gain
- this points to the necessity of our quantitative approach
  - just by looking at the empirical characteristics, it is not possible to tell whether prospect theory can explain a given anomaly
- e.g., in the case of small-cap stocks
  - their greater volatility leads prospect theory to predict a higher average return for them than for large-cap stocks
  - their greater skewness and more negative overhang leads prospect theory to predict a lower average return for them than for large-cap stocks

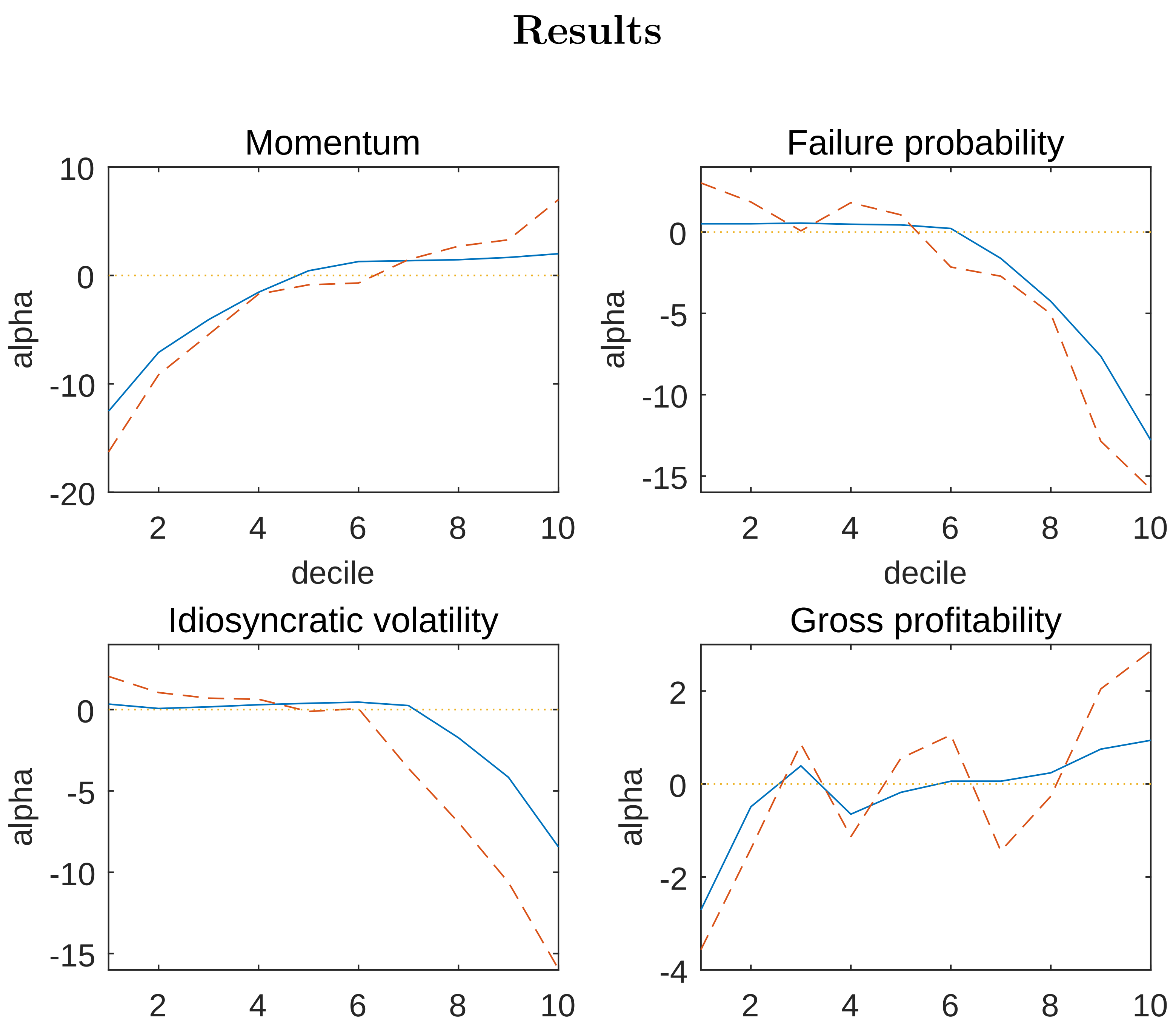

Anomalies
skewness
standard deviation
gain overhang
standard deviation
gain overhang
skewness

Parameter values

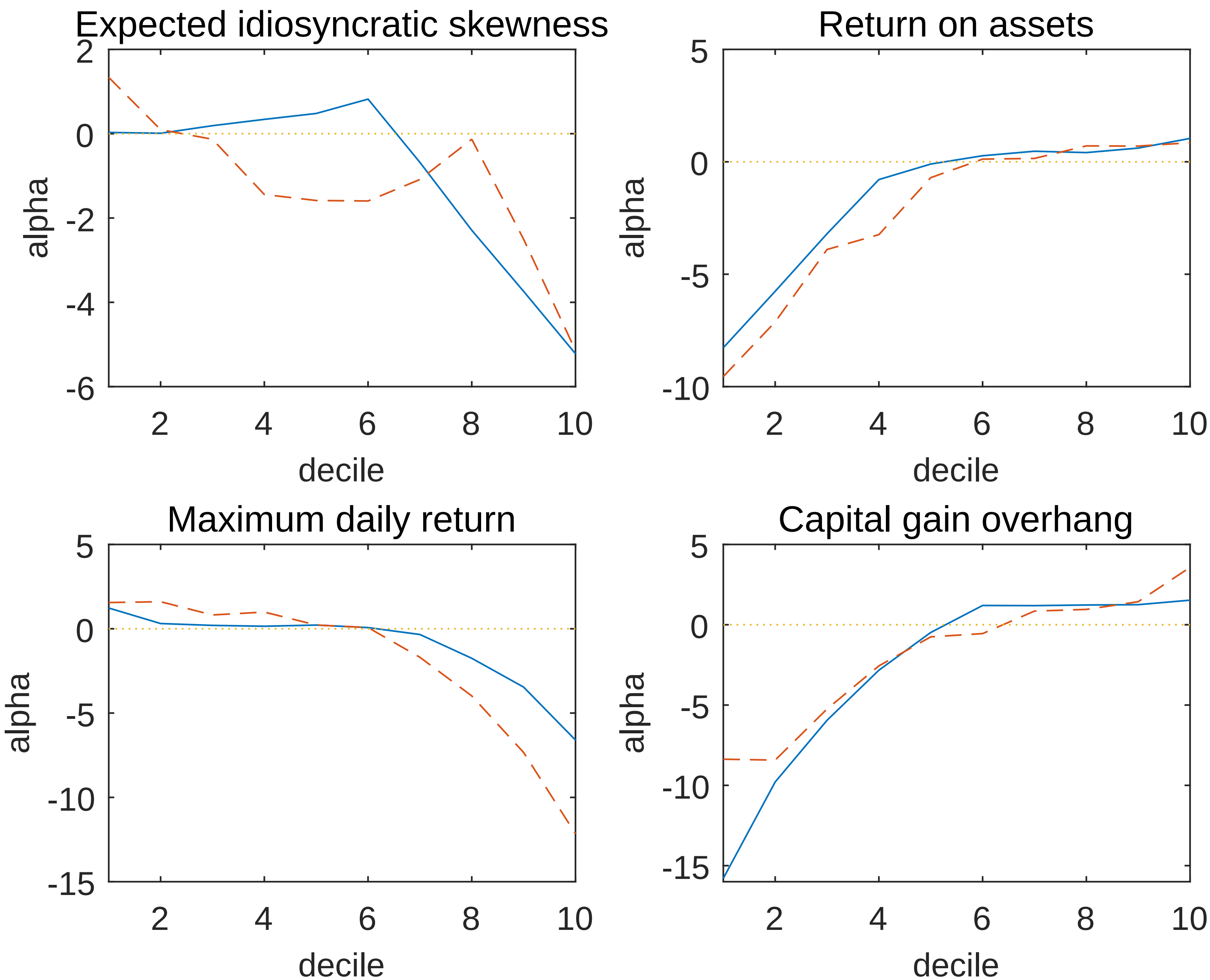
- the model has several parameters, but all are disciplined by either field or experimental data
- the parameters of the GH skewed t distribution are set to match the empirical estimates of volatility and skewness
- the prospect theory parameters governing loss aversion, diminishing sensitivity, and probability weighting are set to the median values estimated in experiments

Results

- we find that the model is helpful for thinking about a majority of the anomalies
  - momentum, failure probability, idiosyncratic volatility, profitability
  - idiosyncratic skewness, return on equity, maximum daily return, Z-score
  - external finance, composite equity issuance, net stock issuance
  - post-earnings announcement drift, difference of opinion
- for these anomalies, the typical stock in the extreme decile with the lower average return is:
  - more highly skewed; more volatile; and has a more negative gain overhang
- the higher skewness and negative gain overhang lead investors to charge a *lower* average return, all else equal
  - while the higher volatility leads investors to charge a *higher* average return, all else equal
- the former effect dominates, quantitatively


Results
Momentum
Failure probability
Idiosyncratic volatility
Gross profitability
alpha
decile

Results



Expected idiosyncratic skewness
Return on assets
Maximum daily return
Capital gain overhang
alpha
decile
2
0
-2
-4
-6
5
-5
-10
-15
4
6
8
10

Results

- for several anomalies, the model is able to explain a large fraction of the empirical alpha spread
- it also captures the "concavity" seen in several of the empirical alpha lines

Results

- for some anomalies, the model makes counterfactual predictions
  - size
  - value
  - long-term reversals, short-term reversals, accruals, asset growth, investment
- for these anomalies, the extreme decile that contains more volatile and skewed stocks with a more negative gain overhang has a *higher* average return
- we consider two explanations for why the model fails for these anomalies
  - one within the context of our framework
  - and one that goes beyond it

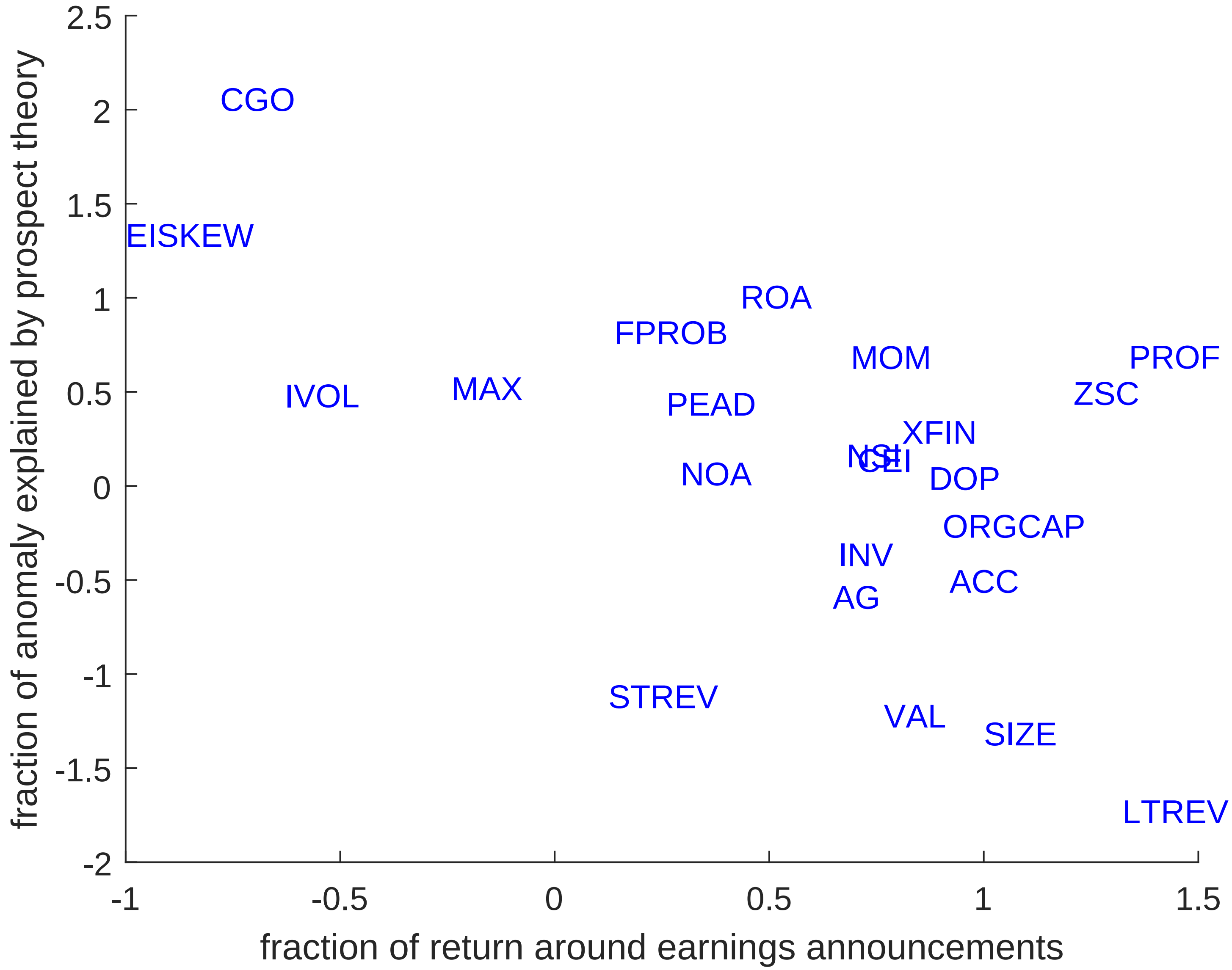
Results

- we have assumed that investors have correct beliefs about stocks' volatility, skewness, and gain overhang
- but, in reality, investors may have mistaken beliefs about these quantities
- for example, value stocks are more highly skewed than growth stocks
  - leading our model to incorrectly predict a lower average return for value stocks
- but investors may mistakenly think that *growth* stocks are more highly skewed
  - when incorporated into our model, this improves the model's prediction

Results

- we have tried to understand anomalies as the result of risk attitudes
  - however, some anomalies may instead be driven by incorrect *beliefs* about future earnings growth or returns
- several of the anomalies where the model fails appear to be driven by such beliefs
  - much of the anomaly return comes around earnings announcements
- more generally, there is a strong negative correlation between the fraction of an anomaly return that can be explained by prospect theory
  - and the fraction of the anomaly return that comes around earnings announcements
- this suggests that many anomalies can be placed in one of two categories
  - those driven by risk attitudes of the type captured by prospect theory
  - those driven by incorrect beliefs about future earnings or returns

Results


fraction of anomaly explained by prospect theory
fraction of return around earnings announcements
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
-0.5
0
0.5
1
1.5
CGO
EISKEW
ROA
FPROB
MOM
PROF
IVOL
MAX
PEAD
ZSC
XFIN
NSI
CEI
NOA
DOP
ORGCAP
INV
ACC
AG
STREV
VAL
SIZE
LTREV

Results

- we evaluate the model more formally by computing pricing errors for 23 long-short portfolios
  - for the prospect theory model and for five factor models
- the prospect theory model performs better than the CAPM and three-factor models
  - and similarly to the four-factor model

Model	Average absolute pricing error
Prospect theory	0.57
CAPM	0.82
Three-factor model	0.83
Four-factor model	0.55
Five-factor model	0.47
Six-factor model	0.31

Time variation

- we have used the model to make sense of anomaly alphas over the full sample from 1963 to 2015
- the model is also able to explain time-variation in anomaly alphas across four subperiods
- it attributes this to time-variation in the key empirical inputs: volatility, skewness, and gain overhang

Out of sample performance

- McLean and Pontiff (2016) show that anomaly alphas decline post-publication
  - due to arbitrage
  - or data mining
- the 14 anomalies that prospect theory can help explain should perform better post-publication
  - they are less likely to be data mined
  - their mispricing is harder to arbitrage
- this is confirmed in the data
  - the average pre- vs. post-publication alphas for the 14 anomalies that prospect theory explains are 13.8% and 7.9%
  - the average pre- vs. post-publication alphas for the 7 anomalies that prospect theory does not explain are 9.3% and 1.8%

Another set of anomalies

- we chose our 23 anomalies to be a representative set of the anomalies that academic researchers and practitioners are most interested in
- we repeated our analysis for another set of 23 anomalies used by Novy Marx et al. (2016) to study transaction costs
  - i.e., for reasons unconnected to prospect theory
- we obtain similar results
  - our model can help explain 13 of the 23 Novy Marx et al. (2016) anomalies

Prospect theory and real-world investors

- one might think that prospect theory primarily describes the risk attitudes of retail investors
  - should we then expect it to have a major influence on stock prices?
- even if prospect theory primarily applies to retail investors, it may still have a significant impact on prices
  - if retail investors trade in a correlated way
  - and if institutional investors do not fully absorb their demand
  - e.g., because the mispricing is concentrated in volatile, small-cap stocks
- in addition, several studies suggest that prospect theory is relevant for some institutional investors as well
  - experiments with institutional investors document all three elements of prospect theory
  - studies of institutional trading exhibit features of prospect theory
  - prospect theory influences are found even in high stakes situations

Prior work on prospect theory and asset prices

- our results are based on three intuitions:
  - due to loss aversion, more volatile stocks should have higher average returns
  - due to diminishing sensitivity, stocks with a higher gain overhang should have higher average returns
  - due to probability weighting, stocks with higher skewness should have lower average returns
- each of these three ideas has been developed in prior research
  - our contribution is to quantitatively combine them
- to do so, we need a model that incorporates all the elements of prospect theory
  - and also accounts for investors' prior gains and losses

Prior work on prospect theory and asset prices

- most prior models consider only a subset of the elements of prospect theory
  - only loss aversion
  - only loss aversion and diminishing sensitivity
  - only loss aversion and probability weighting
- the few models that consider all the elements ignore prior gains and losses

Our model in the context of behavioral finance

Beliefs

- over-extrapolation of past fundamentals, or of past returns
- but also
  - overconfidence
  - differences of opinion

Preferences

- prospect theory
- but also
  - ambiguity aversion

Summary

- we try to make sense of 23 prominent stock market anomalies using a model with psychologically-grounded assumptions about investor risk attitudes
  - specifically, prospect theory risk attitudes
- we find that the model can help explain a majority of the 23 anomalies
- overall, the paper:
  - answers a long-standing question: What does prospect theory predict for stock market anomalies?
  - offers a psychological explanation for multiple stock market puzzles
  - represents the first time a "behavioral" model of either beliefs or preferences has been used to make quantitative predictions about a wide range of anomalies